Decoupling between torsion and magnetic fields in bouncing cosmology

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Abstract. Slow down of decaying cosmic magnetic fields in QED scalar electrodynamics is obtained.

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INTRODUCTION

K Enqvist *et al* [1] place limits on neutrino masses from galactic dynamo mechanism. Since neutrino masses are important in extending the standard model of particle physics, it seems worth to investigate the relation between Lorentz Violation (LV) and galactic dynamos in torsion fields [2, 3]. Knowledge of the dynamics between torsion and cosmic magnetic fields may reveal if dynamo mechanism is a powerful mechanism to feed the galactic magnetic fields of nano-Gauss observed in nature. In this letter, by using a scalar electrodynamics in the context of quantum electrodynamics (QED) [4], it is possible to show that magnetic field decays when torsion is fast amplified. Torsion needed to seed galactic dynamo is of the order of $10^{-18}\mathrm{cm}^{-1}$ and can be found in nature and is even weaker than value estimated in the Early Universe. In previous work [5] we noticed that semi-minimal coupling has been used on a Lagrangian of the type $\frac{1}{4}R_{ijkl}F^{ij}F^{kl}$, [i, j, k, l=0, 1, 2, 3]. This has provided further constraints on torsion up to $10^{-31}GeV$. Here though semi-minimal coupling is preserved, we shall use another gravitational sector in the Lagrangian given by the coupling $RF^{ij}F_{ij}$ [6]. Term $R_{ijkl}F^{ij}F^{kl}$ displays the same symmetries of LV term. Here Riemann-Cartan curvature tensor, includes torsion terms which plays the role of the Higgs sector constants k_{ijkl}. In this paper, we show that the use of this photon sector coupled semi-minimally with torsion mode, in scales of 10 kpc, would require a not very strong torsion field that might exist in nature, so we must conclude that this necessarily implies that galactic magnetic fields can be seeded by such torsion models also in the Mazziteli et al scalar electrodynamics. Some physicists [7] argue that torsion is very weak to have time to seed magnetic fields, but actually from Mazziteli et al scalar QED we show that the torsion field may grow exponentially in regions of weak primordial magnetic field is not strong enough to seed galactic dynamos.

FLAT SEMI-MINIMAL TORSION-PHOTON COUPLING OF RF^2 LAGRANGIAN

Though torsion effects are highly suppressed in comparison with curvature ones of Einstein gravity sector, we do not consider here Minkowski space since as can be easily shown from the field equations torsion vanishes in Minkowski space. Mazziteli et al Lagrangian [6] is

$$S = \frac{1}{m^2}\int d^4x(-g)^{\frac{1}{2}}(-\frac{1}{4}F^2 + (m^2+\varepsilon R)\phi\bar{\phi} - D_j\phi D^j\bar{\phi}), \tag{1}$$

where $D_i = \partial_i - \sqrt{-1}eA_i$ is the covariant derivative for the scalar fields. Mazzitelli *et al* [6] have computed an effective Lagrangian for the e.m field by integrating the quantum scalar field. Via dimensional regularisation they obtain the effective Lagrangian [6]

$$\mathscr{L}_{eff} = -\frac{1}{4}F^2 + \frac{1}{2}\frac{1}{4\pi^{\frac{d}{2}}}(\frac{m}{\mu})^{d-4}\sum a_j(x)m^{4-2j}\Gamma(j-\frac{d}{2}). \tag{2}$$

The first Schwinger-De Witt (SDW) coefficients, computed by Mazzitelli *et al*, are $a_0 = 1$,

$$a_1 = -(\varepsilon - \frac{1}{6})R, \tag{3}$$

$$a_2 = \frac{1}{180}(R_{ijkl}R^{ijkl} - R_{ij}R^{ij}) + \frac{1}{2}(\varepsilon - \frac{1}{6})^2R^2 + \frac{1}{6}(\varepsilon - \frac{1}{5})\Box R - \frac{e^2}{12}F^2, \tag{4}$$

$$a_3 = ... + \frac{e^2}{60}R_{ijkl}F^{ij}F^{kl} - \frac{e^2}{90}R_{ij}F^{ik}F_{kj} + (\frac{1}{6} - \varepsilon)RF^2 + ... \tag{5}$$

Due to the use of semi-minimal coupling, torsion appears only in a_2 as first term, since there torsion does not appears in the covariant derivative and consequently not in the electromagnetic field. Actually, following this reasoning, torsion appears only in the curvatures for the first time in a_1. Following Mazzitelli *et al*, we shall consider the following effective Lagrangian in Riemann-Cartan spacetime, through the minimal coupling as

$$\mathscr{L}_{eff} = -\frac{1}{4}F^2(1 + \frac{b}{m^2}R), \tag{6}$$

where b is the coupling constant, m the electron mass and R the Ricci curvature. This is similar to Widrow and Turner Lagrangian [10]. From (6) we obtain the field equations for the Friedmann spatially flat metric $ds^2 = a^2(-d\eta^2 + dx^2)$ as

$$\partial^i(F_{ij}(1 + \frac{bR}{m^2})) = 0, \qquad i,j = 0,1,2,3. \tag{7}$$

From these equations we obtain, with appropriated approximations,

$$(\ddot{A}_i + k^2A_i)(1 + \frac{bR}{m^2}) + \frac{b\dot{R}}{m^2R}\dot{A}_i = 0, \qquad i = 0,1,2,3, \tag{8}$$

where A is the electromagnetic potential and k the wave number. We may approximate for high coherence scales, where $k^2 << 1$, and the fact that in Riemannian case in inflationary epoch $R >>> m^2$, so the last equation would be reduced to

$$[\ddot{A}_i + \frac{\dot{R}}{R}\dot{A}_i] = 0, \tag{9}$$

where R is the Ricci scalar. This shows that, although there is no inflation, here we consider that torsion has a similar behaviour, so actually $\dot{K} >>> m^2$.

GALACTIC DYNAMO SEEDS IN RF^2 SEMI-MINIMAL COUPLING

In this section equation (9) is solved in the case of curved spacetime performing the semi-minimal coupling, where the Ricci scalar is approximated taken as $2\dot{K}$, where K is the time component K^0 of contortion. To simplify matters only homogeneous component of contortion is used. Here we adopt linearisation of the Ricci-Cartan scalar [8] where

$$R = g_{ij}R^{ij} = R^* + 2\nabla_i K^i - K^2, \tag{10}$$

where $K^j = K^{rj}{}_r$, represents the trace of contortion and R^* is the Riemannian Ricci scalar that here shall be taken as constant like in de Sitter or Einstein space. Let us now perform the variation of the Lagrangian density $\sqrt{g}\mathscr{L}$ with respect to the scale cosmological factor a, and contortion K, to complete the system of Einstein-Cartan-Maxwell equations of course with propagating torsion. This can be done easily by computing the Euler Lagrange equations

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathscr{L}}{\partial\dot{a}} - \frac{\partial\sqrt{g}\mathscr{L}}{\partial a} = 0, \tag{11}$$

$$\frac{d}{dt}\frac{\partial\sqrt{g}\mathscr{L}}{\partial\dot{K}} - \frac{\partial\sqrt{g}\mathscr{L}}{\partial K} = 0. \tag{12}$$

Let us start from the last equation to determine K in terms of the scale factor a. This yields $K = -\frac{3\dot{a}}{a}$. Before applying this result to the expression for the Ricci-Cartan scalar, let us express this scalar in terms of a and K. This yields the following expression

$$R = g_{ij}R^{ij} = R^* + 2\dot{K}^i - K^2 + \partial_t ln\sqrt{g}K \tag{13}$$

or

$$R = g_{ij}R^{ij} = -6[\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2] + 2\dot{K}^i - K^2 + (\partial_t lna^3)K, \tag{14}$$

which yields

$$\dot{R} = \dot{R^*} + \ddot{K} + (\frac{\dot{a}}{a} + 2K)\dot{K} + 3[\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}]K. \tag{15}$$

The expression for $\ddot{K}$ is

$$\ddot{K} = -3[\frac{\dddot{a}}{a} - 3\frac{\ddot{a}\dot{a}}{a^2} + (\frac{\dot{a}}{a})^3] \tag{16}$$

and expression for Ricci-Cartan scalar Lagrangian $\sqrt{g}R$ is $a^3R = -3[3\ddot{a}a^2 + 7\ddot{a}^2a]$. Substitution of this expression into the Euler-Lagrange equation above one has

$$\dddot{a}a - 4\ddot{a}\dot{a} = 0. \tag{17}$$

By making use of the ansatz $a \sim t^n$, where n is a real number, one obtains the following algebraic equation $n(n-2) - 4n^2 = 0$, which yields immediately $n = -\frac{2}{3}$, and $a \sim t^{-\frac{2}{3}}$, which represents a contracting phase of the cosmological model with torsion. Therefore from the above expression for K one obtains $K \sim -2t^{-1}$. Dynamo effect can be easily seen by computing the ratio $\frac{\dot{R}}{R}$ as

$$\frac{\dot{R}}{R} = \frac{[3\ddot{K} - \frac{2}{3}(K^2)^{\cdot}]}{[3\dot{K} - \frac{2}{3}K^2]}. \tag{18}$$

Since the torsion is a very weak field, this can be approximated to

$$\frac{\dot{R}}{R} \approx \frac{\ddot{K}}{\dot{K}}. \tag{19}$$

Substitution the ansatz $A_i \sim t^n$ into the Maxwell like equation above and solving it, taking into account the expression $B = \sqrt{-1}kA$ where B is the magnetic field and k is the wave number one obtains the estimate for $B_+ \sim t$ or $B_- \sim t^{-2}$ depending on the n sign. Therefore one may conclude that the torsion decays while the magnetic field grows in the contracting phase of the universe [9] exactly like in the general relativistic version investigated by Salim *et al* [10]. One also notes however that the decaying solution is faster than the growing solution so if one superposes both solution the overall magnetic field actually decays, imposing a damping final solution.

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