

A TOUR ON THE LIFE AND WORK OF A. M. NAVEIRA

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Dedicated to A. M. Naveira

In this paper we will summarize a lecture we gave with pleasure on the life of our advisor, professor Antonio Martínez Naveira, at the "Differential Geometry, Valencia 2001, an International Meeting on the occasion of the 60th birthday of A. M. Naveira"

1 Introduction

It should be quite easy for us to write this paper, because Professor Naveira is our teacher and friend. However, we are not sure to be able to communicate you, to get into your souls, a series of facts absolutely necessary to understand why the Naveira phenomenon and why many of us love him.

We are sure that anyone of the Organizing Committee was able to prepare a very nice article on this subject. However, we have been chosen according to the following three reasons: (i) we are who less time have spent to organize this event; (ii) we are probably the closest to Antonio and his family; and (iii) we are Naveira's oldest, but not the best, disciples. Be sure that we will do our best to reach our goal.

In this article, a tribute to our friend, we intend to illustrate his virtues as a person and as a mathematician, trying to give a true image of who Antonio is. We will try to keep it as close as possible to the presentation we gave, in the inaugural lecture of the International Conference, that was organized by all of his disciples in his honour.

2 Some biographical data of A. M. Naveira

Antonio Martínez Naveira was born in La Coruña, but he spent his childhood in the village of Churío, in the corunian county of Aranga where he spent his childhood.

His parents, farmers, had scarce economic resources. For that reason, Antonio begins his studies in the Unitarian school of his town, where there

is only one teacher for all children regardless of their age, Doña Raquel Rey de Castro, who would have a decisive influence in his life. During the first year of high school (there were seven grades in all) he studied in the town of Aranga, having to take exams at the high school of La Coruña. At that

time his teacher was Professor Mosquera. This professor's son still remembers perfectly that studious boy whom they tried to help so that he could continue his studies in some school of La Coruña as an intern student. The brother of Doña Raquel, Don Gumersindo Rey de Castro was one of the owners and the principal of the Academia Galicia school which had the prestige of being one of the best schools of La Coruña, but it had a very high tuition, especially for the means of people dedicated to agriculture. Antonio is received in the school as a direct relative of the principal with the right to free education. A great friendship blossoms between them, to such a point that, when talking about Antonio, Don Gumersindo's mother refers to him as her grandson. The confidence bestowed on him by the Rey de Castro brothers is highly rewarded. Thus, in the year 1959, he graduates with honours as a Bachiller.

While he was in elementary school, Antonio studied to be a school teacher (in those days you could do it after completing four years of high school education) and when he passed the admission exam to go to university, he also obtained a teacher's degree. We remember that Antonio always says that the subject that most trouble gave him was calligraphy, which had to be done with quill and ink. Anybody that has seen his writing realizes how hard it must have been to him to pass.



Antonio in Santiago (1960)

se ven por μ para μ \dots
 de tipo constante primitiva. Si M es de tipo
 constante primitiva, \exists μ para todo μ por de campos
 vectoriales $\langle X, Y \rangle = \langle \partial X, \partial Y \rangle = 0$ y $\|X\| = \|Y\| = 1$, la fun
 ción $\|(\partial X, \partial Y)\|$ es constante, entonces se dice
 que M es de tipo constante global

A sample of Antonio's writing

In June of 1960 he passes the exam which will allow him to begin his studies in mathematics, and, in October of that same year, the first course, common for all science students, begins. Antonio spends this course in a typical boarding house of Santiago, which later would become the San Clemente's college, a men only facility at the time, it becomes his residence until completing his studies. There are numerous anecdotes of his life at San Clemente's college that are recalled by his mates. Thus, very famous, were Antonio's broadcasts of imaginary soccer games, which were told so passionately that one could even see the ball run.

He also stood out by being the most methodical student of the entire College which helped him enormously in his studies. Since all the rooms of the College were double, even if the room was full of people, he would take a nap or go to bed early, not minding the noise all around him.



Antonio with the Director of his College
and the Rector of his University (1960)

Concerning the mathematical studies of the time, it is worth mentioning that Antonio's class was the third

since the creation of the mathematics section of the School of Science. The number of students in his class was nine, something unthinkable these days; but the number of professors was also very low, standing out amongst them, Prof. García Rodeja (in charge of algebra), Prof. Aguiló (analysis), Prof. Vidal Abascal (geometry) and Professor Aller (astronomy). It is with Professor Enrique Vidal Abascal that Antonio clicks with and he will be Antonio's advisor for his doctoral dissertation and one of the most influential people of his mathematical development. Antonio graduates in June of 1965. Around that time D. Enrique was working in foliations theory and suggested Antonio

to work on a related problem in this area. As a result, his doctoral thesis, defended in January 1969, titled "Foliated manifolds with almost-fibred metric". The board of examiners was composed of Dr. Vidal Abascal, Dr. Etayo, Dr. Vaquer, Dr. Viviente and Dr. García Rodeja who granted it with the highest grade of outstanding cum laude. It was the second thesis defended in Santiago, the first one being that of Professor Echarte, currently a Full Professor at Sevilla.

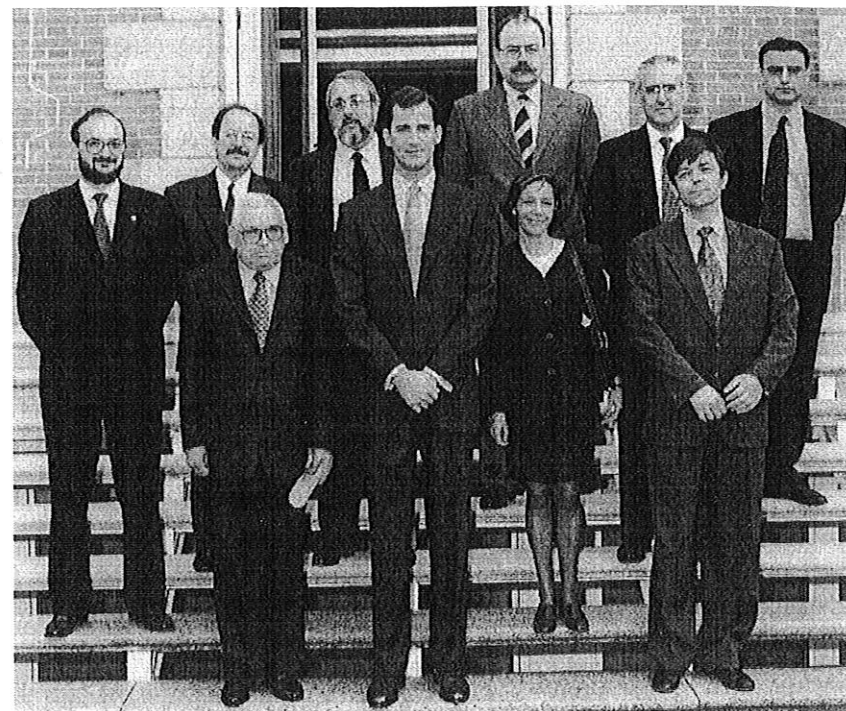
While he prepared his doctoral thesis, Naveira had a position as a hire professor in the mathematical section from October 1965 until the year 1973, when he is appointed Associated Professor in a famous scholarly ceremony where all the candidates for Associated Professor had to go to Madrid to swear loyalty to the National Movement in the Royal Theater. Afterwards he prepares exams for a position as an Full Professor and in the year 1975 he obtains the position of Full Professor in differential geometry at the University of Granada.



Antonio's doctoral dissertation.

From right to left: Profs. Viviente, Vaquer, Vidal-Abascal, Etayo and Rodeja.

In 1976 he obtains via a merit contest the position of Full Professor at the University of Valencia, appointment that he currently holds. In the year 1996 he is appointed President of the Royal Mathematical Society of Spain. *Thanks to his effort we can now say that the RSME (its Spanish acronym) is alive once again and tries to occupy its corresponding place. Conclude with that office in the year 2000.*



Antonio with the Prince of Spain and other members of the RSME (2000)

3 Naveira at Santiago, by L. Hervella

My relationship with Antonio Martínez Naveira begins when I was a student on my second year in mathematics, in the academic course of 1966-67. He was our Analysis teacher and one of my classmates was Isabel Vázquez Paredes, today Antonio's wife, who at the time was his girlfriend, and with whom I was bound in great friendship. This friendship quickly led to one with Antonio and at the end of the course, together with another friend, we were the three of us bringing serenade to Isabel. I must say that Antonio is a good mathematician but as a singer he still needs more practice. Once Antonio and Isabel were married, when she finished her third school year, their house was my favorite restaurant. They invited me uncountable times so that I could forget how bad I ate in my Residence.

Once I graduated, I started working in the Department of Geometry and Topology which, at the time, was run by Professor Vidal Abascal, be-

ing Naveira his closest collaborator. I remember that D. Enrique handed me Haefliger's thesis on foliations theory, so that I would read it and start getting involved with the subject. I told Antonio that it wasn't what I liked and he began studying the work of Alfred Gray on Almost Hermitian Geometry to advise me on this subject. That is how I completed my doctoral dissertation, which I defended in July 1974 and which was officially under the supervision of D. Enrique Vidal because in those days only full Professors could appear advisors, but in the thesis I acknowledged Dr. Naveira for his unvaluable help. This is why Antonio always says that I was his first student. I am thankful for and share this thought.

From 1970, the year I graduated, my relationship with Antonio grows stronger. I must point out, among other things, the following:

- i) His ability to work. I have never met anyone capable of surpassing him. When I was working on my thesis, we would stop discussing at 11 p.m. and already he would ask me to arrive at nine in the morning the next day with all the computations finished. I should point out that he would always bring them solved to compare them with mine.
- ii) His hopes to solve any problem he would find himself working on. I will always say that the Royal Mathematical Society of Spain is alive again because of the luck they had to be able to appoint Antonio as their President. Besides, he transmits this hope to everyone that surrounds him.

During the five years we worked together, he left in 1975 to Granada, I could tell many stories of things we've gone through together. For instance, those occurred on the travels to the congress we attended. Since in those days life at the University was not as good, we had to travel in our own cars and, for that, Antonio's Seat 600 helped us a great deal. But then again, we had to leave at three in the morning to go to the Piedrafita pass before all the trucks did. Also, during spring of 1973 we went to Paris to go to the presentation of his third cycle thesis at the University of Paris VI, supervised by Prof. René Deheuvels, and which received the highest grade.

Besides the scientific work done in that journey, I remember that Antonio would subject his wife and me to long walks. He had a theory: if you want to get to know a city you have to walk it. Today I am thankful of it, but in those days I hoped he would sprain an ankle so that we could sit down and keep exploring Paris via subway.

To finish my journey through Antonio's life, I would like to relate the Durham congress of 1974 to which we both went to. We were going, basically,

with the sole purpose of meeting Alfred Gray, since he had been the referee for our work on Schur's lemma on Nearly Kähler manifolds, published in the Proceedings of the A.M.S. He had asked us to go. I believe this trip was fundamental for differential geometry in Spain, since besides meeting Alfred and beginning a strong relationship with him, we also met Lieven Vanhecke who has done so much in helping geometers from Santiago, Granada, La Laguna... Also in this conference, Professor Gray showed us the work of one of his collaborators, which Naveira and I refuted since the new structure under consideration were in reality nothing more than Kähler ones.



Antonio and Luis in Paris (1973)

We attended this congress invited by Professor Willmore who was a great friend of the University of Santiago, who in numerous occasions had visited us to participate in congresses organized by Professor Vidal.

One year after these events, Prof. Gray visited Granada and from 1976 we started stays at Maryland extending for periods of six months or more, most often by Antonio's students. Antonio himself also visited Maryland several times, and his classification of Almost Product structures was produced while in that american university.

4 Naveira at Granada and Valencia, by A. Ferrández

I decided to look for amenity, keeping of course seriousness, to explain the excellent work of Prof. Naveira, which is nothing but the answer of someone devoted himself to studying; someone always having blind faith of his doing. And everything done with a bit of geniality that makes him unique, singular and unreproducible. Antonio, as Spain, is different^a. I will point out a series of key facts all through his life which characterize Prof. Naveira and become him a great guy.

I always like to name him El Maestro (The Teacher), and this fact is so important for me that I will forever proclaim it provided that I keep a little

^aa touristic slogan of the 70's in Spain

bit of breath. On occasion of his 60 birthday, I, we, dedicated him our best paper^b.

The organizers of this conference believe that the scientific work of Prof. Naveira is so important that I am sure that it is our duty to spend some of our time talking on it. Furthermore, I think you will agree with me, **the main project carried out by Antonio was the creation of a Differential Geometry School.** This is, probably, the very reason of this meeting.

The fact is that this tribute to Prof. Naveira is a pretext to talk about some recent advances in Differential Geometry. However, if this were the only aim of this meeting, I confess that probably I would not have attended it. I strongly bet for this celebration, its inspiration and philosophy, because two, among the principles governing my life, are, on one hand, to be happy with the success of my friends; and, on the other hand, to apply myself that saying of Sancho to Don Quijote: "It is only well born who is able to give thanks" (free translation). So then, besides to speak highly of Naveira's work in the past ten years, I will point out some details, moments, sentences and anecdotes that we will bring ourselves near to the human profile of Prof. Naveira, so that you can find him more understandable and you can then understand why certain unconditional adhesions. The core of my speech is going to anybody that want to know the very personality of Antonio.

I met Naveira in September 1975 in the University of Granada. A few young men of the Department of Geometry and Topology had decided to study Geometry in order to make a thesis. It seemed, at that time, an impossible task because we had no direction, no open problems, no objectives, no ..., no ... However, we found a never open pair of books: K-N I & II^c. The first time we talked about working with Prof. Naveira we spoke different languages. Otherwise, we thought he were coming from another world. He arrived, to the department, the first, at 8 a.m.; he leaved it the latter; he had a cup of coffee in 3 minutes; he took lunch in 45 minutes; and so on. As a matter of fact, we cannot arrive until 11 a.m., because we had to close all pubs in the Campo del Príncipe^d. Antonio was seeking desperately for any young mathematicien to start a thesis. He came to Granada with a lot of open problems, and almost solved. But, please, believe me, we were afraid of the working way of Antonio and such an amount of strange things such as

^bM. Barros, A. Ferrández and P. Lucas; Conformal tension in string theories and M-theory, *Nuclear Physics B* 584 (2000), 719-748.

^cS. Kobayashi and K. Nomizu, *Differential Geometry*, vol. I and II, Interscience 1963, 1969.

^da well known square of Granada

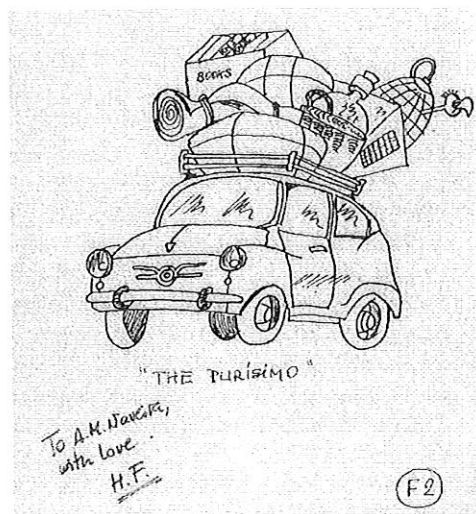
Kählerian, nearly, almost, quasi, G_1 and G_2 , don Enrique, Gray, Vanhecke, Santiago, Paris, Lovaina, Maryland,... It seemed we were in Hollywood.

As a matter of fact, we yearned for working in our thesis. However, I wish to point out a key fact: we had an unbreakable agreement, accordingly Manolo Barros should be the first one to begin a thesis with Antonio. It was a little difficult to explain it to Antonio, since he had not yet met Barros. In January 76, Manolo came back from military duties and, hardly a year, those open problems, that Antonio brought himself, were nicely well solved. This was the first thesis of Naveira out of Galicia, which was finished in the University of Valencia, although its reading took place in Granada about the middle of December 76. It deserves a special mention, since this is the root, under the direction of Prof. Barros, of the recent and brilliant history of the Granada Differential Geometry team. Following Barros' tracks, A. Ramírez, F. J. Carreras and myself went to Valencia looking for a promising future. The time has been said to us that we were right.

I do not pretend to talk about Granada University, but Naveira is directly concerned with the success of the current Differential Geometry group. I like to speak of "Granada before Naveira" and "Granada after Naveira".

But how did Naveira land in Granada? In a Boeing 747? In an Airbus? Look at the right

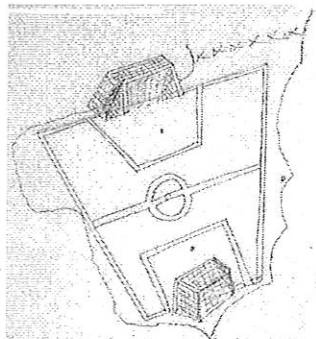
Antonio changed radically our way of life. However, he also learnt from us. I attended his classes, took personal notes, put in a clean and easy to read form and he asked me for them. It was for me a great pleasure. He never used to have personal notes to teach, up to those taken from Prof. Deheuvels classes in Paris. They have a sad history: we need them for my thesis and a bad day, cleaning people throw them out the rubbish.



We followed a postgraduate course where Prof. Naveira taught us the first classification of almost-hermitian manifolds. There he felt completely at home, and very soon we were able to get up and down with the almost-

complex structure, i.e., with the familiar J . In such a way that in our argot we used to say "to dance the J ", like a quite typical Spanish dancing. It is well known that the best choreography of the J -dancing has been, without doubt, carried out by the couple Gray-Hervella.

We found Antonio's genialities anywhere and anytime. For instance, also, in that glorious time (the 70's), Antonio had a favourite sentence: "I introduced the curvature in Spain". Indeed, it is easy to imagine the following situation

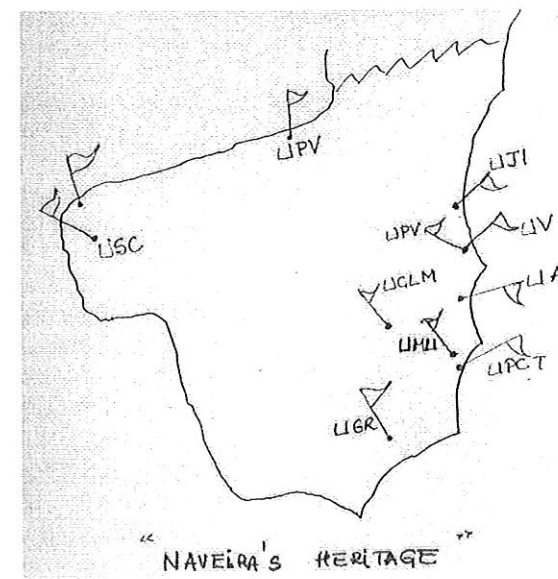


Spain before Naveira



and a piece of Spain after Naveira

I will be very happy if, through my talk, I have been able to show you that Prof. Naveira is a person that I hold in the highest regard. This is my little tribute to a man of whom I learned the way to do research and the very spirit of the university. He knows very well these are not only words, but also the understanding of a philosophy and its application. That means that the old school has grown year out and we are very proud to see how the young researchers outshine the teachers. It is well known the selfsatisfaction of Antonio talking about his "sons", because the Spanish Differential Geometry is going through a sweet period and all of them have to do with. This is a short list: M. Barros, A. Ramírez, V. Miquel, O. Gil, F. Carreras, A. H. Rocamora, S. Segura, X. Gual and myself. (L. M. Hervella, of course, completes the list). The fruits of Granada's seed yield to second and third generations, so that Naveira has "grandsons" and "great-grandsons" in the universities of Santiago, Coruña, País Vasco, Castellón, Valencia, Politénica de Valencia, Alicante, Murcia, Politénica de Cartagena, Castilla-La Mancha and, of course, Granada. I'm very proud to tell you that, in the las 25 years, "Naveira's sons" have been written more than 200 papers, published in the first ranked journals.



5 Mathematical activity of Prof. Naveira

Next we will briefly report on the Mathematical activity of A. M. Naveira. After his graduation in 1965 he started to do research in Differential Geometry under the supervision of prof. Vidal Abascal. For the sake of simplicity, we will scheduled Naveira's research as follows

- 3.1. Foliations.
- 3.2. Almost Hermitian structures.
- 3.3. Almost product structures.
- 3.4. Volumes of geodesic spheres and tubes.
- 3.5. Integral Geometry.

All his work was beneficted by collaborations and conversations with many mathematicians (indeed, friends). Among them, professors Lichnerowicz, Deheuvels, Gray, Vanhecke and Santaló will occupate a distinguished place in Antonio's life. Here we should also mention Naveira's students: Hervella, Barros, Ramírez, Ferrández, Miquel, Gil Medrano, Carreras, Segura, Rocamora, Gual....

Ferrández, A., Naveira, A. M.; Normal form of the NK-curvature operators. *Czechoslovak Math. J.* **32** (1982), no. 3, 404–417.

Gray, A., Barros, M., Naveira, A. M., Vanhecke, L.; The Chern numbers of holomorphic vector bundles and formally holomorphic connections of complex vector bundles over almost complex manifolds. *J. Reine Angew. Math.* **314** (1980), 84–98.

Naveira, A. M., Vanhecke, L.; Two problems for almost Hermitian manifolds. *Demonstratio Math.* **10** (1977), no. 1, 189–203.

Barros, M., Naveira, A. M.; Décomposition des variétés presque kählériennes vérifiant la deuxième condition de courbure. *C. R. Acad. Sci. Paris Sér. A-B* **284** (1977), no. 22, A1461–A1463.

Naveira, A. M.; Caractérisation des variétés à courbures sectionnelles holomorphes généralisées constantes. *J. Differential Geom.* **9** (1974), 55–60.

Naveira, A. M., Fuertes, C.; The zeroes of nonnegative holomorphic curvature operators. *Trans. Amer. Math. Soc.* **210** (1975), 139–147.

Naveira, A. M., Fuertes, C.; Les zéros des opérateurs courbure holomorphes non-négatifs. *Géométrie différentielle (Colloq., Univ. Santiago de Compostela, Santiago de Compostela, 1972)*, pp. 61–63. Lecture Notes in Math., Vol. **392**, Springer, Berlin, 1974.

Naveira, A. M.; On the higher order sectional curvatures. *Illinois J. Math.* **19** (1975), 165–172.

Naveira, A. M., Hervella, L. M.; Schur's theorem for nearly Kähler manifolds. *Proc. Amer. Math. Soc.* **49** (1975), 421–425.

5.3 Almost product structures

Motivated by the visits of prof. R. Deheuvels at Santiago at the end of the 60's, the study of almost product structures attracted the attention of some of the young students of E. Vidal. So, two Doctoral Thesis are presented in 1971 closely related to this subject. In both of them Antonio participated in a very active way.

Recall that an *almost product structure* on a Riemannian manifold (M, g) is a pair of orthogonal plane fields \mathcal{V} and \mathcal{H} on M . If $\pi_{\mathcal{V}}$ and $\pi_{\mathcal{H}}$ are the corresponding projections from TM , then $P = \pi_{\mathcal{V}} - \pi_{\mathcal{H}}$ is a $(1, 1)$ -tensor field on M such that $P^2 = id$ and $g(PX, PY) = g(X, Y)$ for all vector fields X, Y on M . In analogy with the theory of almost complex structures, the tensor field P is called an *almost product structure*. Clearly any foliation \mathcal{F} on a Riemannian manifold (M, g) defines an almost product structure.

During the year 1979, Naveira started to think on a possible classification of the almost product structures by following the techniques previously used in the classification of almost Hermitian manifolds. Therefore, for a given almost product metric structure (g, P) on a manifold M , Antonio considers the associated tensor field $\varphi(X, Y) = g(PX, Y)$ and studied the decomposition of the space of tensors with the same symmetries of $\nabla\varphi$ under the action of $O(p) \times O(q)$. This way, Naveira obtains 36 different classes of almost product structures.

Major part of this work was done while Antonio visited prof. A. Gray at Maryland University in 1980, and it appeared in the "Rendiconti di Matematica e Applicazione" in 1983. Since then, it became a reference for many people working on almost product structures. As a constant on Antonio's activity his research strongly influenced his students' works as it can be observed in

Gil-Medrano, O., Naveira, A. M.; Some remarks about the Riemannian curvature operator of a Riemannian almost-product manifold. *Rev. Roumaine Math. Pures Appl.* **30** (1985), no. 8, 647–658.

Naveira, A. M., Rocamora, A. H.; A geometrical obstruction to the existence of two totally umbilical complementary foliations in compact manifolds. *Differential geometric methods in mathematical physics (Clausthal, 1983)*, 263–279, Lecture Notes in Math., **1139**, Springer, Berlin, 1985.

Carreras, F. J., Naveira, A. M.; On the Pontrjagin algebra of a certain class of flags of foliations. *Canad. Math. Bull.* **28** (1985), no. 1, 77–83.

Naveira, A. M.; A classification of Riemannian almost-product manifolds. *Rend. Mat. (7)* **3** (1983), no. 3, 577–592.

Gil-Medrano, O., Naveira, A. M.; The Gauss-Bonnet integrand for a class of Riemannian manifolds admitting two orthogonal complementary foliations. *Canad. Math. Bull.* **26** (1983), no. 3, 358–364.

and other papers that his students and collaborators wrote on those years.

5.4 Volumes of small geodesic spheres and tubes

Motivated by the work of A. Gray on the characterization of special classes of Riemannian manifolds by the volumes of their geodesic spheres and further work by Gray and L. Vanhecke on geodesic tubes, Naveira started to study the geometrical significance of the coefficients in the power series expansions of the different functions above. The so-called *volume conjecture*: "A Riemannian manifold where the volume of all sufficiently small geodesic spheres coincides with that of the Euclidean space must be locally flat" diverted Antonio's research into some fields related to isoperimetric problems, volumes of tubes

and geodesic spheres in symmetric spaces of rank > 1 , total curvature and absolute total curvature of certain submanifolds and so on.

It is worth mentioning at this point that, besides a lot of work on it, the volume conjecture above remains still open. We would like to mention here some contributions by Antonio:

Gual-Arnau, X., Naveira, A. M.; Volume of tubes in noncompact symmetric spaces. *Publ. Math. Debrecen* **54** (1999), no. 3-4, 313–320.

Carreras, F. J., Naveira, A. M.; On the volume of a small extrinsic ball in a hypersurface of the hyperbolic space. *Math. Scand.* **83** (1998), 220–234.

Naveira, A. M., Gual, X.; The volume of geodesic balls and tubes about totally geodesic submanifolds in compact symmetric spaces. *Differential Geom. Appl.* **7** (1997), no. 2, 101–113.

Gual, X., Naveira, A. M.; The volume of geodesic balls and tubes about totally geodesic submanifolds in $S^n(1) \times S^m(1)$. *Proceedings of 23rd Conference on Geometry and Topology (Cluj-Napoca, 1993)*, 71–79, "Babeş-Bolyai" Univ., Cluj-Napoca, 1994.

Gual Arnau, X., Naveira, A. M.; Total curvatures of compact complex submanifolds in CP^n . *Ann. Global Anal. Geom.* **13** (1995), no. 1, 9–18.

Naveira, A. M., Segura, S.; Some isoperimetric inequalities for the space forms. *Rev. Roumaine Math. Pures Appl.* **35** (1990), no. 3, 249–259.

Naveira, A. M., Segura, S.; Some remarks about the isoperimetric problem. *Differential geometry and its applications (Dubrovnik, 1988)*, 243–252, Univ. Novi Sad, Novi Sad, 1989.

Naveira, A. M., Segura, S.; The isoperimetric inequality and the geodesic spheres. Some geometric consequences. *Differential geometry, Peñíscola 1985*, 235–242, Lecture Notes in Math., **1209**, Springer, Berlin, 1986.

Miquel, V., Naveira, A. M.; Sur la relation entre la fonction volume de certaines boules géodésiques et la géométrie d'une variété riemannienne. *C. R. Acad. Sci. Paris Sér. A-B* **290** (1980), no. 8, A379–A381.

Naveira, A. M.; Note sur des volumes en espaces de Riemann fibrés. *Rend. Circ. Mat. Palermo* (2) **18** (1969), 276–280.

5.5 Integral geometry

Why Naveira engaged in studying integral geometry? We never talked about this question with Antonio, but we know him and he was probably guided by, on one hand, the great personality of Santaló, and his book; on the other

hand, according time flies, one needs to do something special, such as to see an application of your early own research.

The notion of averaged sectional measures (ASM), defined on convex bodies K , also known as "quermassintegrals", given by Minkowski, yields to total curvatures. The recent work of Naveira, in the field of integral geometry, is based on that notion. By assuming certain differentiability conditions on ∂K , we find the ASM of K in terms of the integral over ∂K of the elementary symmetric functions of principal curvatures. By giving a local expression of the total curvatures, we find a relationship among them and the formula of the volume of a tube around a submanifold. This is the key fact that yielded to Naveira to studying formulas for the volume of tubes around certain submanifolds in compact symmetric spaces.

The branch where integral geometry applies has been called Stereology, defined as "the set of methods to explore the 3-space by means of the knowledge of 2-dimensional sections or projections over planes". A recognized expert in stereology is Prof. L. M. Cruz Orive (University of Cantabria), an old friend of Prof. Naveira, who spent more than 20 years as a researcher in the University of Berna. They shared, a few years ago, military duties and now, since 91, they also share applied geometry projects.

We would like to mention here the following contributions by Antonio Naveira and his students in these fields:

Gual, X., Naveira, A. M., Tarrío, A.; Integral geometry in Euclidean and projective quaternionic spaces. *Bull. Math. Soc. Sci. Math. Roumanie (N.S.)* **43(91)** (2000), no. 3-4, 267–277.

Naveira, A. M., Tarrío, A.; Some properties of integral geometry of generalized flag manifolds. *III International Conference in "Stochastic Geometry, Convex Bodies and Empirical Measures", Part I (Mazara del Vallo, 1999)*. *Rend. Circ. Mat. Palermo* (2) Suppl. No. **65**, part I (2000), 233–246.

Masó, R., Naveira, A. M.; A note about some algebraic relations among the cross-sectional measures of convex bodies in \mathbb{R}^n . *Bull. Math. Soc. Sci. Math. Roumanie (N.S.)* **43(91)** (2000), no. 3-4, 299–311.

Gual, X., Naveira, A. M., Tarrío, A.; An introduction to integral geometry in the n -dimensional quaternionic space. *Proceedings of the Third International Workshop on Differential Geometry and its Applications and the First German-Romanian Seminar on Geometry (Sibiu, 1997)*. *Gen. Math.* **5** (1997), 175–181.

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SUMMARIZING. Naveira is intuition, hopefulness, faith, geniality, energy. He is tireless and does not know the discouragement. He is faithful, a teacher and a friend. He is very fond of his teachers and old friends: D. Enrique, Professors Deheuvels, Lichnerowicz, Gray, Vanhecke, Bourguignon, Aubin. He is also very fond of France and French people.

Antonio, have a long life and let us enjoy it with you.

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