Black Holes

Theory & Astrophysics

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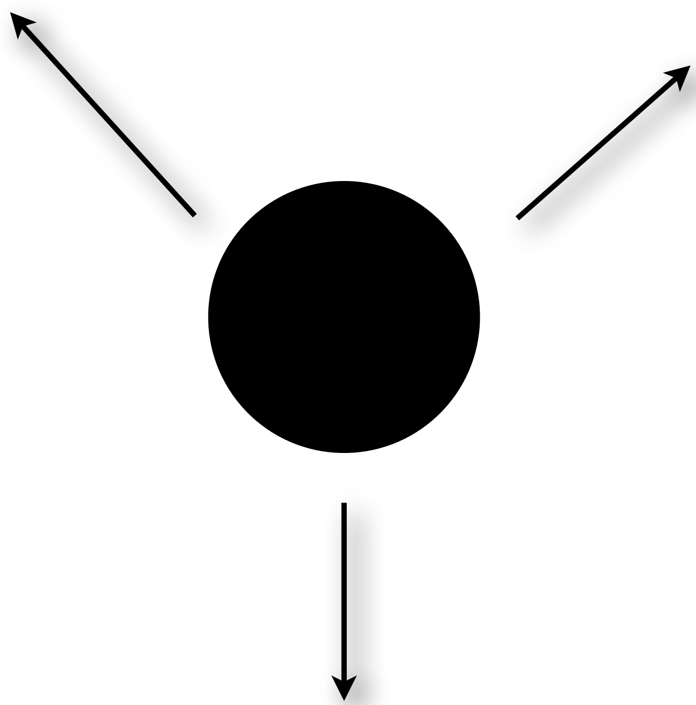
Contents

What are Black Holes?

They are black

You can’t see them,
they trap light



They are holes

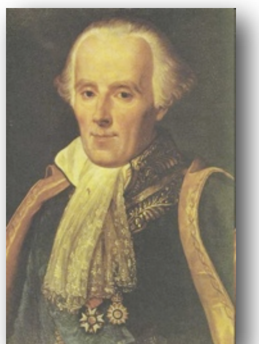
When you fall in
you can’t get out

They are strange

Time is “frozen” near them,
there is a ‘singularity’ inside
a black hole

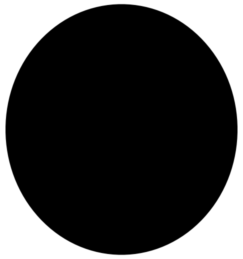
Origins of the idea of BHs

Gravity affects light

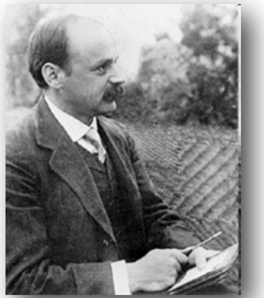

Death of stars

Once they run out of nuclear fuel, stars cannot support themselves and collapse. They can become white dwarfs (Mmax = 1.4 Msol) or neutron stars (Mmax ~ 3 Msol). But what happens if the star is too heavy?




General Relativity

The solution for the gravitational field of a point mass predicts infinite spacetime curvature (at the location of the mass) and an 'event horizon'. Initially believed to have no connection to the real physical world.

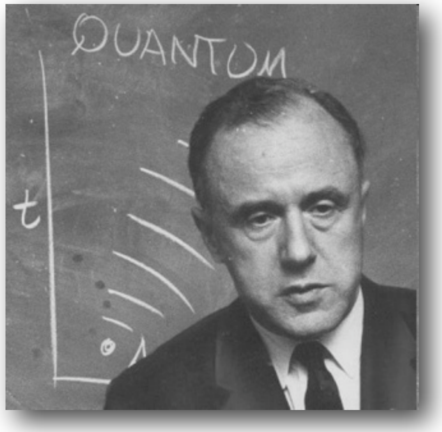



A bit of history

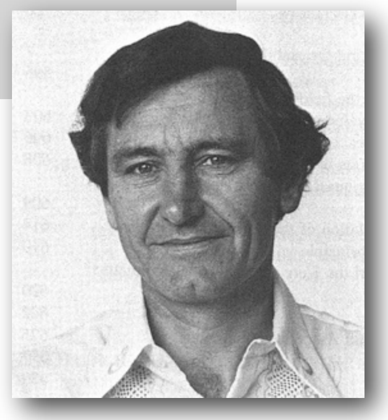
1915: Albert Einstein's General Relativity is formulated.

1916: Karl Schwarzschild discovers the first BH solution.

1960s: John Wheeler coins the name 'black hole', replacing the earlier term "frozen stars".



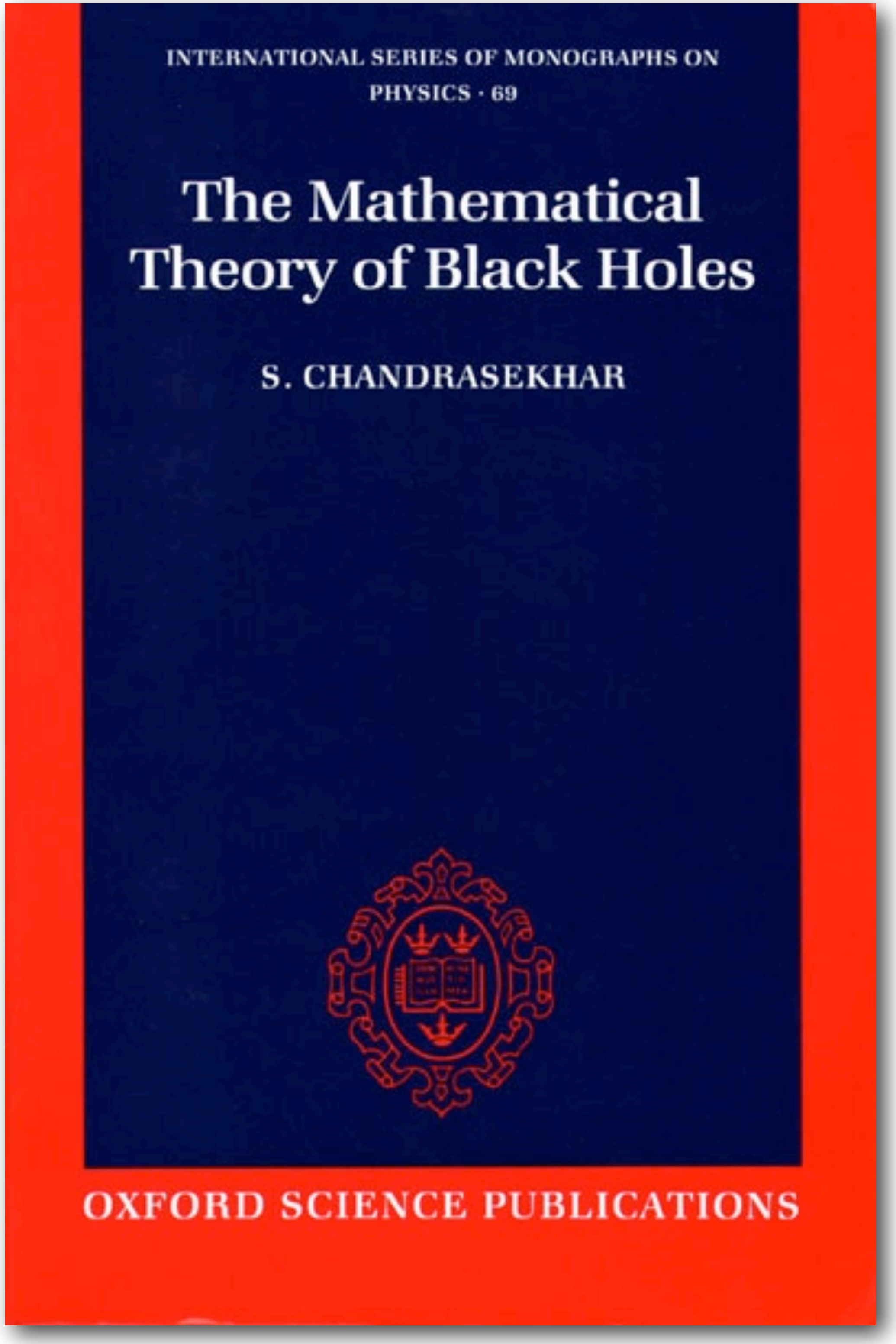
1963: Roy Kerr discovers the eponymous solution.



1960s-1970s: The theory of BHs is firmly established.

Part I

Theory of Black Holes



General Relativistic gravity

- The field equations of GR are:

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

Einstein tensor,
depends on the metric

Stress-energy tensor,
describes the matter

- The spacetime metric is given by: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

- BHs are vacuum solutions: $G^{\mu\nu}[g_{ab}] = 0$

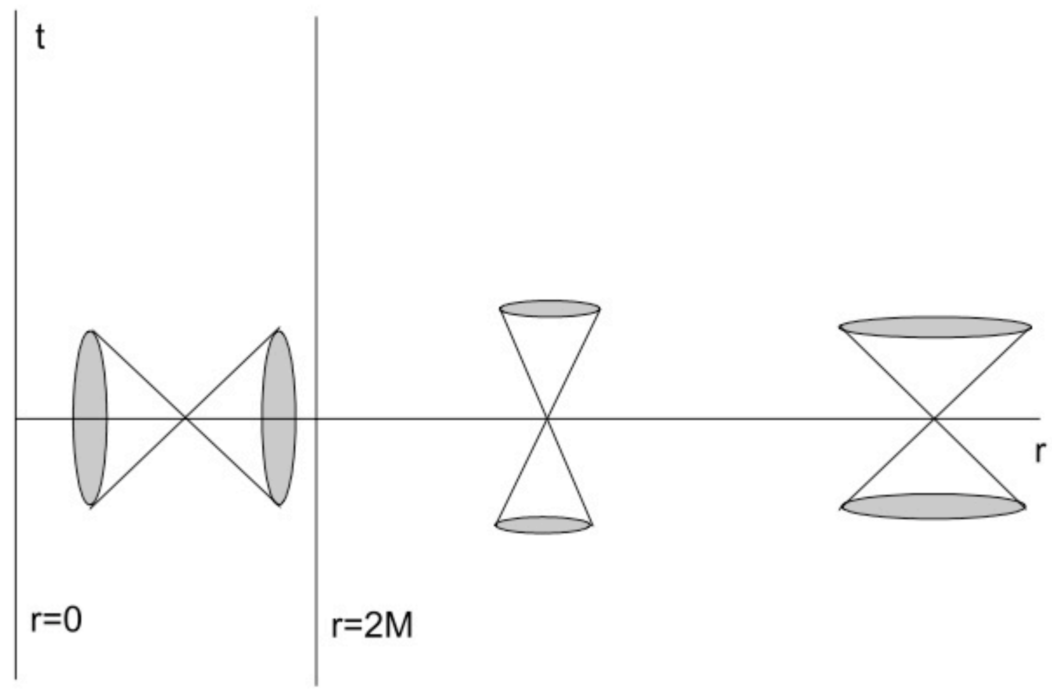
- Some symmetry is also required: time-independence and spherical symmetry (Schwarzschild solution) or axisymmetry (Kerr solution).

The Schwarzschild solution

- The Schwarzschild BH solution is the simplest one:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

- There is a horizon at: $g_{rr} = \infty \rightarrow r = 2M$
  and a singularity (infinite curvature) at: r=0.



Light cone structure in the Schwarzschild metric

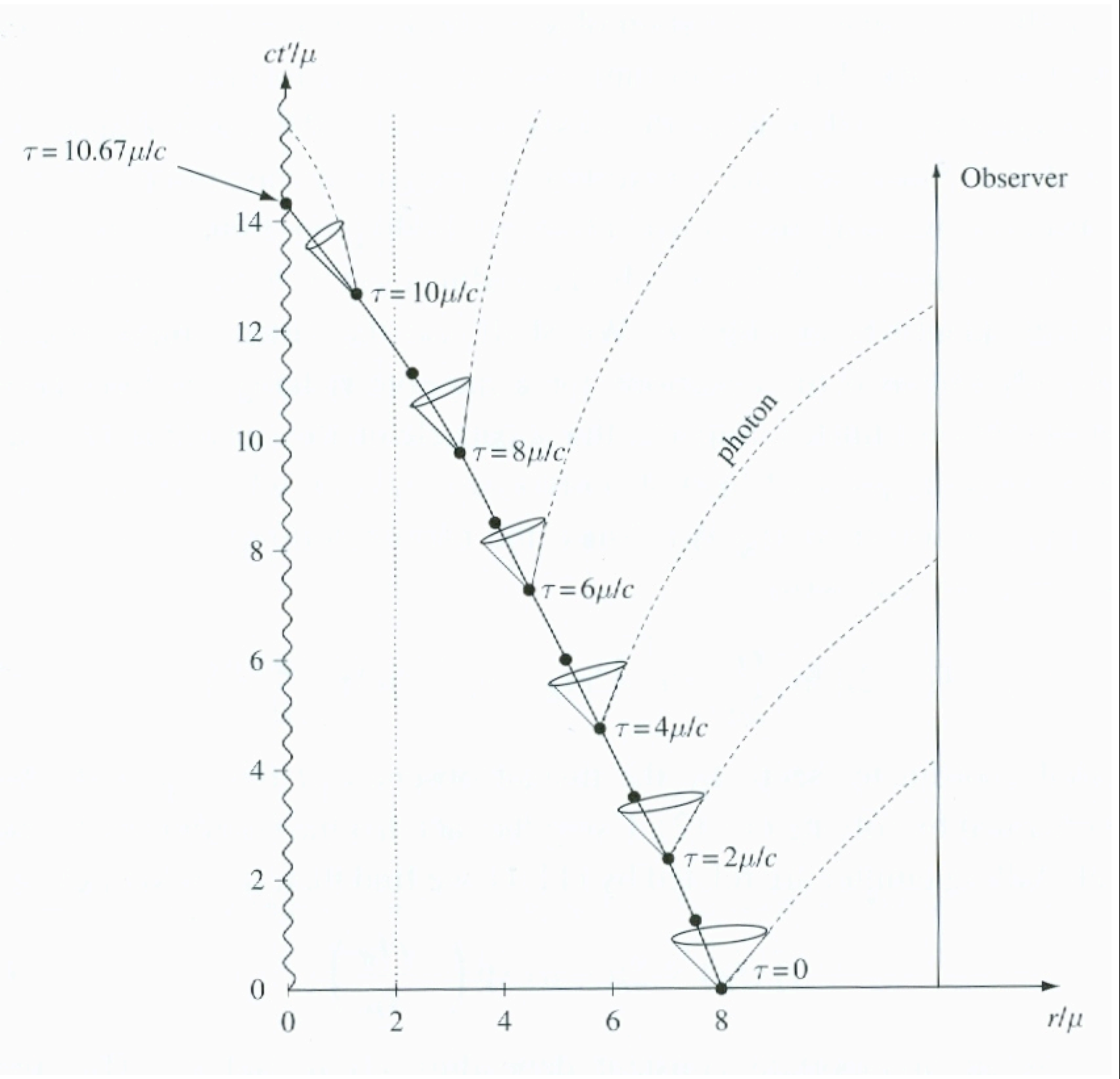
Infalling light source

- The horizon is a surface of infinite redshift (because $g_{tt} = 0$ there).
- Imagine a light source falling into a BH.
- Light frequency is increasingly red-shifted. At a fixed distance r:

$$f_\infty = f_{\text{local}}(r)\left(1 - \frac{2M}{r}\right)^{1/2}$$

(derive this!)

- No signal escapes from r < 2M.

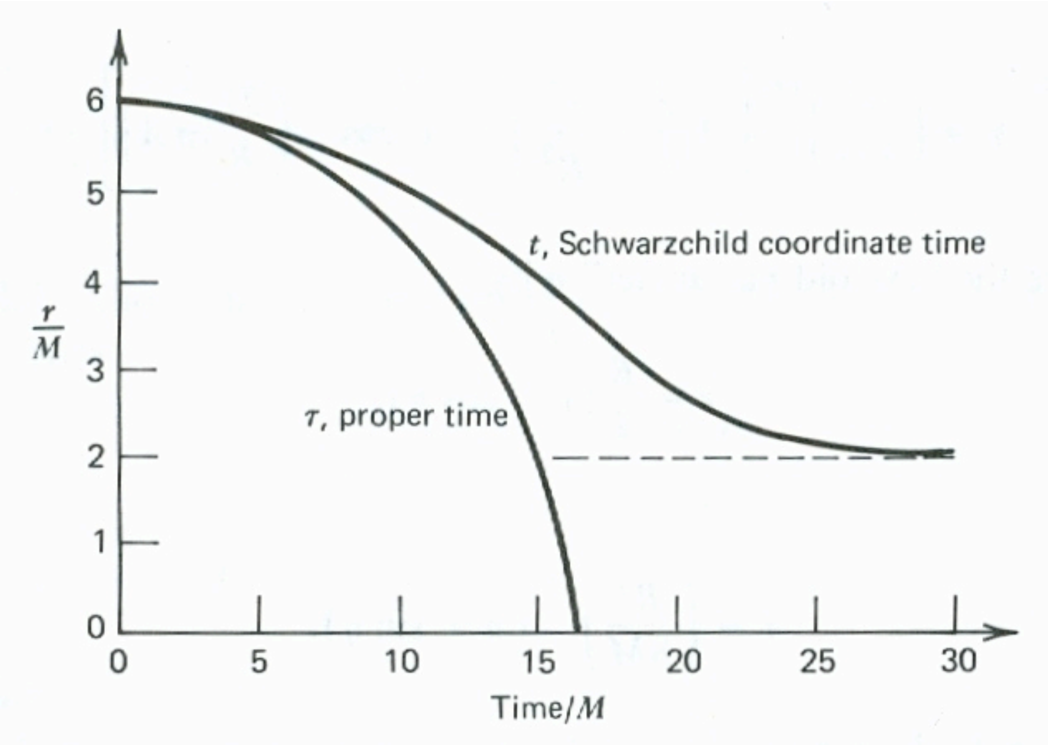


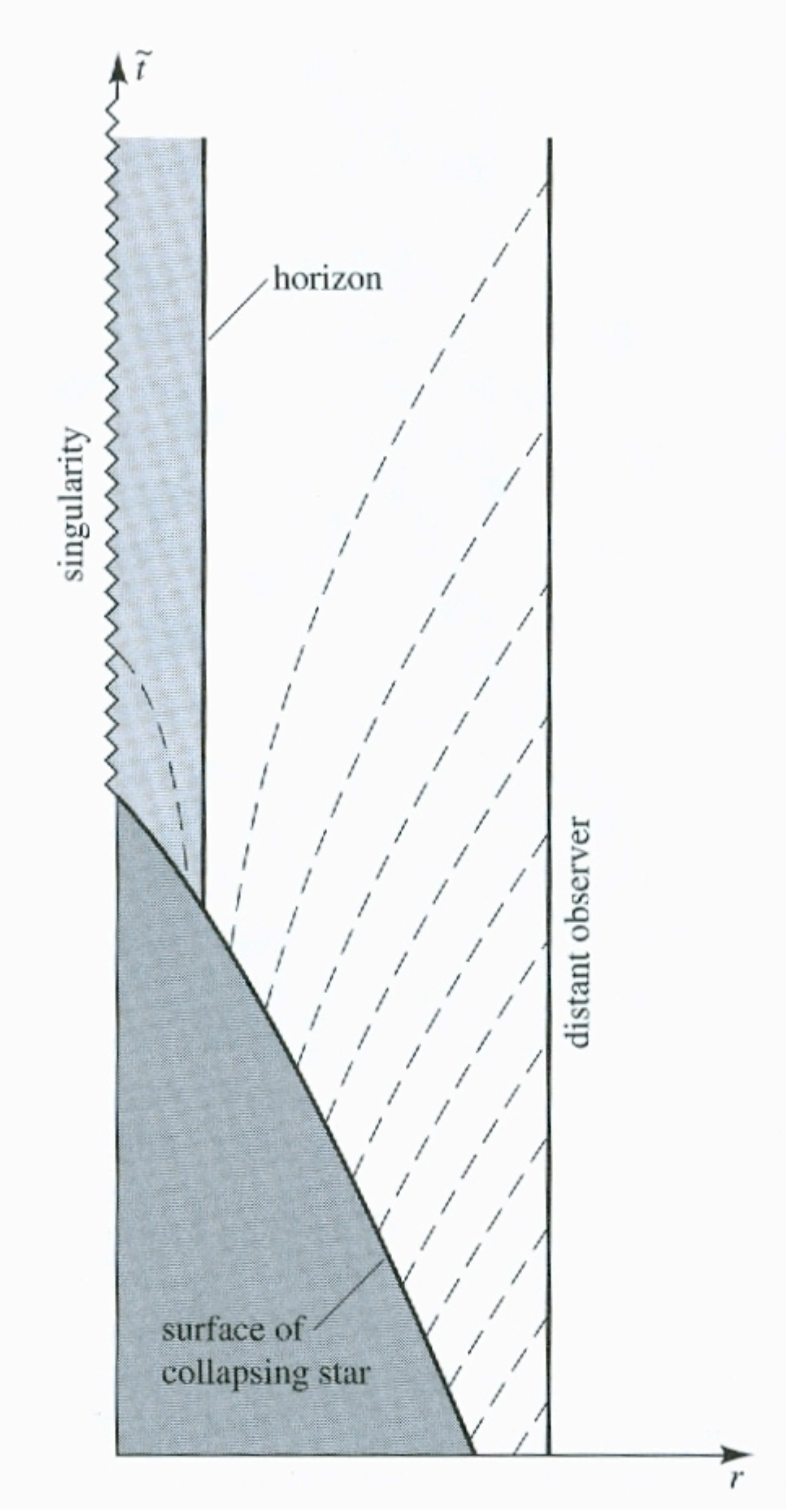
Gravitational collapse

- The collapse of (very) massive star can give birth to BHs. The surface of the collapsing star behaves as our infalling light source.

- Any emission coming from the star is increasingly red-shifted => surface becomes fainter.

- Time "freezes" as the surface approaches r= 2M.





The Kerr BH solution

- The Kerr metric is (using Boyer-Lindquist coordinates {r, θ, φ}):

$$ds^2 = -(1 - 2Mr/\Sigma)dt^2 - (4Mar\sin^2\theta/\Sigma)dtd\varphi$$

$$+(\Sigma/\Delta)dr^2 + \Sigma d\theta^2 + (r^2 + a^2 + 2Ma^2 r\sin^2\theta/\Sigma)\sin^2\theta d\varphi^2$$

$$\Delta = r^2 - 2Mr + a^2, \qquad \Sigma = r^2 + a^2\cos^2\theta$$

- This may seem complicated but it only depends on the mass M and spin $\alpha = J/M$. The limit $\alpha=0$ takes us back to the Schwarzschild spacetime.

- This simple thing is expected to describe real astrophysical BHs !
  In Chandrasekhar's words:
  "*The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time*"

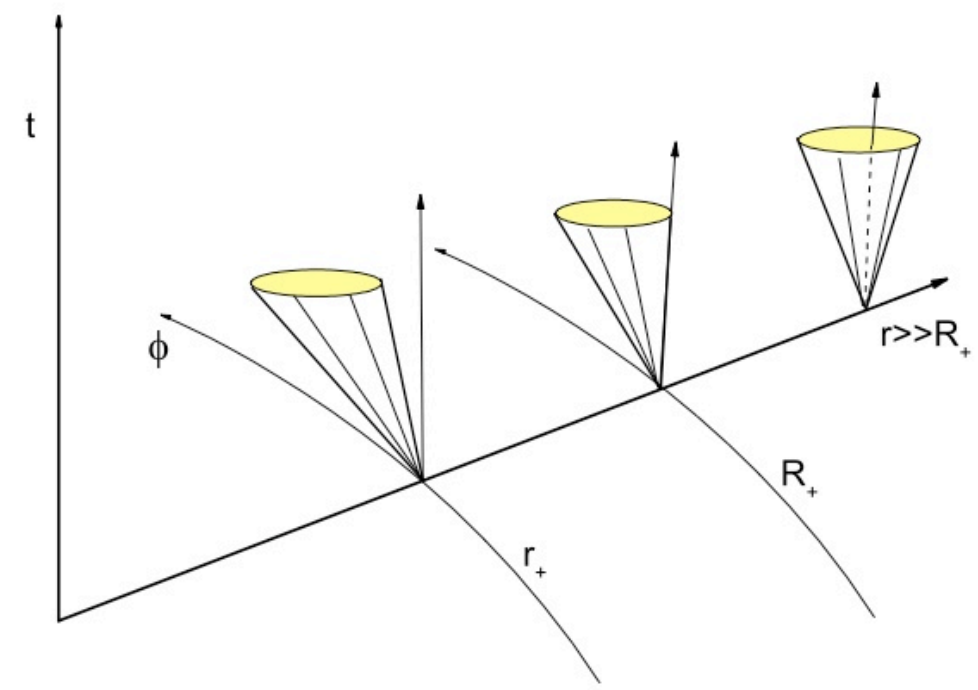
Kerr spacetime: properties

- The Kerr solution describes rotating BHs with angular momentum J = αM.

- A key difference with respect to the (non-rotating) Schwarzschild BHs is the presence of the metric component $g_{t\varphi}$.

- This leads to the phenomenon of "frame dragging". This can be seen by looking at the structure of the light cone:

- Light rays (and particles) are "dragged" along the direction of the BH's rotation!



Anatomy of a Kerr BH

- The event horizon is located at: $g_{rr} = \infty \quad \longrightarrow \quad r_{\rm H} = M \pm \sqrt{M^2 - a^2}$

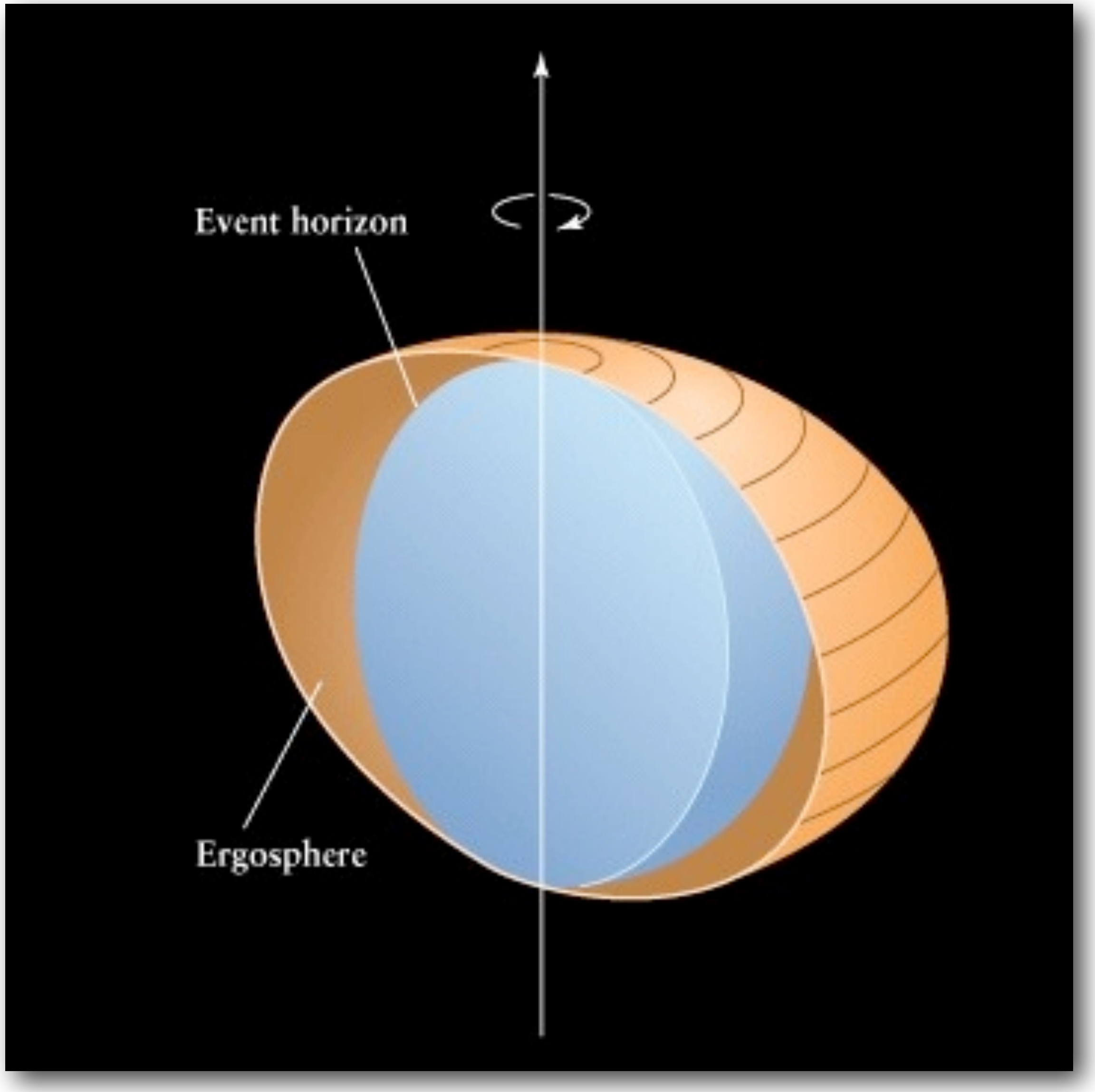
- The infinite redshift surface $g_{tt} = 0$ is at

$$r_{\rm erg}(\theta) = M + \sqrt{M^2 - a^2 \cos^2\theta}$$

- The region $r_{\rm H} < r < r_{\rm erg}$ is the ergosphere.

- Frame dragging inside the ergosphere is so strong that prevents motion against the direction of rotation!

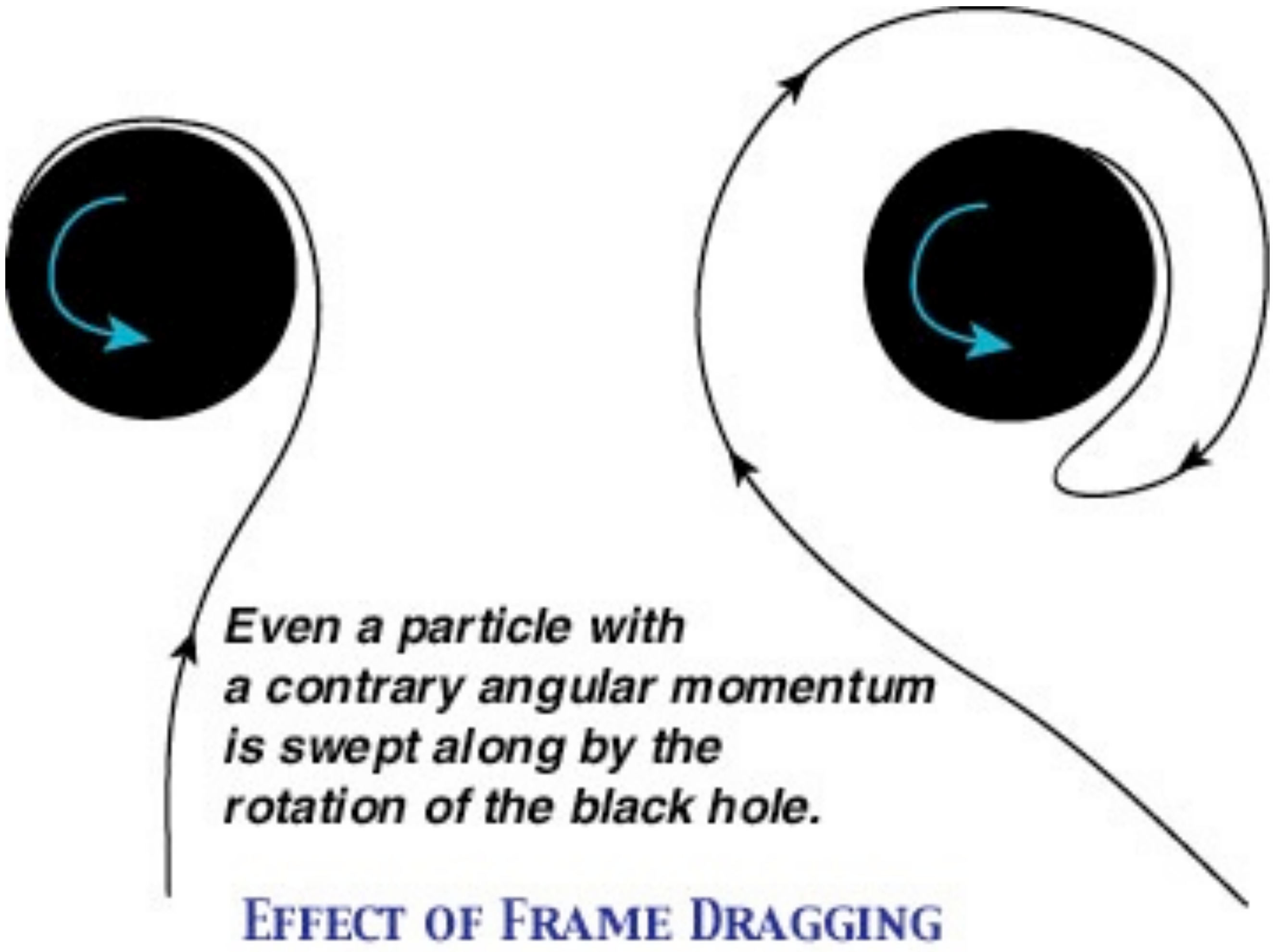
- The boundary $r_{\rm erg}$ is also known as the static limit.



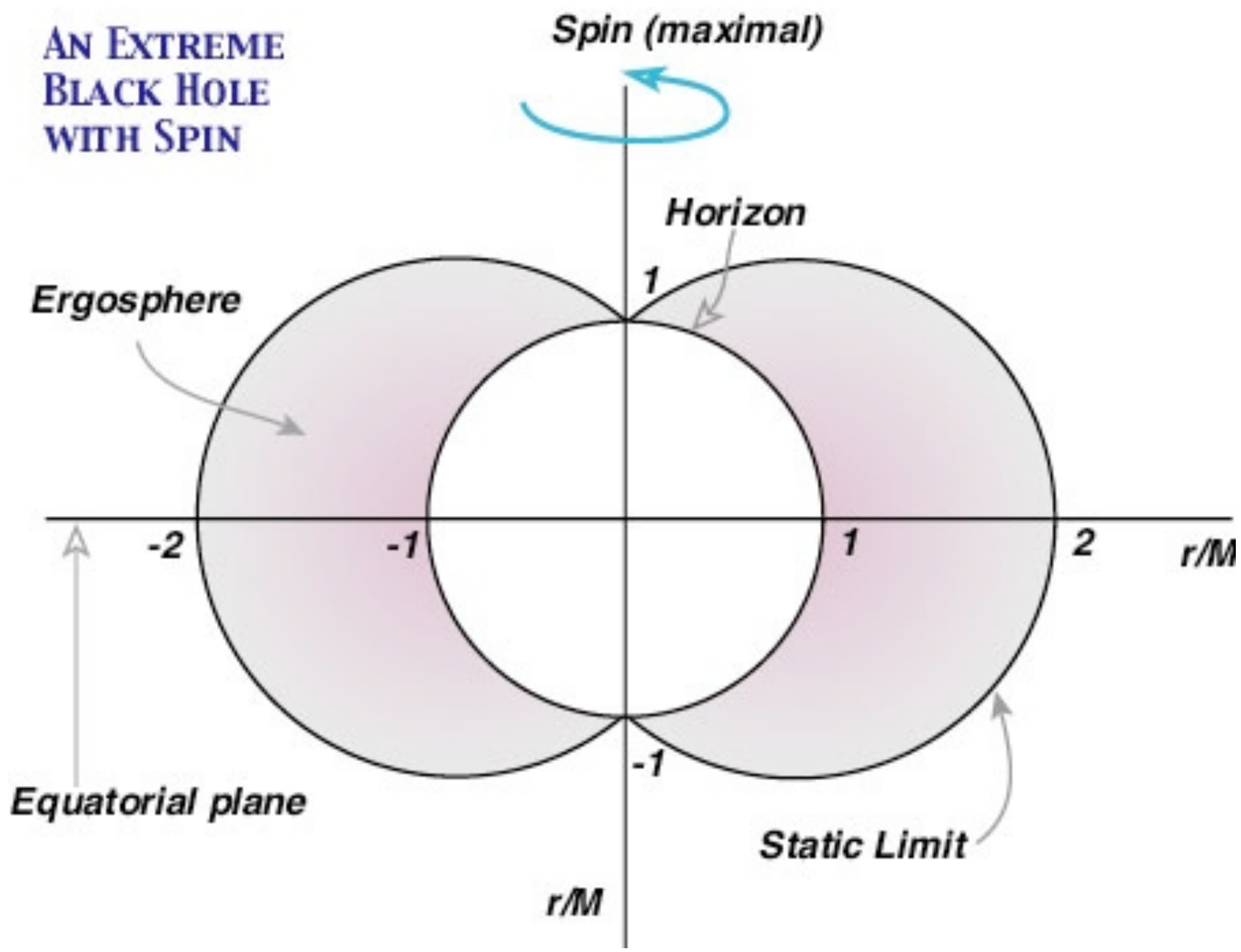
Frame dragging

- Frame dragging is not unique to BHs.

- For instance, Earth's gravity field also causes frame dragging. It has been measured to a precision of ~ few % (and is obviously in agreement with the predicted GR result)!



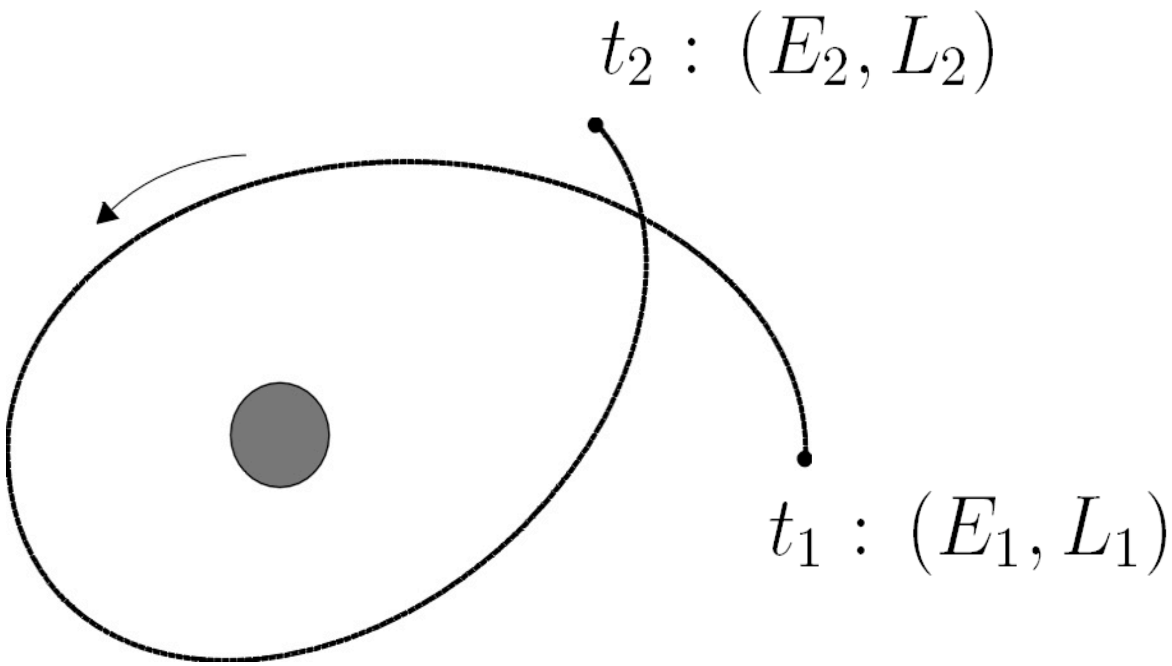
Maximum spin and beyond



Comment: The horizon disappears for α > M leading to a "naked singularity"; the "cosmic censorship" theorem forbids that. Hence, the maximum spin of a "real" BH is α = M.

Part II

Celestial Mechanics



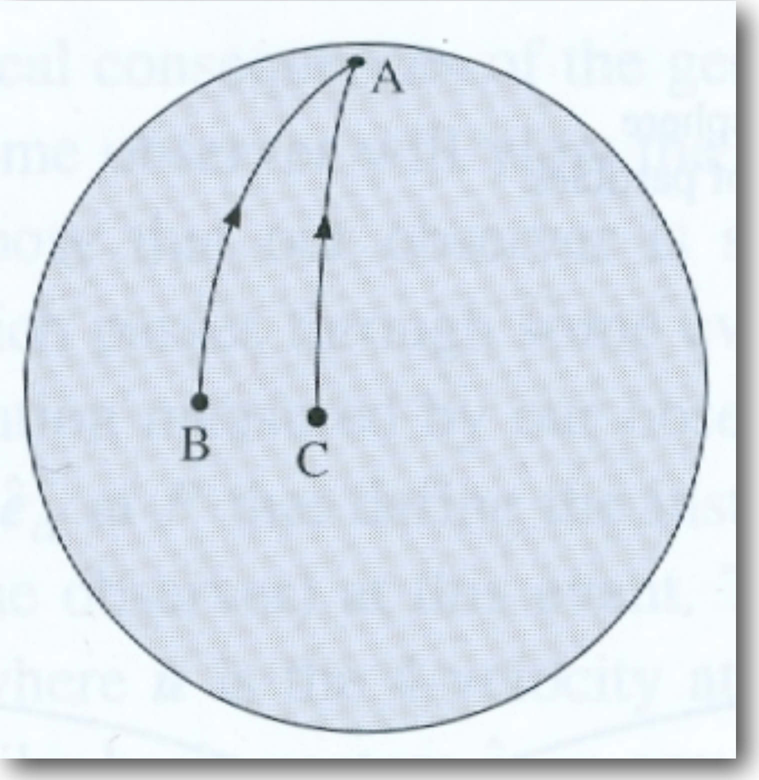
Geodesic motion

- In GR, the motion of a test body under the action of gravity alone is geodesic, that is a minimum distance curve in a given spacetime.

- The equation of geodesic motion is:

$$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0, \qquad u^\mu = dx^\mu / d\tau$$

The Christoffel symbols are functions of the metric derivatives



Orbital motion (N)

- Recall how we calculate the motion of a small mass m around a big mass M in Newtonian theory:

✓ The time-independence and spherical symmetry of M's gravitational field leads to a conserved energy E and angular momentum L (integrals of motion).

$$E = \frac{1}{2}m(u_r^2 + r^2 u_\varphi^2) - \frac{Mm}{r}, \qquad L = mr^2 u_\varphi \qquad u^i = dx^i/dt$$

- Using these we can derive 1st order equations of motion (eom):

$$u_r^2 = V_r(r, E, L), \qquad u_\varphi^2 = V_\varphi(r, E, L)$$

- Resulting orbits: circular, elliptical, parabolic.

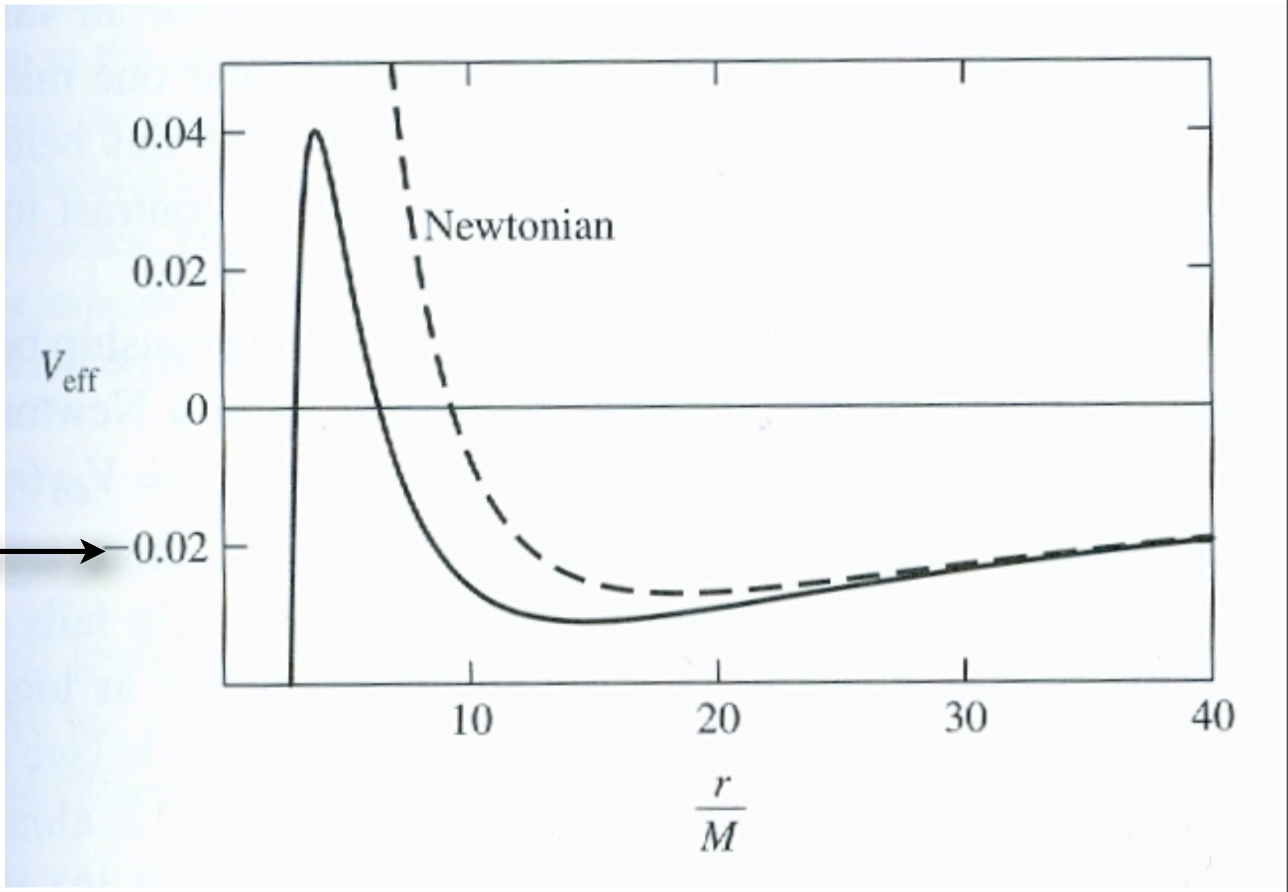
Orbital motion (GR)

- The Kerr/Schwarzschild spacetimes are t-independent and axisymmetric (K) or spherically symmetric (S). There is a conserved energy E, angular momentum L (about the spin axis in Kerr). In addition, we have the conserved product $g_{\mu\nu}u^\mu u^\nu = -1$ and a "Carter constant" Q (this is relevant for non-equatorial motion around a Kerr BH).

- The eom are again of 1st order:

$$u_r^2 = V_r(r, E, L), \qquad u_\varphi^2 = V_\varphi(r, E, L)$$

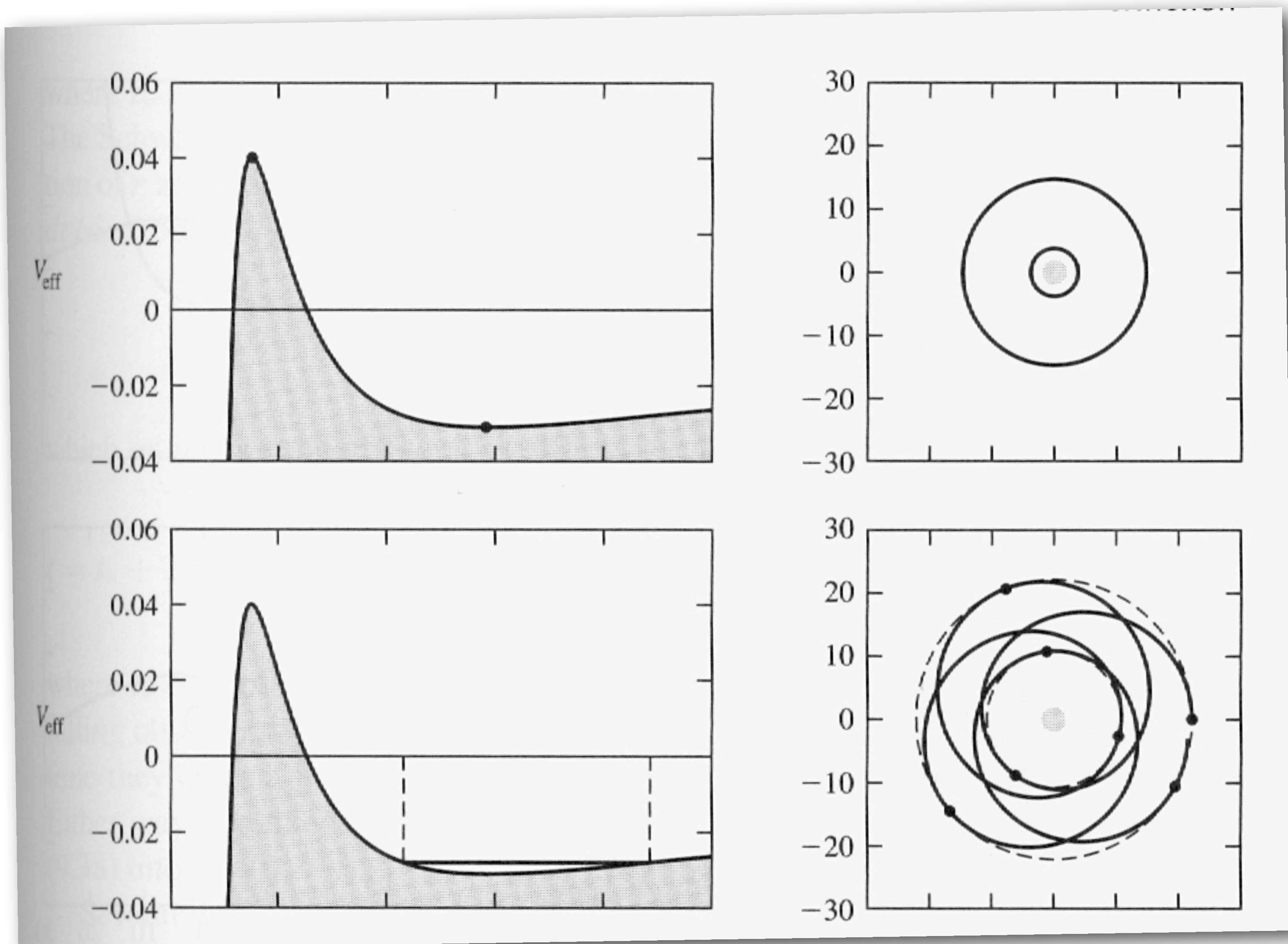
- The radial potential V_r looks like this:



- Example: circular orbit at $r = r_0$

$$V_r(r_0) = 0, \quad dV_r/dr(r_0) = 0 \quad \rightarrow \quad E(r_0), L(r_0)$$

Bound orbits



- Circular orbits: r=constant

  ✓ r ≥ 6M in Schwarzshild spacetime

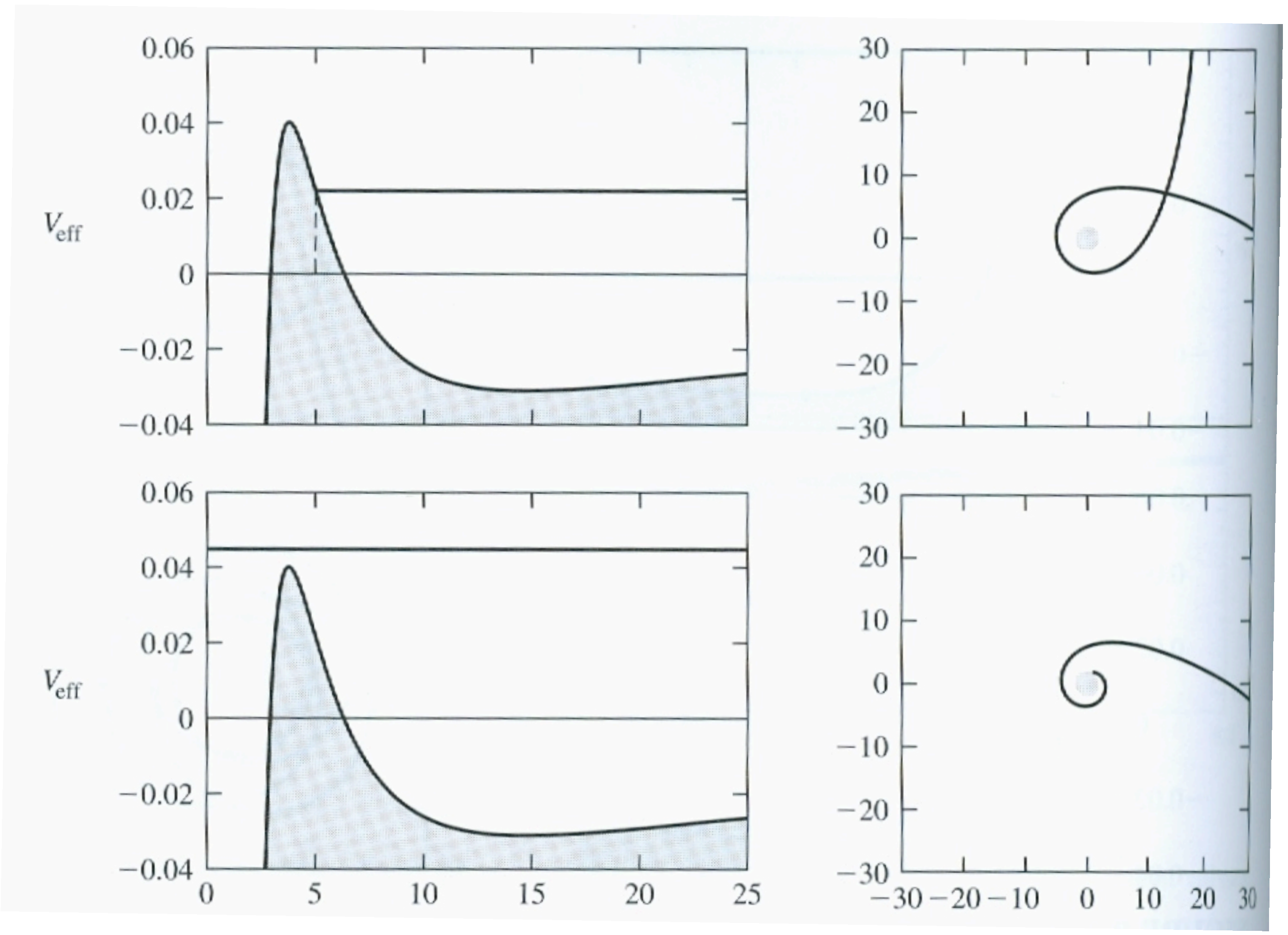
  ✓ Kepler's third law:

$$\Omega_\varphi = \frac{d\varphi}{dt} = \pm \frac{M^{1/2}}{r^{3/2} \pm a M^{1/2}}$$

- Elliptical orbits: $r_1 < r < r_2$

  ✓ Ellipses are not closed as a result of periastron precession.

Unbound orbits

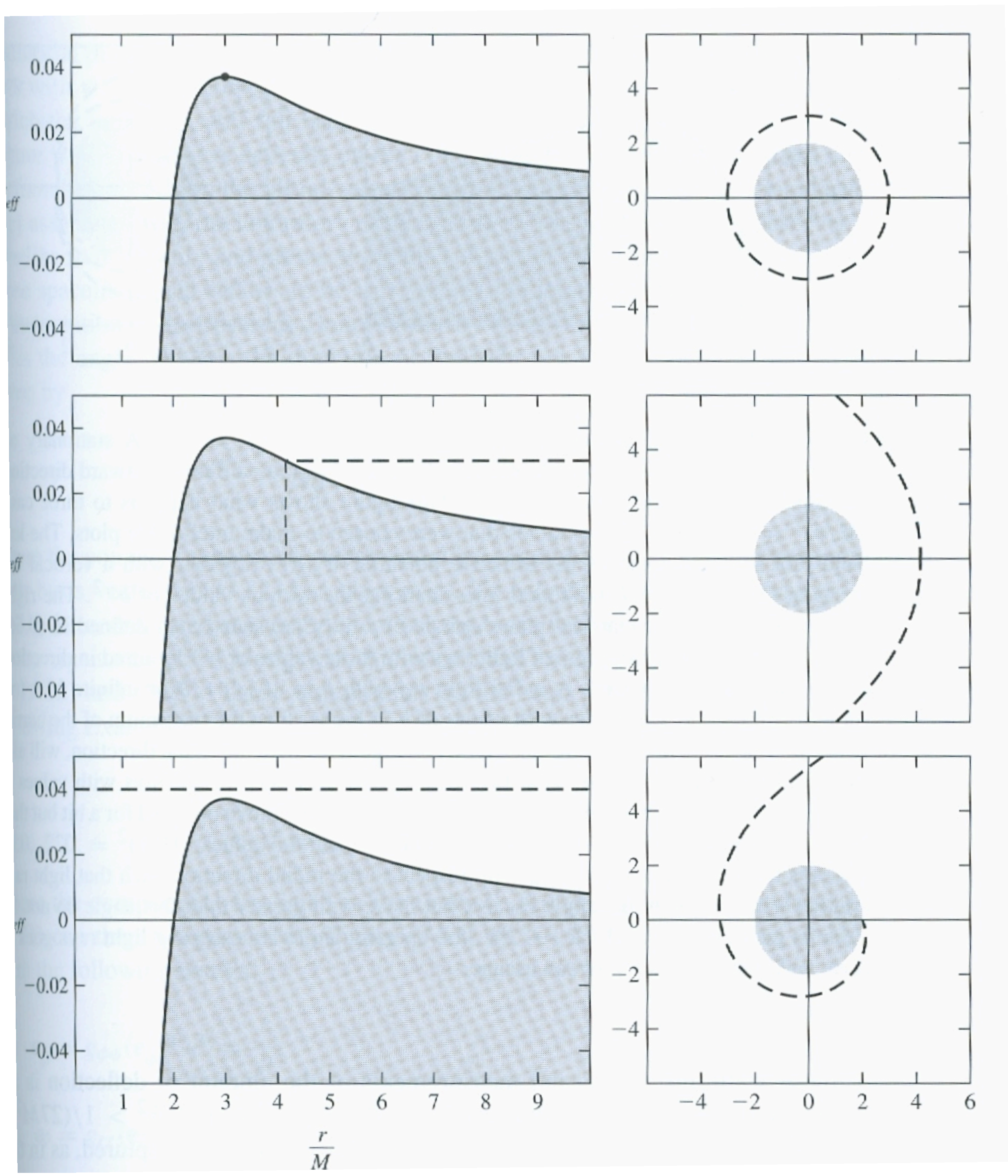


- Parabolic/hyperbolic orbits:

$$r_1 < r < \infty$$

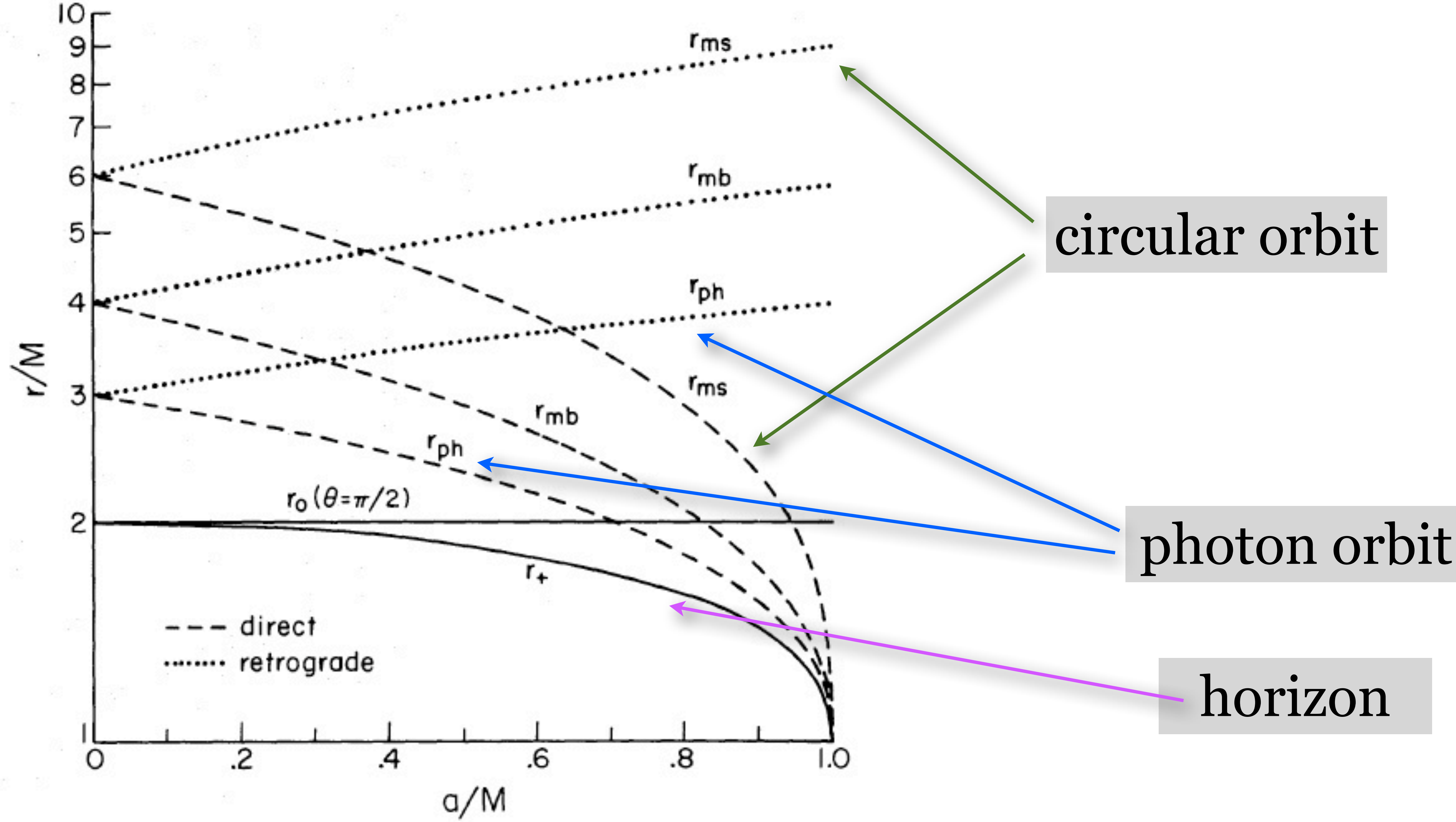
- Capture orbits.

Photon orbits



- Circular (unstable)
  - ✓ r = 3M in Schwarzshild spacetime
- Parabolic (unbound)
- Capture

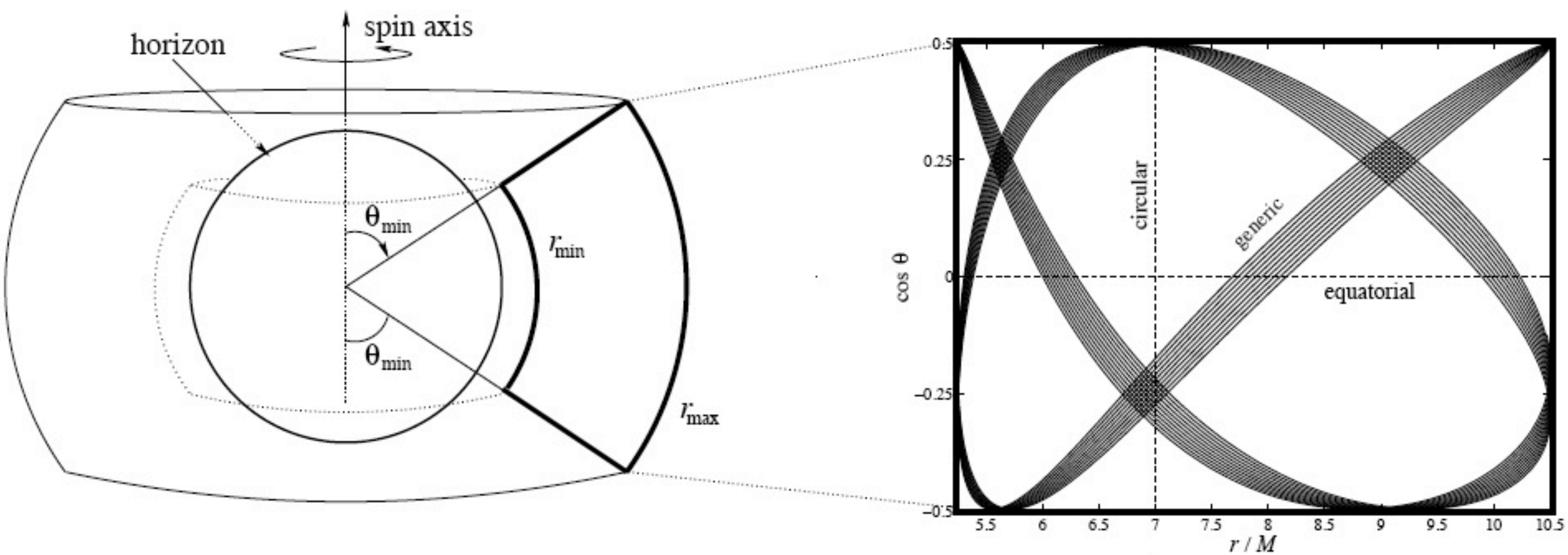
Slow vs fast rotation



BH spin

Non-equatorial orbits

- When motion is non-equatorial geodesic motion in Kerr is significantly different to that in Schwarzschild.
- Orbital plane precesses, a generic orbit looks like this:



Part III

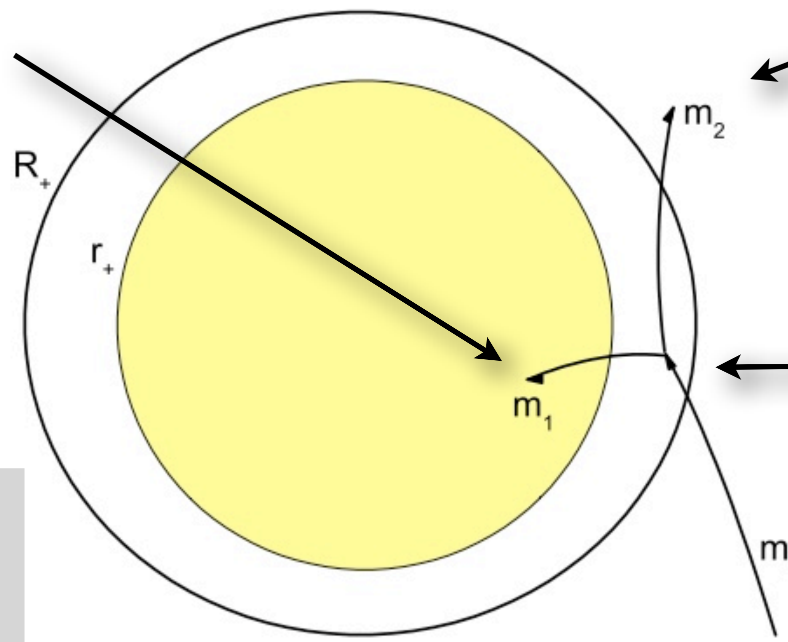
Energy Extraction from Black Holes

The Penrose process

- The ergosphere is a key property of Kerr BHs. Particles inside the ergosphere may have negative energy ($E < 0$).

- This can be used to extract energy from a Kerr BH via the Penrose mechanism:

4.The mass m_1 has $E_1 < 0$ and can only be in a capture orbit.



3.The mass m_2 has $E_2 > 0$ and can escape to infinity

2.Inside the ergosphere m_0 splits into m_1 and m_2

1.The mass m_0 is in a geodesic with energy E_0

Energy conservation:
$E_2 = E_0 - E_1 > E_0$ =>
energy amplification!

- The process can take place until $\alpha = 0$, i.e. the BH is reduced to Schwarzschild.

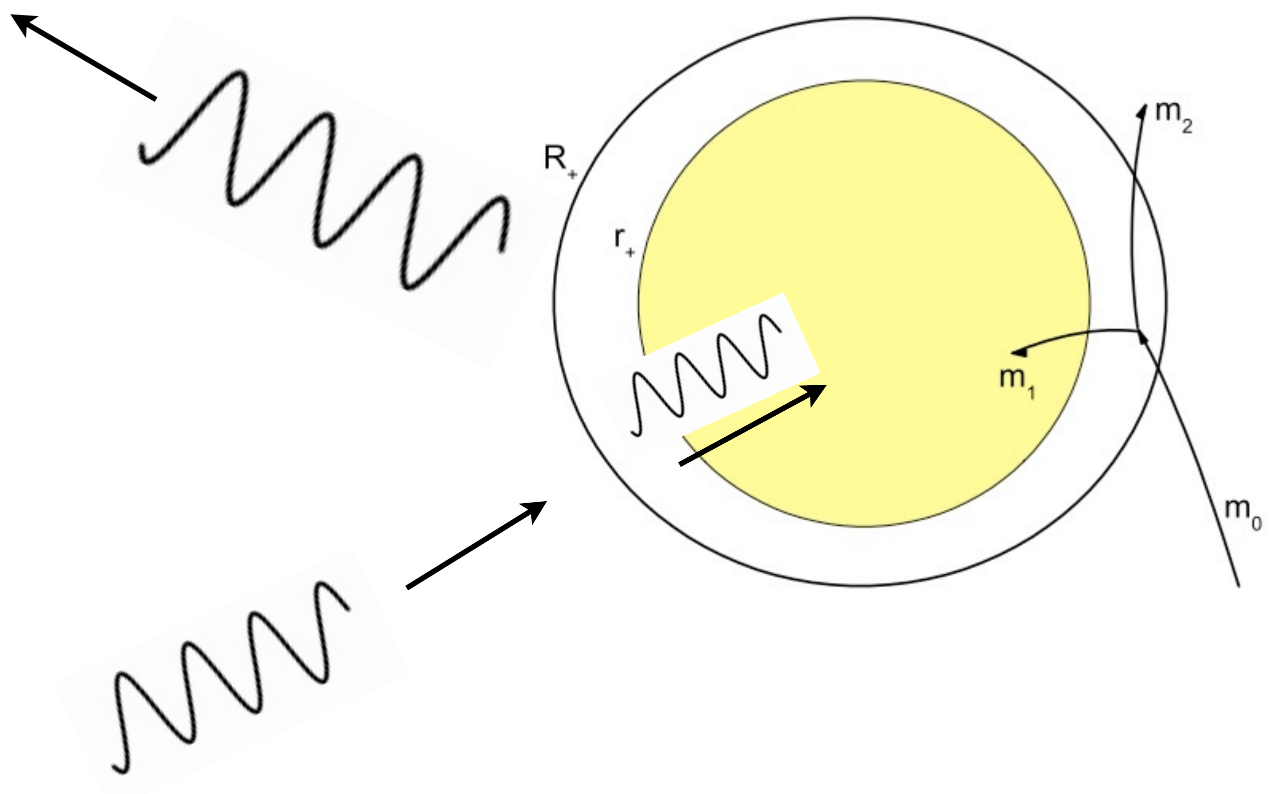
Superradiance

- The same mechanism works if instead of a particle we use a wave-packet.

- The pulse is amplified provided its frequency is: $\omega < m\Omega_{\rm H}$

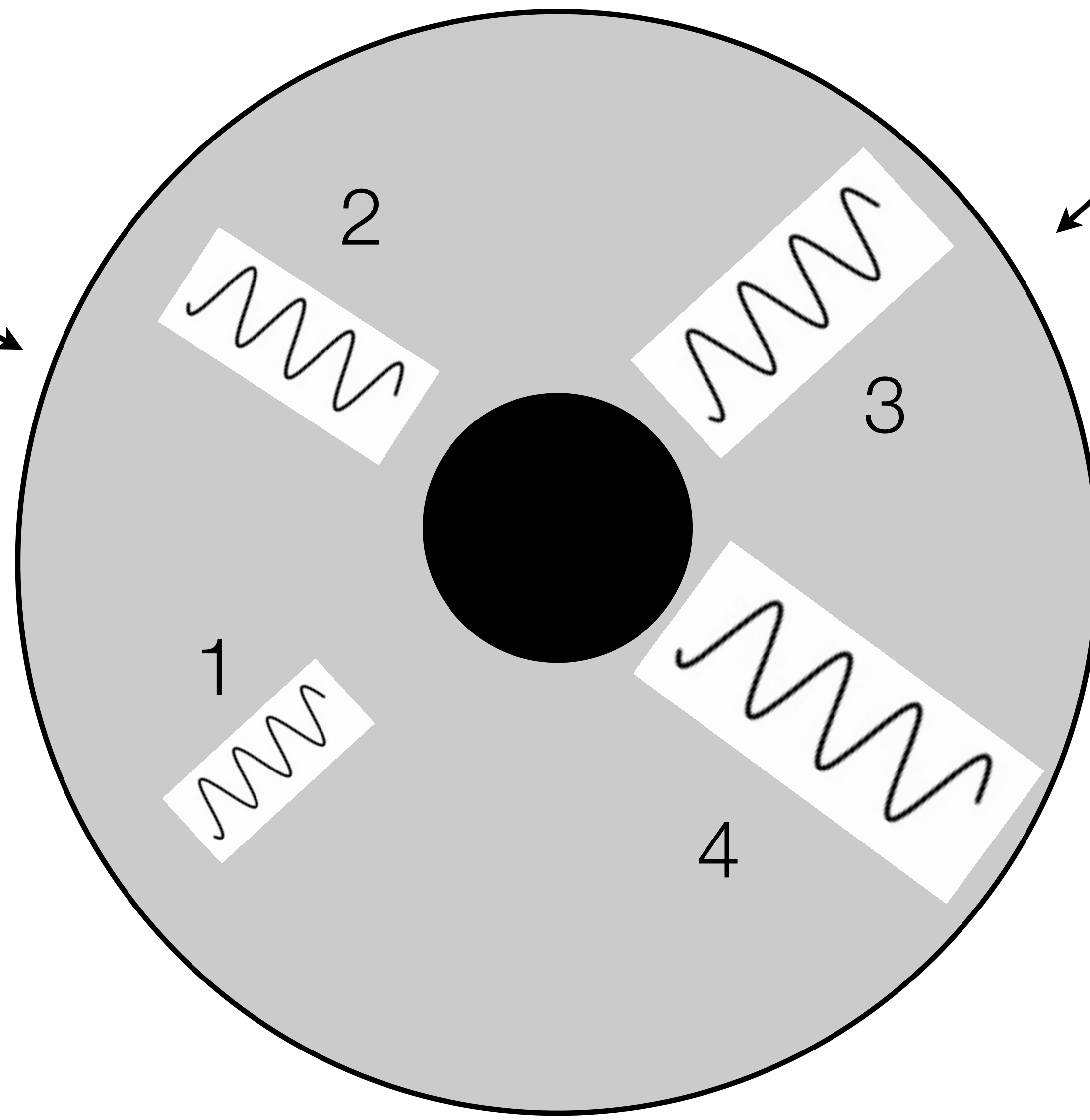
where $\Omega_{\rm H} = \dfrac{a}{2Mr_{\rm H}}$ is the horizon angular frequency and m the

azimuthal multipole number.



The black hole "bomb"

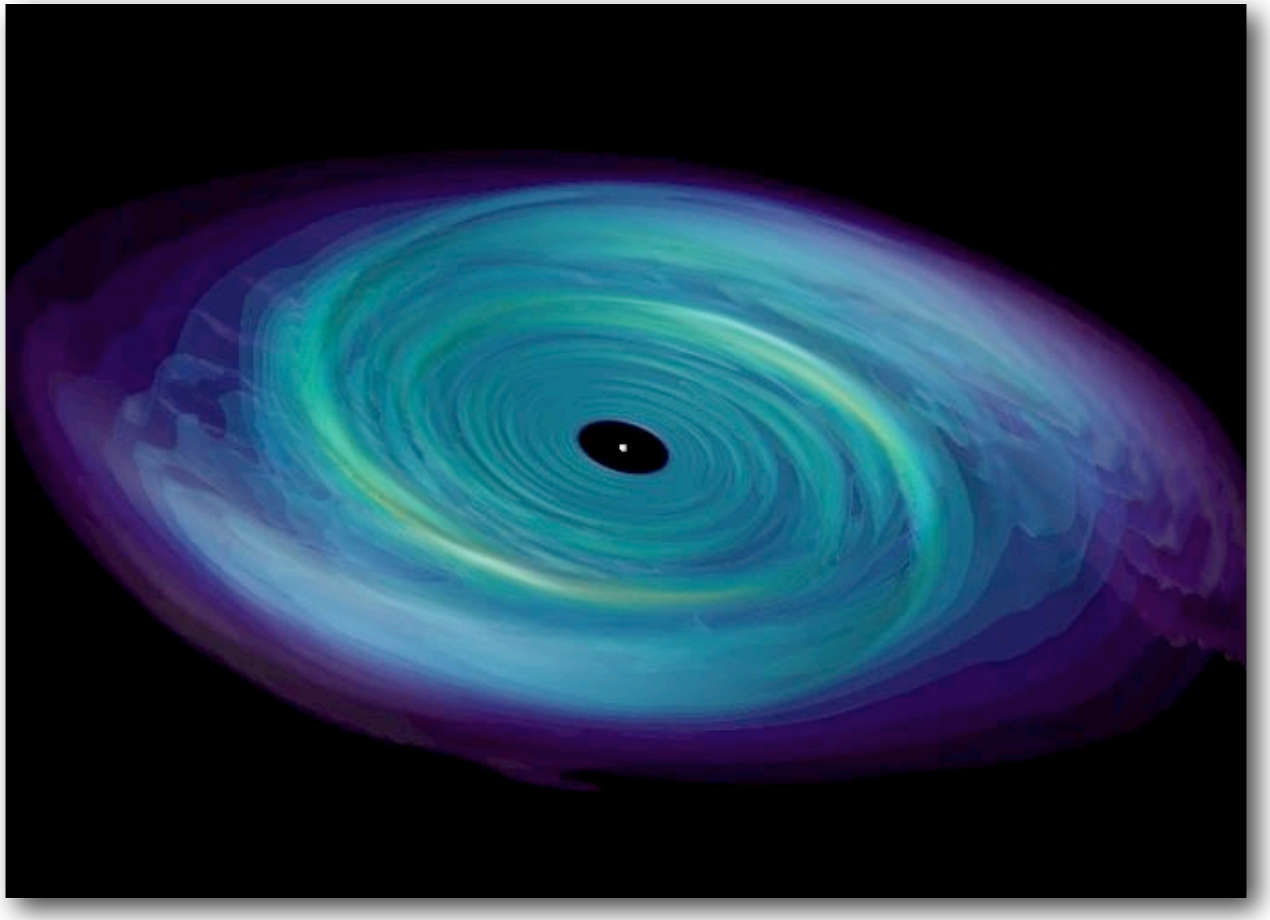
The amplified pulse is reflected back. The amplification goes on until the BH spin = 0



The BH is surrounded by a mirror surface

Part IV

Astrophysical black holes

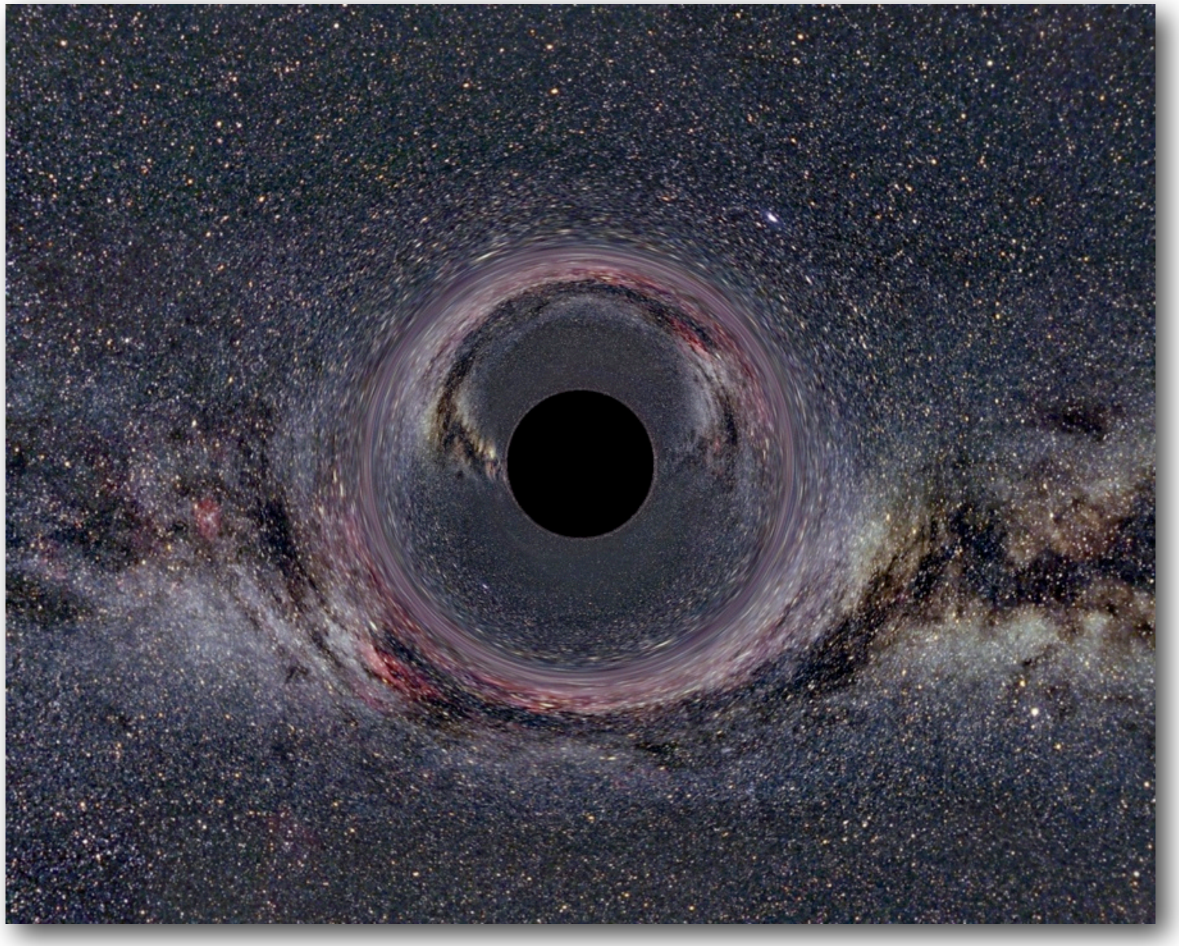


Do black holes exist?

Find the BH in here!

If we can't see BHs, how can we prove their existence?

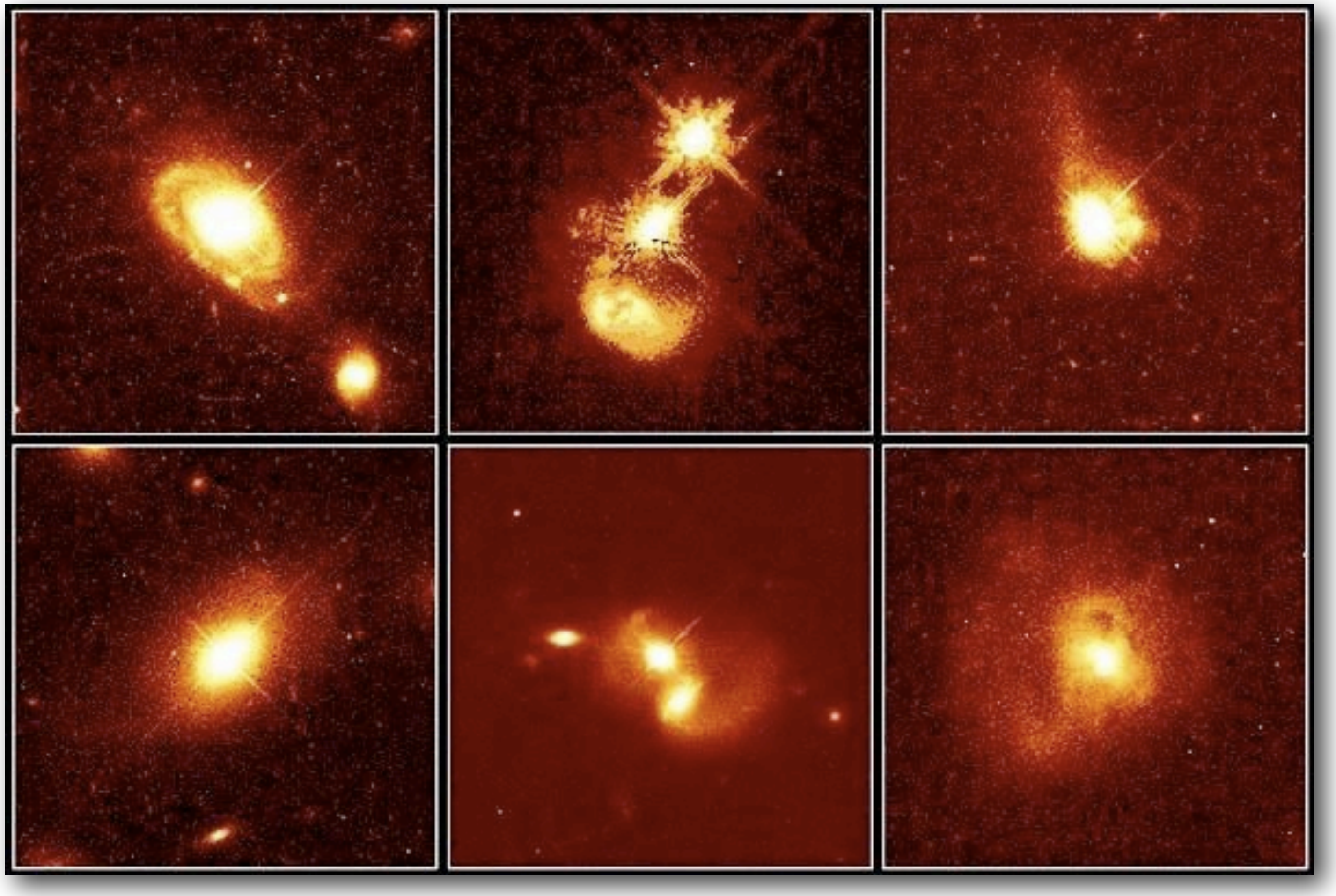
Do black holes exist?



Now add matter/radiation near a BH.
BHs can be “seen” by studying the motion of matter and radiation in their vicinity.

Quasars: the most powerful engines

- Quasars were first observed in the late 1950s and were recognized to be objects with a very high redshift in 1962
- It was a matter of some debate how an object could be sufficiently luminous to be seen at the great distance implied by their high redshift.
- By the early 1970s the basic model of energy production through an accretion disk around a supermassive black hole was widely accepted.

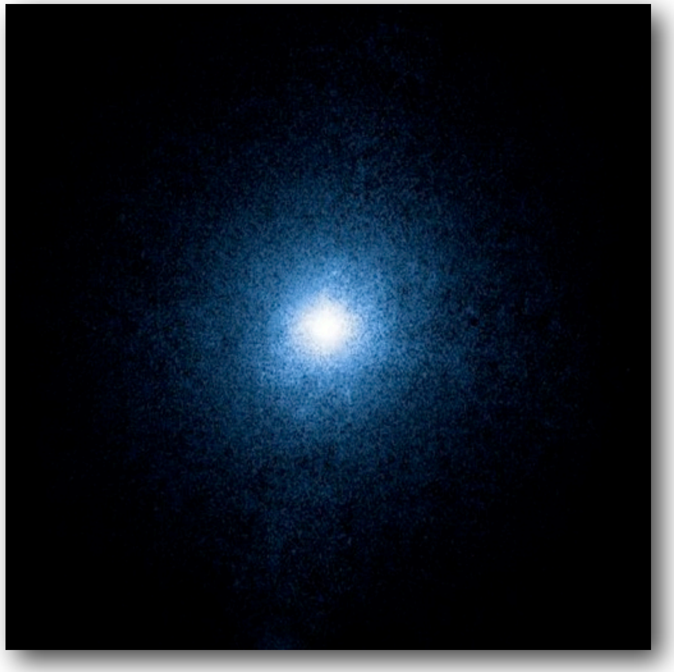


Quasars as seen by the HST

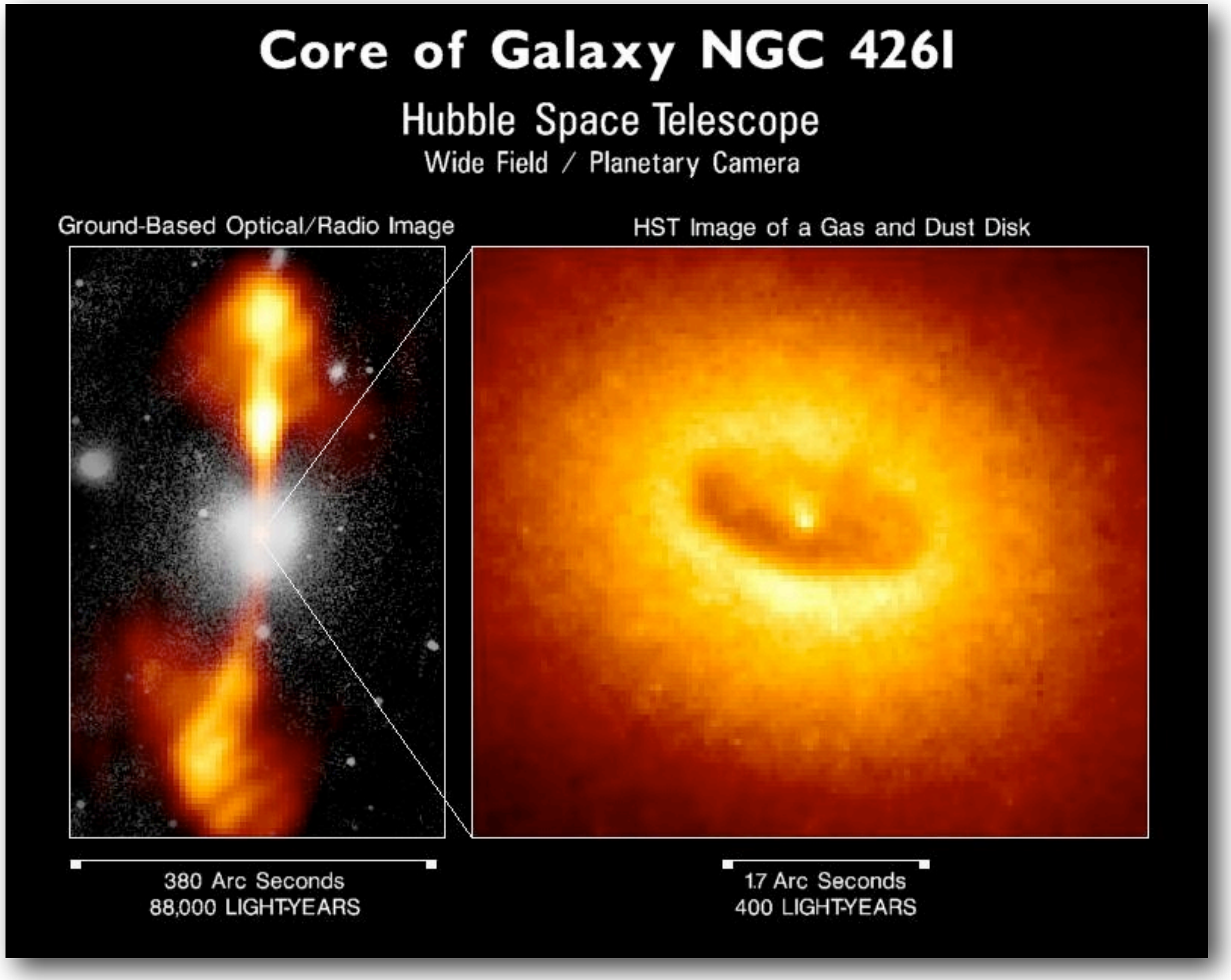
Black Holes of all sizes

- Most galaxies harbor a supermassive BH ($M \sim 10^6 - 10^9\, M_\odot$) at their centers.

- Lighter ($M \sim 10 M_\odot$) accreting BHs are also known to exist.

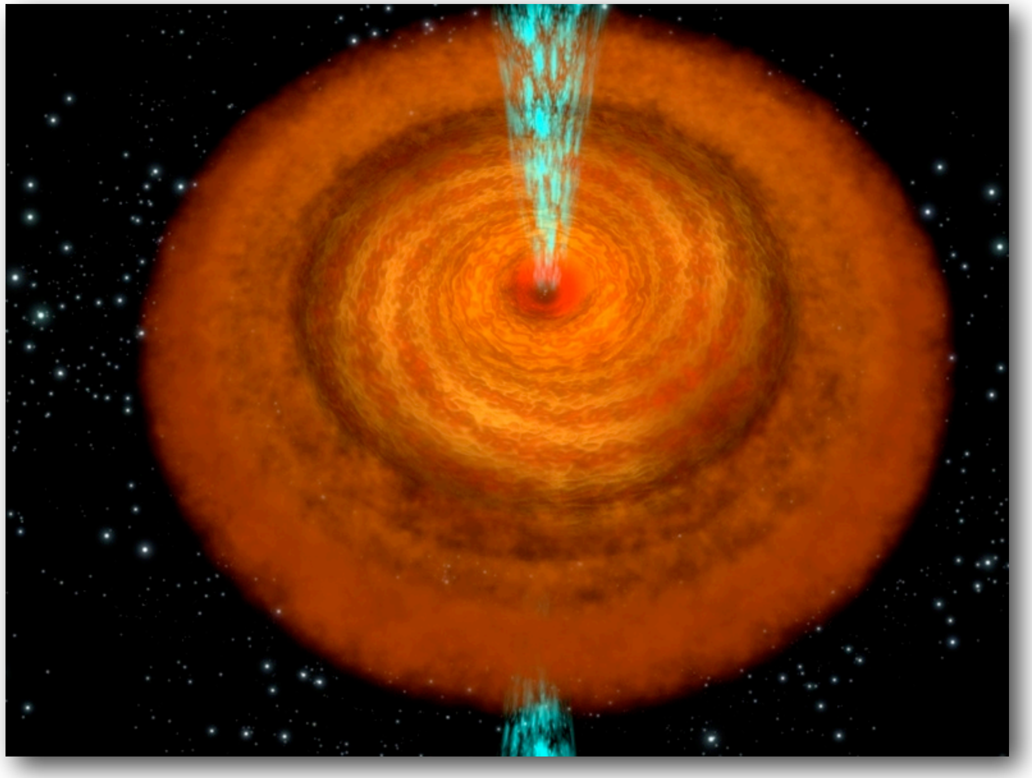


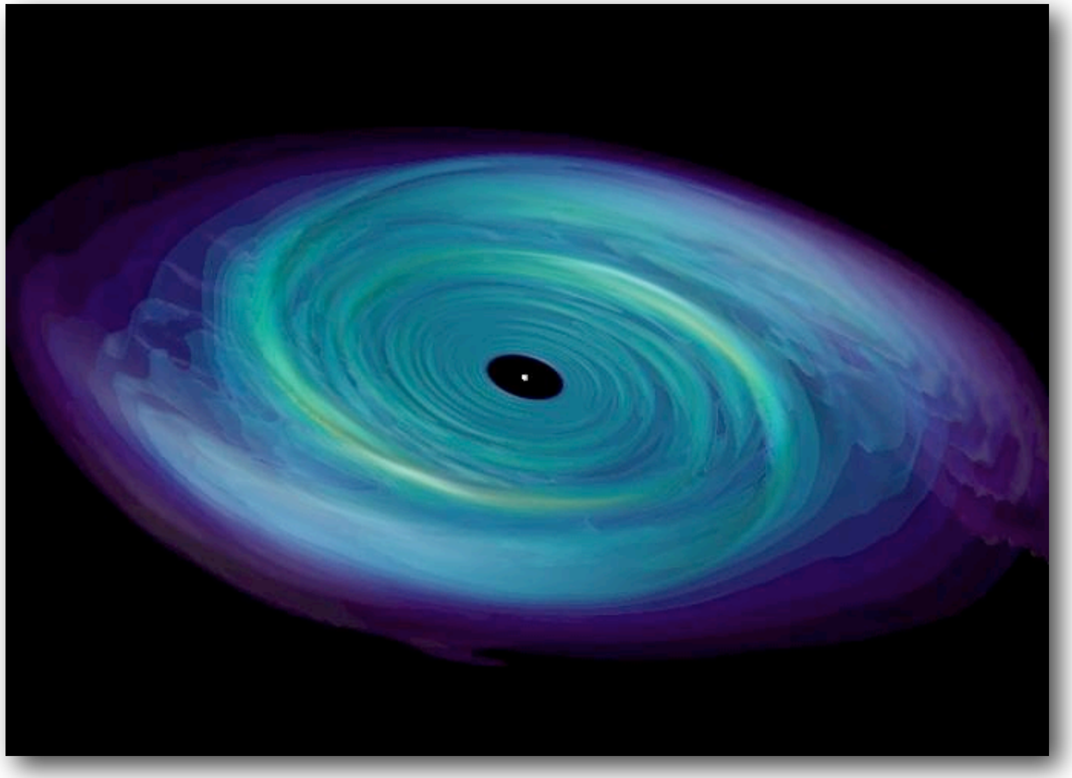
Cygnus X-1



The power of accretion (I)

- As matter falls into a black hole, enormous amounts of heat are often generated. Imagine freewheeling down the longest, steepest hill in the Universe to get an idea of the kinetic energy created. This hot matter accreting onto the black hole generates highly energetic radiation which we can observe.

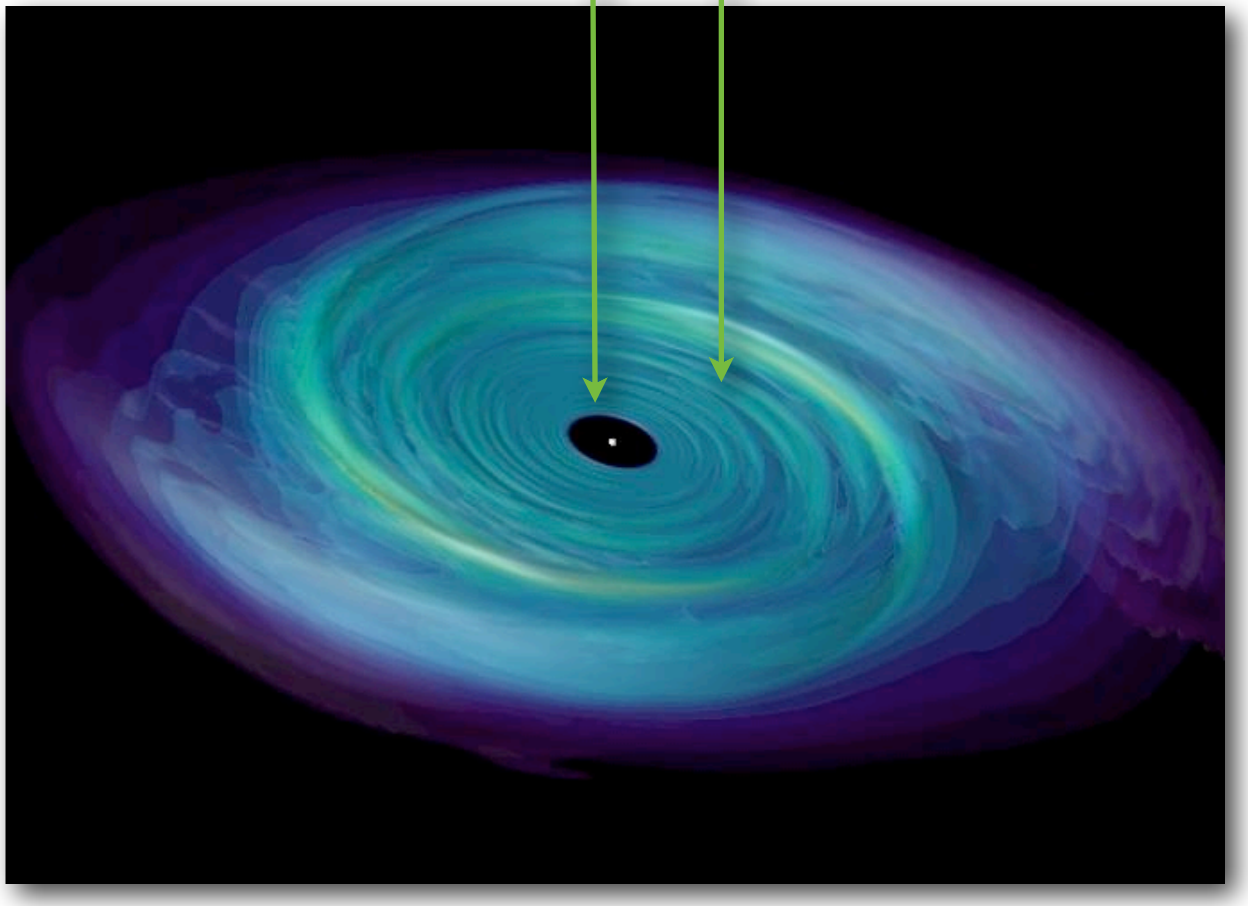




The power of accretion (II)

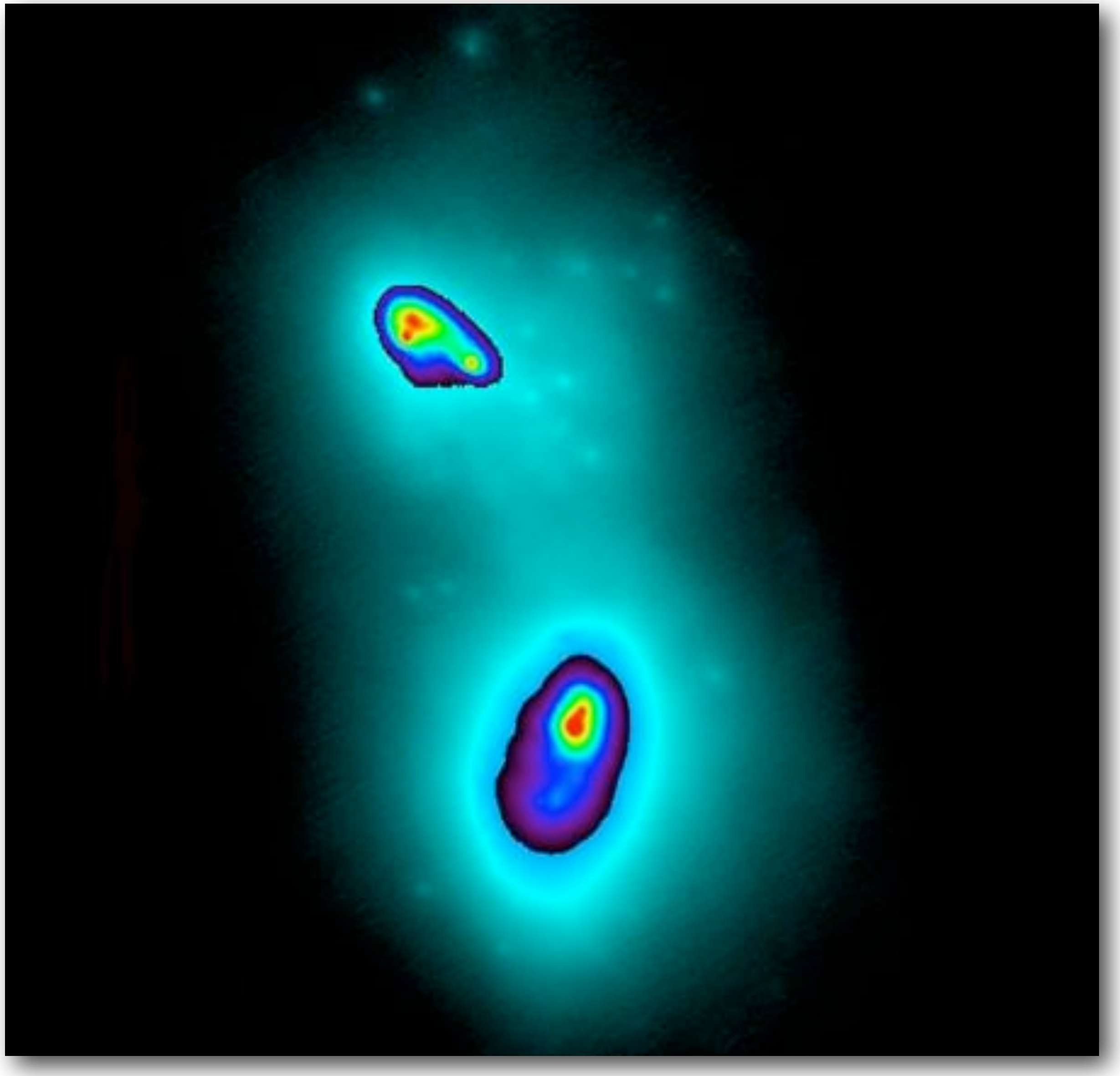
- Models for accretion discs are based on the notion of fluid elements (gas) moving in quasi-circular geodesics plus a slow inspiral as a result of loosing energy to heat.

- The disc is truncated at the innermost stable circular orbit ($r = 6M$ in Schwarzschild).



Merging black holes

- Galaxies frequently merge, so very likely the two black holes at their centers end up merging too.
- This is one way of producing supermassive black holes. An alternative mechanism is accretion.

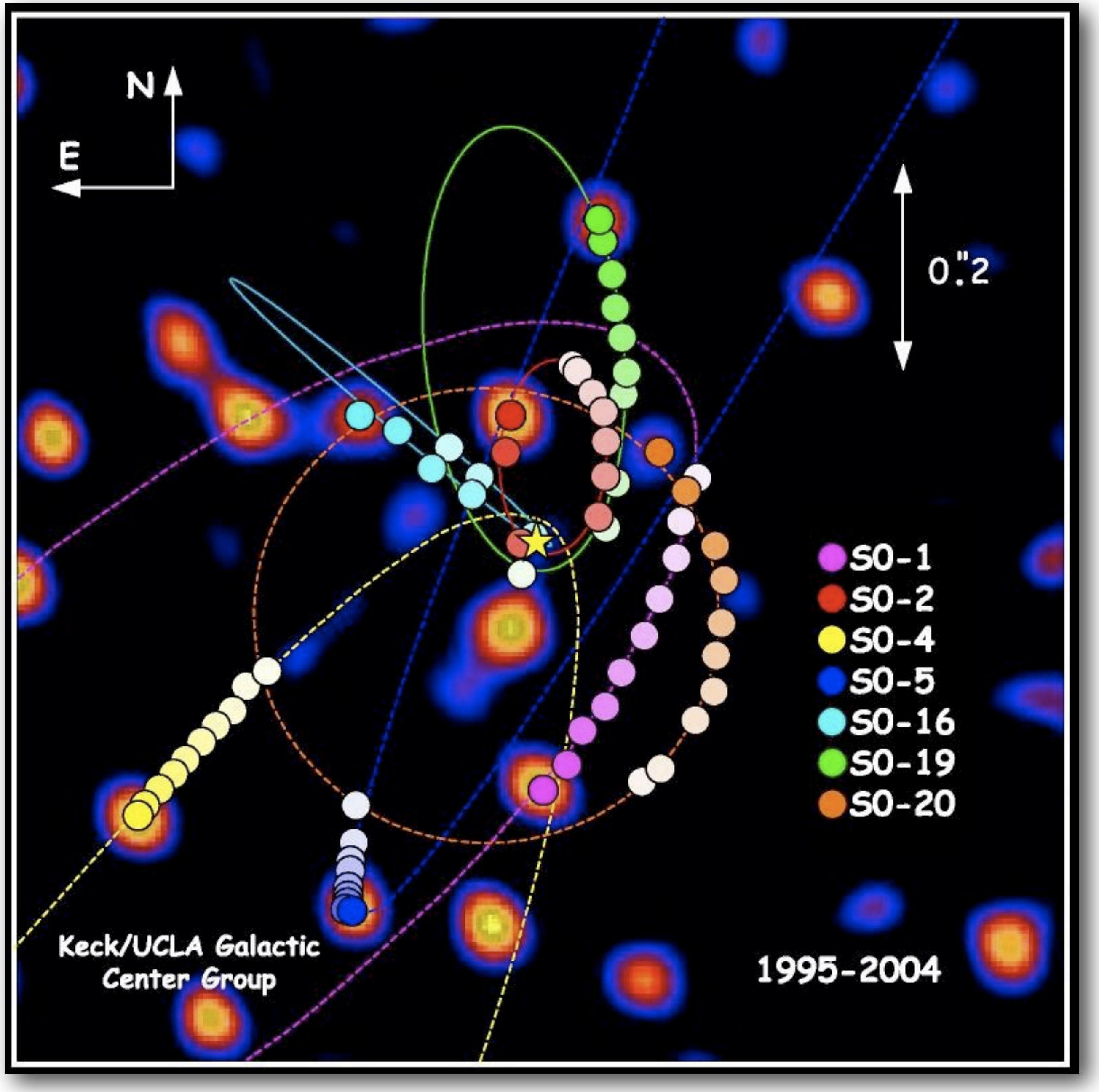


Evidence for merging BHs in NGC 6240

Our Galactic center Black Hole

- In the last decade or so amazing progress has been made in showing that SMBHs do exist at the hearts of nearby galaxies and in measuring the masses of these objects.

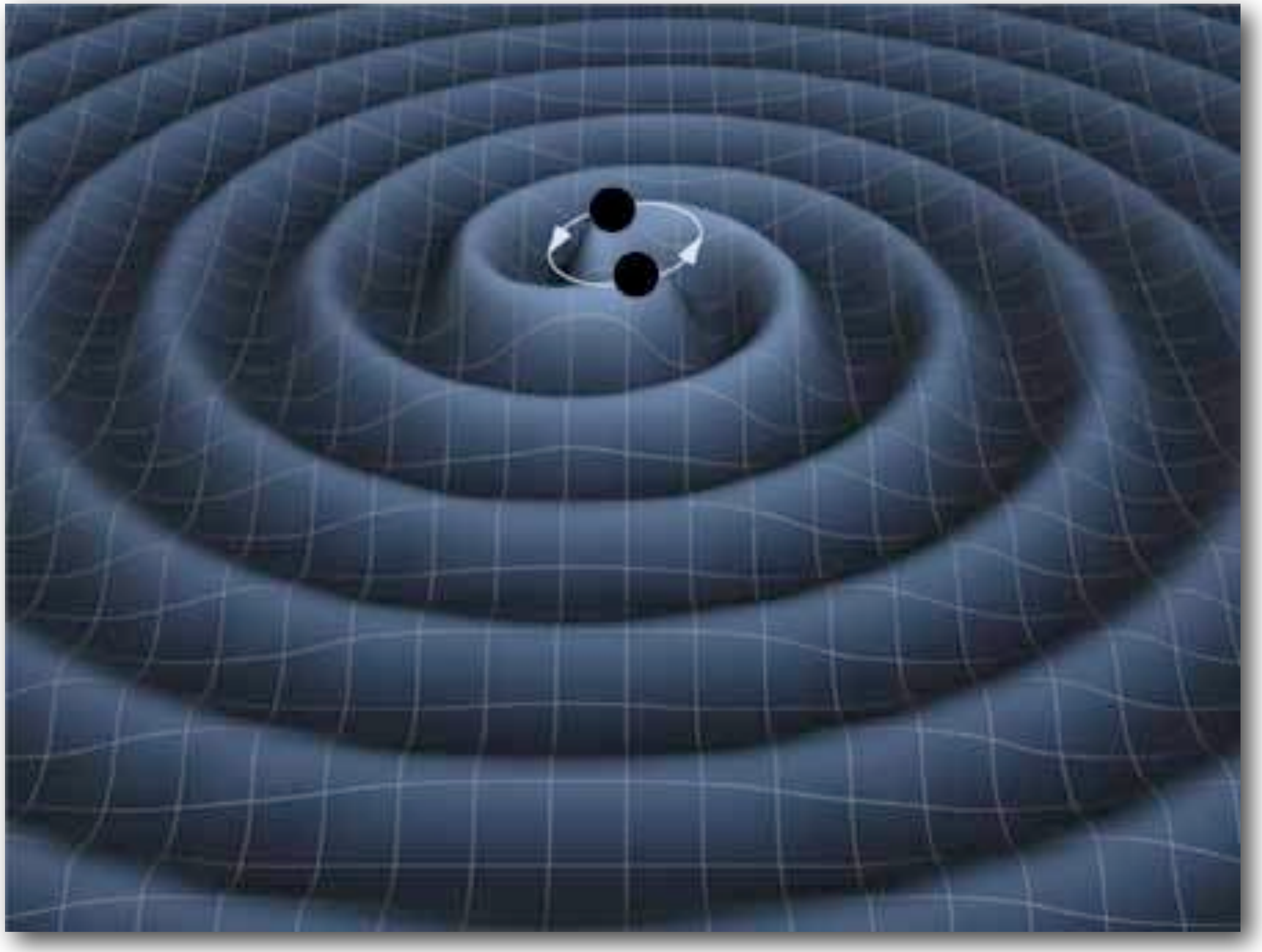
- At right is a depiction of the observations of stellar motion which have permitted the measurement of the mass of the SMBH at the heart of our own Milky Way.



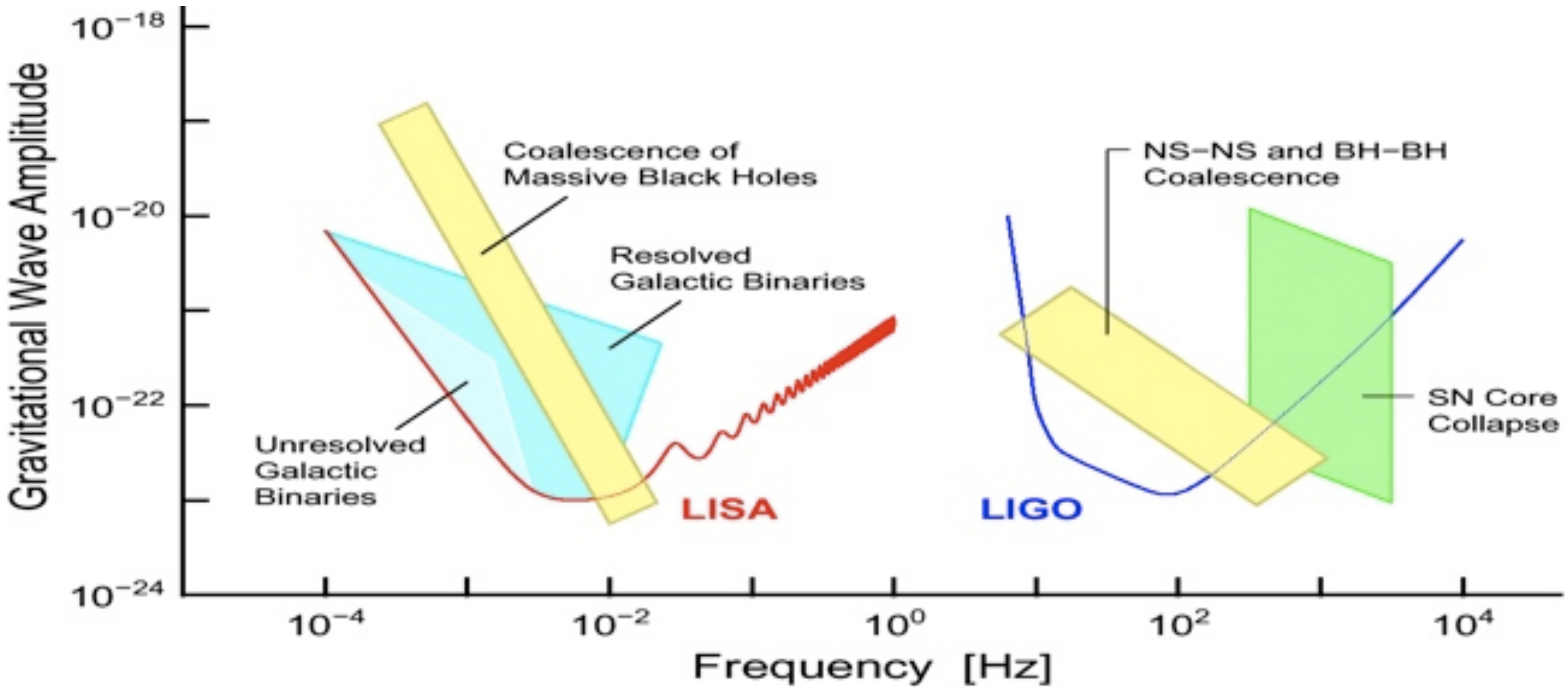
The orbits of the “S-stars” around the SMBH in Sgr A.

Part V

Black holes as sources of gravitational waves

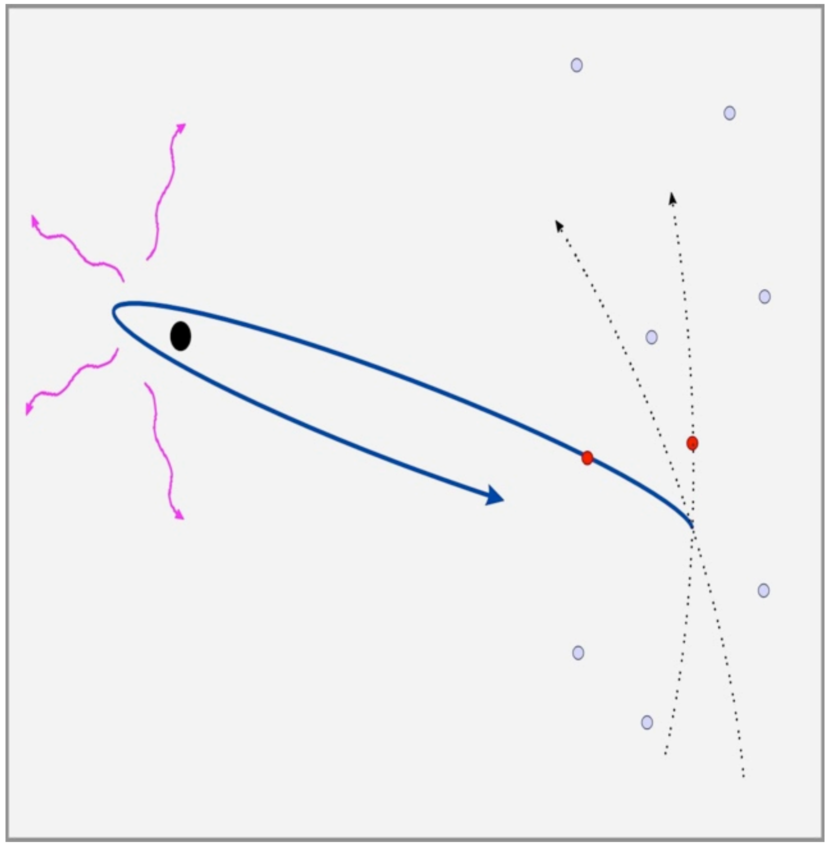


Gravitational waves from Black Holes



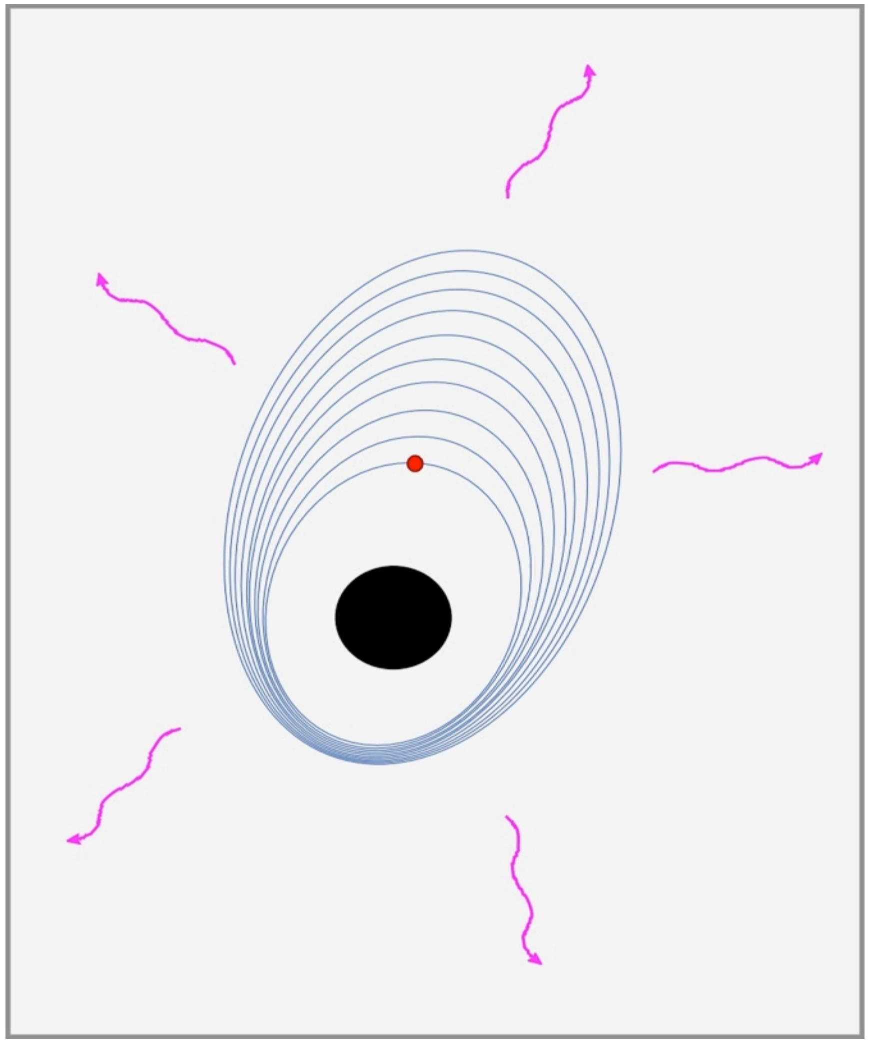
SMBHs are low-frequency GW sources, suitable for LISA. Merging SMBHs can be detected 'anywhere' in the Universe, and LISA will be "listening" all these events simultaneously!

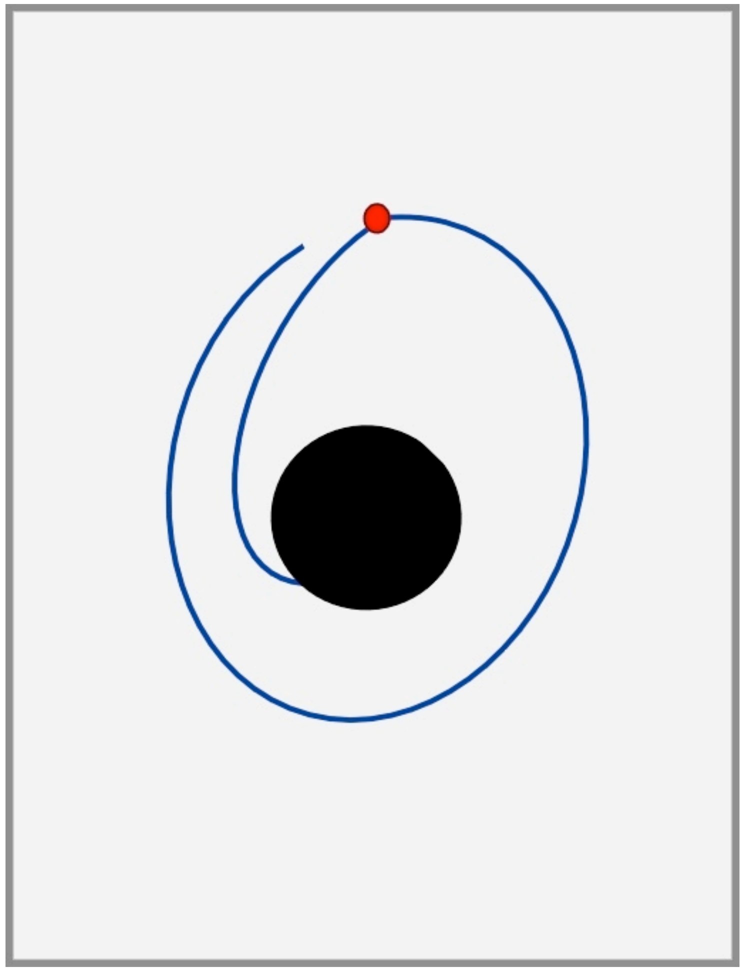
A binary BH system (extreme mass ratio)



1. Capture in bound orbit

2. Inspiral and emission of GWs

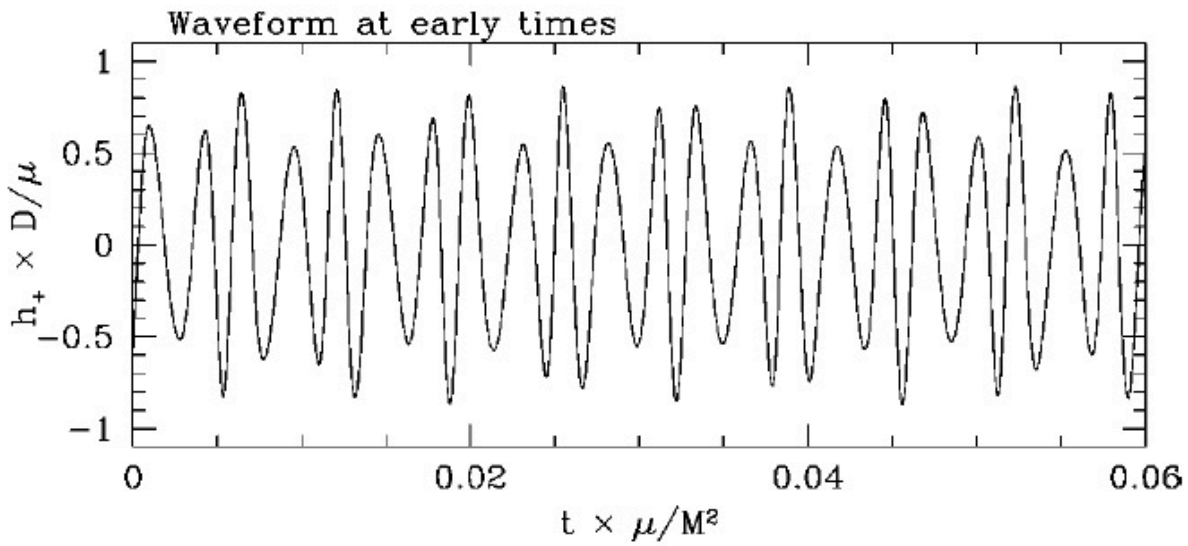


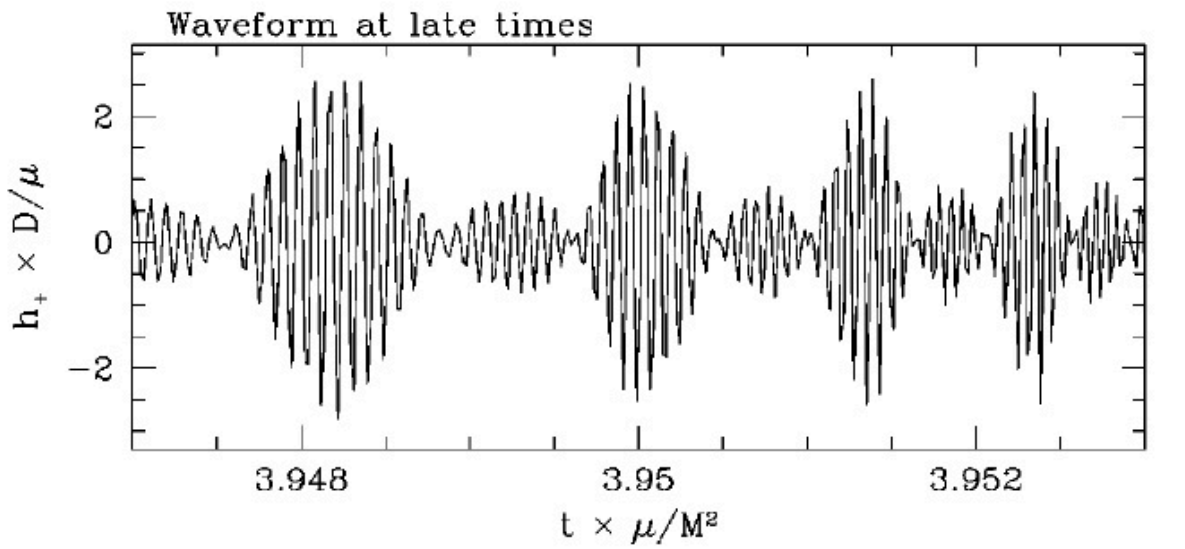


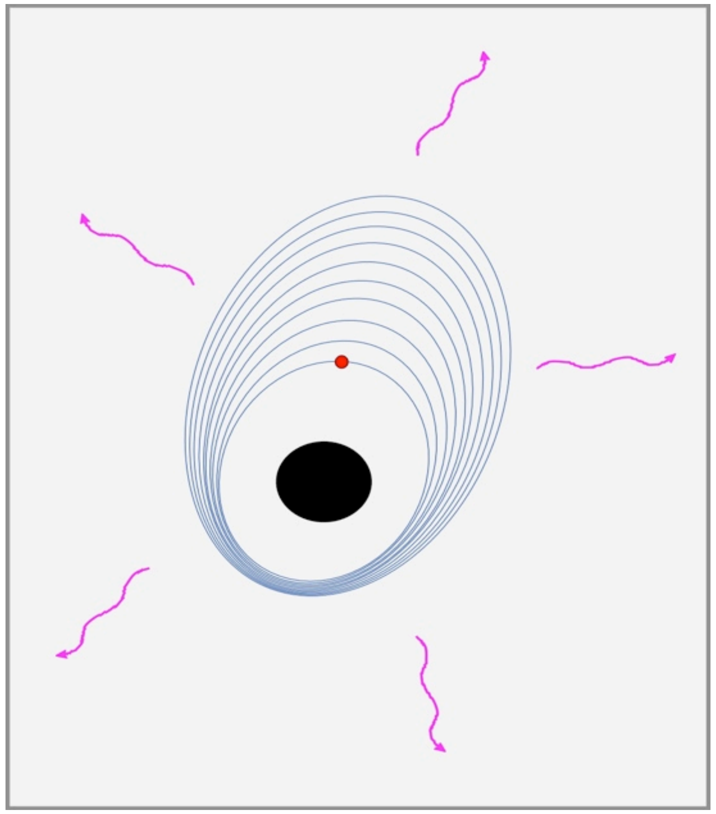
3. Last stable orbit and final plunge.

The gravitational waveform

Observation of gravitational waves from small BHs orbiting around supermassive ones should provide high-precision evidence for the Kerr spacetime.

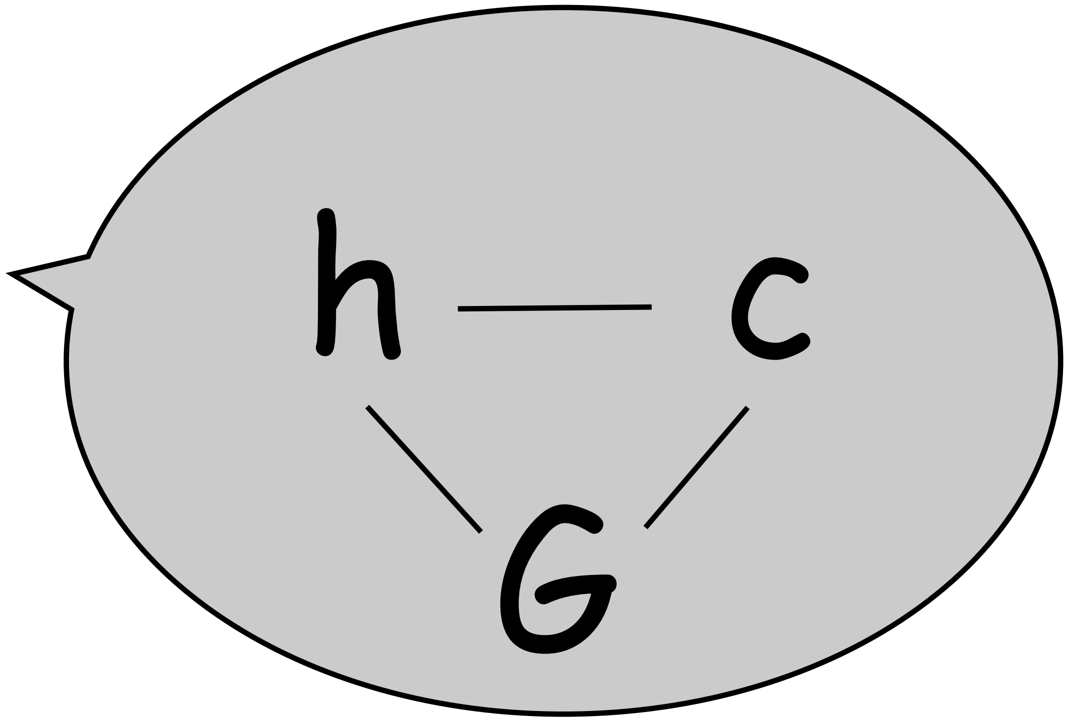






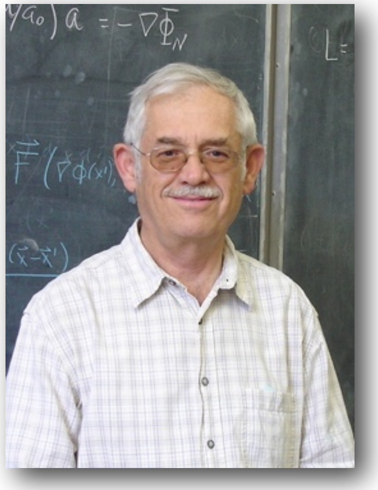
Part VI

Black holes
&
Quantum
physics



Black hole thermodynamics

- The surface area of a BH horizon has the strange property of always increasing (or remaining unchanged) during a physical process (for instance the Penrose energy extraction).



- J. Bekenstein was the first to associate the horizon area with the concept of entropy:

$$S_{\mathrm{BH}} = \frac{c}{4\hbar} k_{\mathrm{B}} A_{\mathrm{H}}$$

- Similarly, a black body temperature can be assigned to a BH. For a non-rotating BH this is:

$$T_{\mathrm{BH}} = \frac{\hbar c^3}{8\pi k_{\mathrm{B}} G M} \approx 6.2 \times 10^{-8} \left(\frac{M_\odot}{M} \right) \mathrm{K}$$

- It turns out that BH obey the usual 3 laws of thermodynamics.

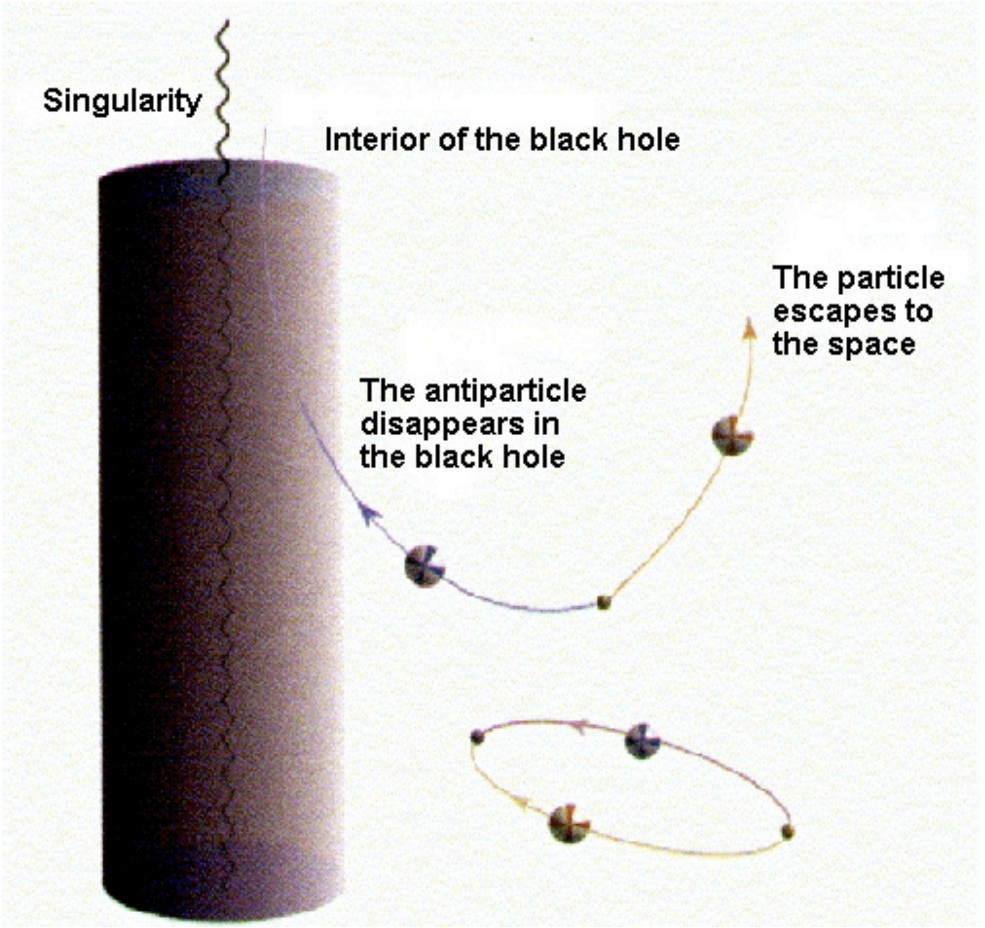
Hawking radiation

- A body with finite temperature should emit radiation. If the BH temperature is “real” then BHs should also emit radiation.

- How is this possible if nothing can escape from them? The famous work by S. Hawking (1974) provided the answer to this.

- Quantum vacuum fluctuations (Heisenberg’s principle) in a BH spacetime produce the desired radiation.

- BH thermodynamics is a remarkable interface between gravitational and quantum physics.



Epilogue: analogies

"*...and I cherish more than anything else the Analogies, my most trustworthy masters.*"

– Johannes Kepler

It wouldn't be too surprising if your mental picture of BHs looks like this picture.

How far can we push this analogy?

Black holes in your bathtub!

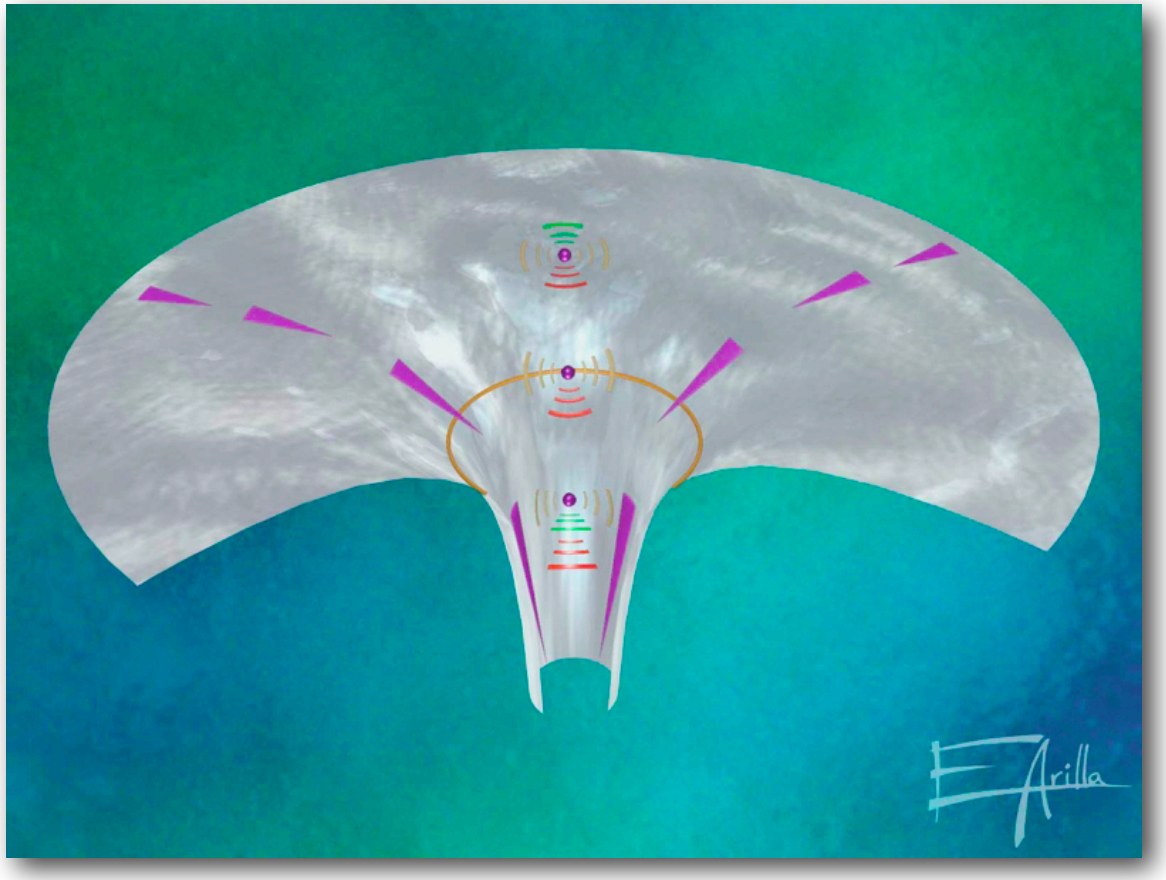
- Remarkably, a fluid flow like the one in a draining bathtub can be mathematically described in terms of an **effective "acoustic" metric** (Unruh 1981). This looks like:

$$ds^2 = -c_s^2 dt^2 + \left(dr - \frac{A}{r}dt\right)^2 + \left(r d\varphi - \frac{B}{r}dt\right)^2$$

(The sound speed plays the role of c)

- Sound waves propagating in such flow can be trapped behind an "acoustic horizon" and/or frame-dragged (for this to happen $v_{\rm flow} > c_s$)

$$r_{\rm H} = \frac{|A|}{c_s}, \qquad r_{\rm erg} = \frac{\sqrt{A^2+B^2}}{c_s}$$



acoustic BH (or "dumb hole")