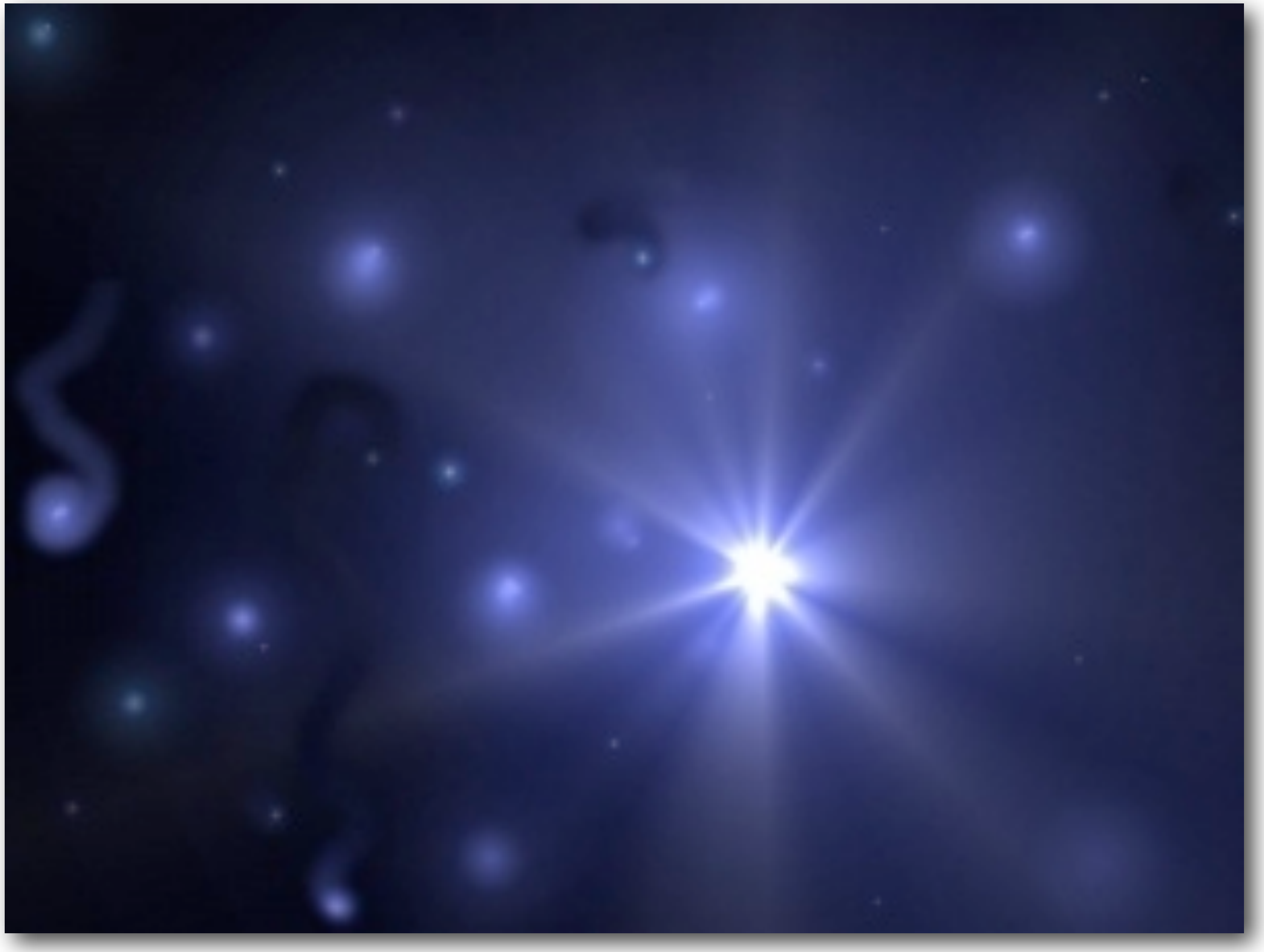
The Accelerating Universe

An Introduction to Cosmology



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Outline of this lecture

- This lecture is an introduction to modern cosmology, as described by the theory of General Relativity.
- The focus is mainly on the gravitational physics of cosmology rather than its particle physics aspect.
- The first part of the lecture is devoted to the review of the various phases of cosmic evolution and the discussion of some key observations like the recession of galaxies (Hubble's law) and the cosmic microwave background radiation.
- The second part of the lecture introduces the key ingredients for building a cosmological model: the cosmological principle, the FLRW spacetime and the Friedmann equations. We discuss the propagation of light and work out the most important cosmological models.
- The third part of the lecture is a discussion of a remarkable property of our universe, namely, its accelerated expansion, believed to be driven by a "dark energy". In this part we also make contact with the idea of inflation and the elusive dark matter.

What is Cosmology?

- Modern Cosmology was born in the aftermath of the formulation of Einstein's General Relativity theory (1915).

- Some general things about Cosmology:

✓ It is the scientific study of the *large-scale structure* of our Universe.

✓ It studies the very distant past, the origin of the Universe, its future evolution and the average distribution of matter (galaxies) and radiation.

✓ The *main player is gravity*, but one also needs quantum physics, properties of matter and radiation, etcetera.

✓ *Modern cosmology is a mixture of well understood physics and some quite uncertain physics that lie at the edge of our knowledge.*

Enter Einstein

- Einstein's General Theory of Relativity views gravity as spacetime curvature.

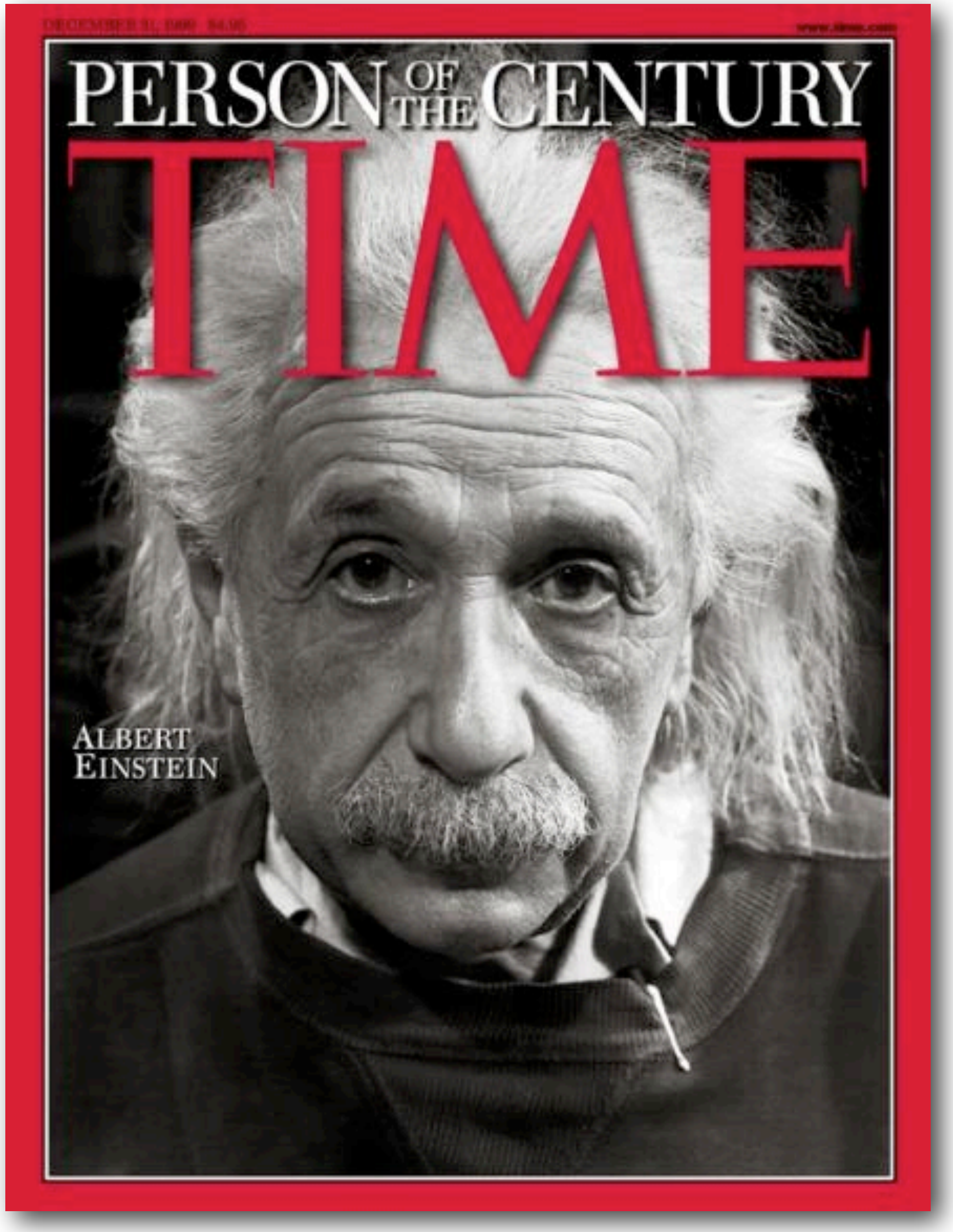
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

- A famous textbook description:

space tells matter how to move ....
and matter tells space how to curve

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

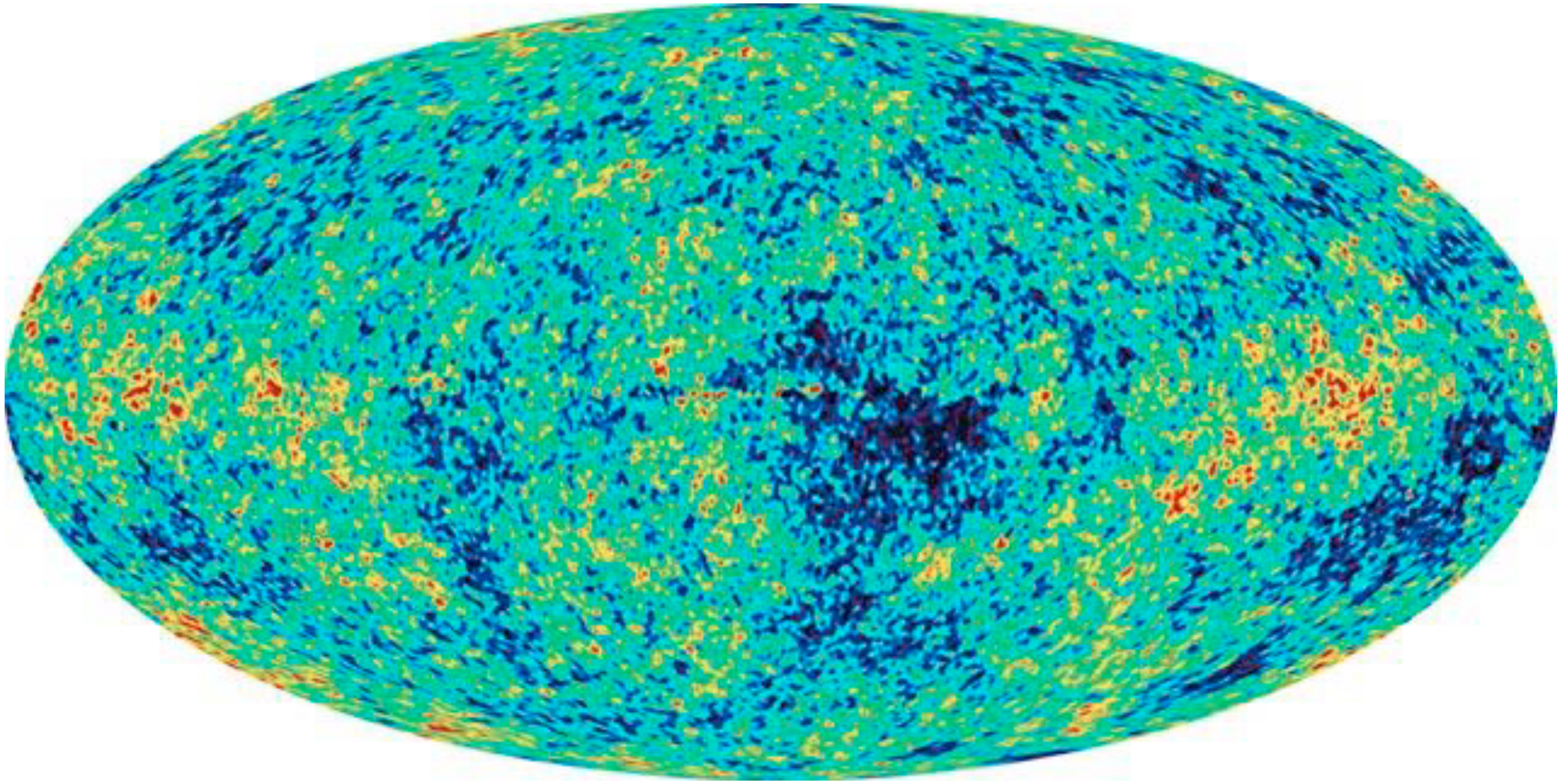
geometry　　matter/radiation



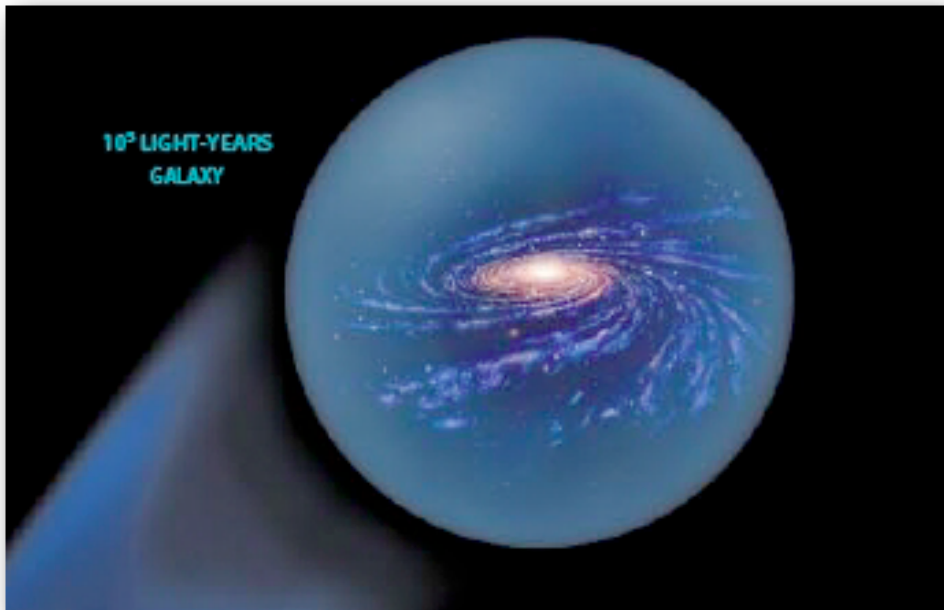
- This may look "simple" but in reality it is a system of 10 coupled partial differential equations.


$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$
LA COLMENA
CHILE
A. EINSTEIN

Part I
Preliminaries & Observations

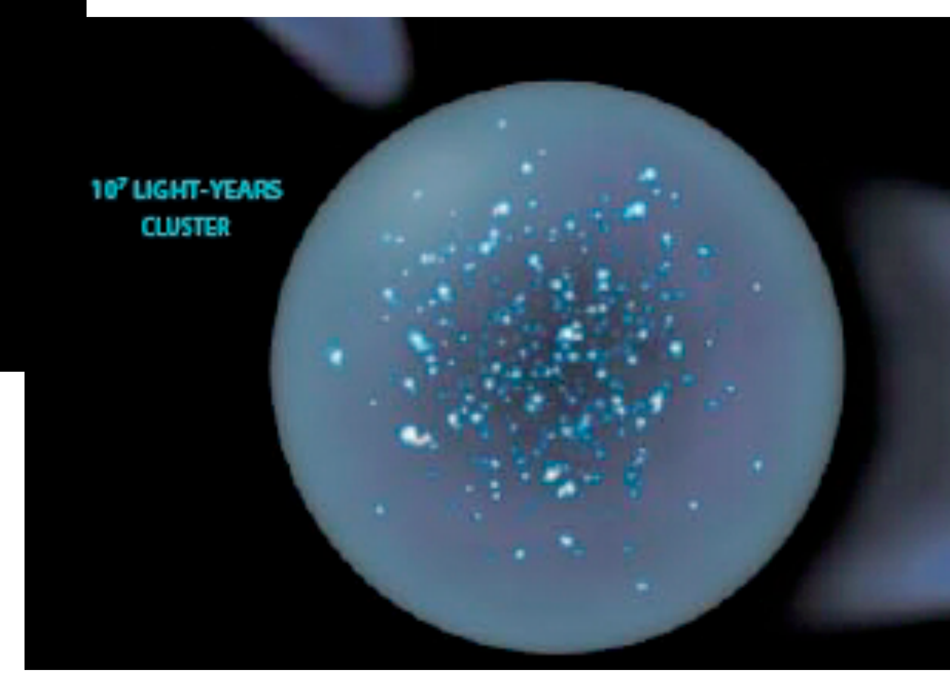


Cosmic length scales (I)



Galaxy: 10^5 light years

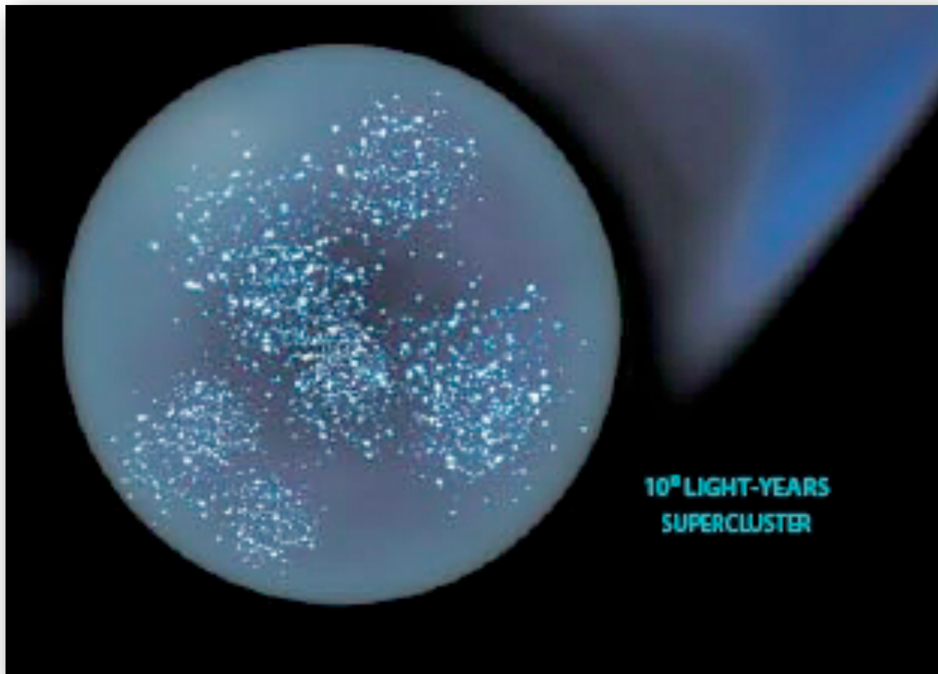
Group of galaxies: 10^6 light years



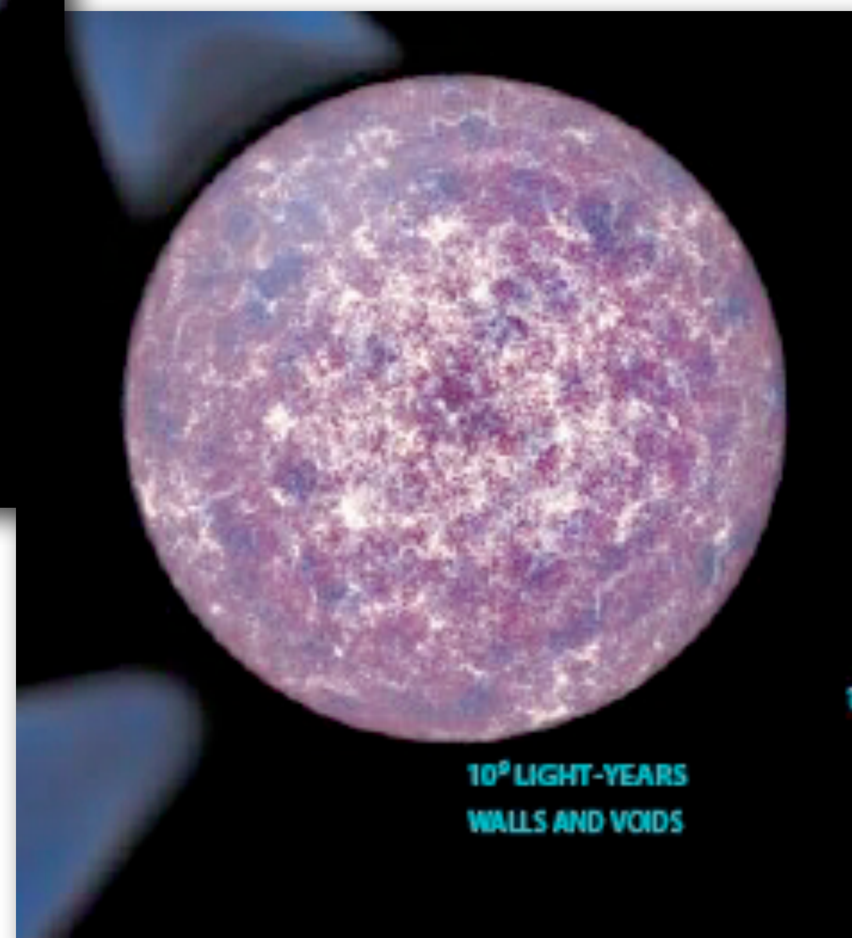
Distance Sun-Earth : 8 light *minutes*

Cluster of galaxies: 10^7 light years

Cosmic length scales (II)



Super-cluster: 10^8 light years



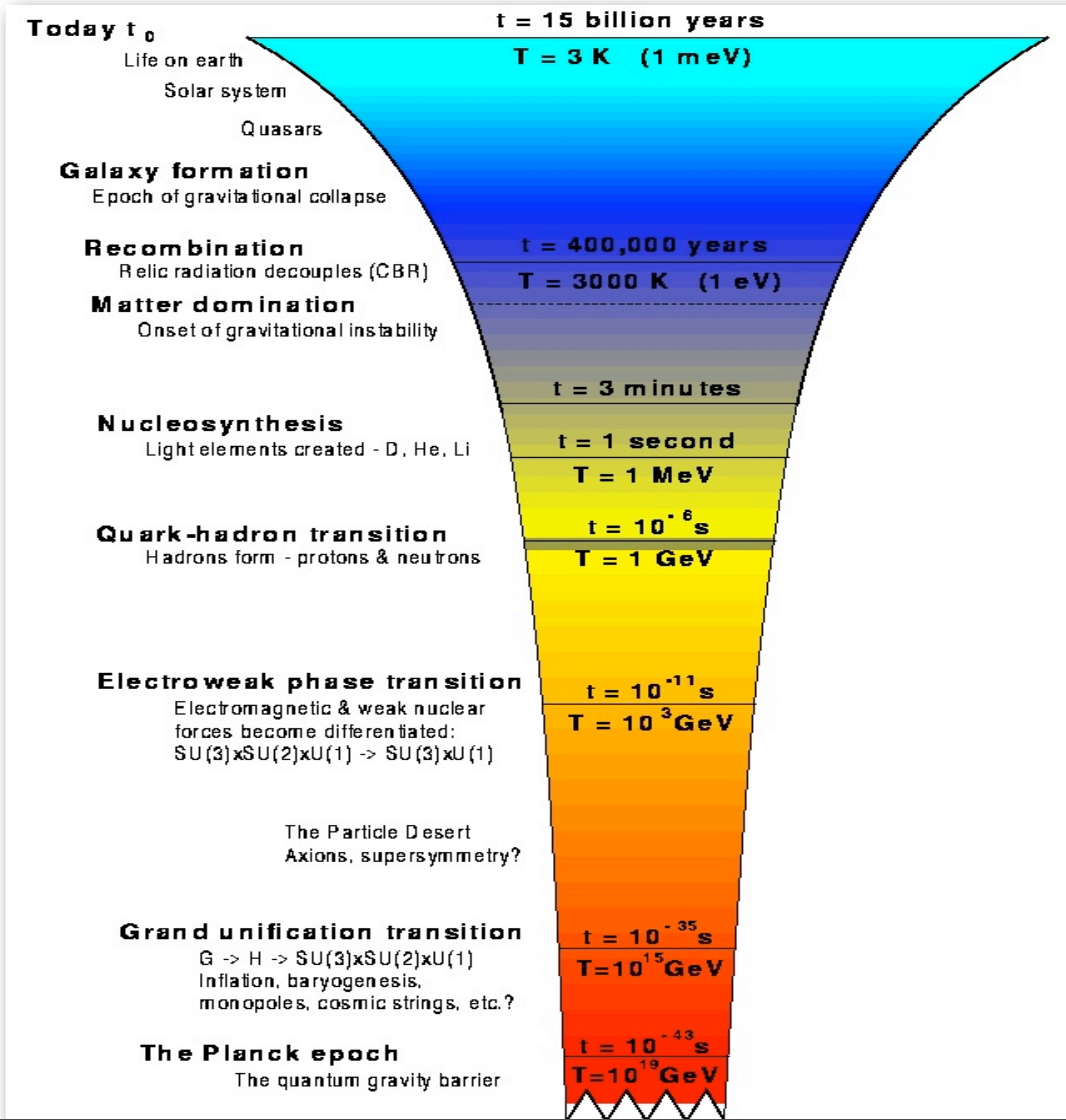
"Walls" and "Voids": 10^9 light years

Visible Universe: 10^{10} light years

Phases of the cosmic evolution

matter dominates

radiation dominates

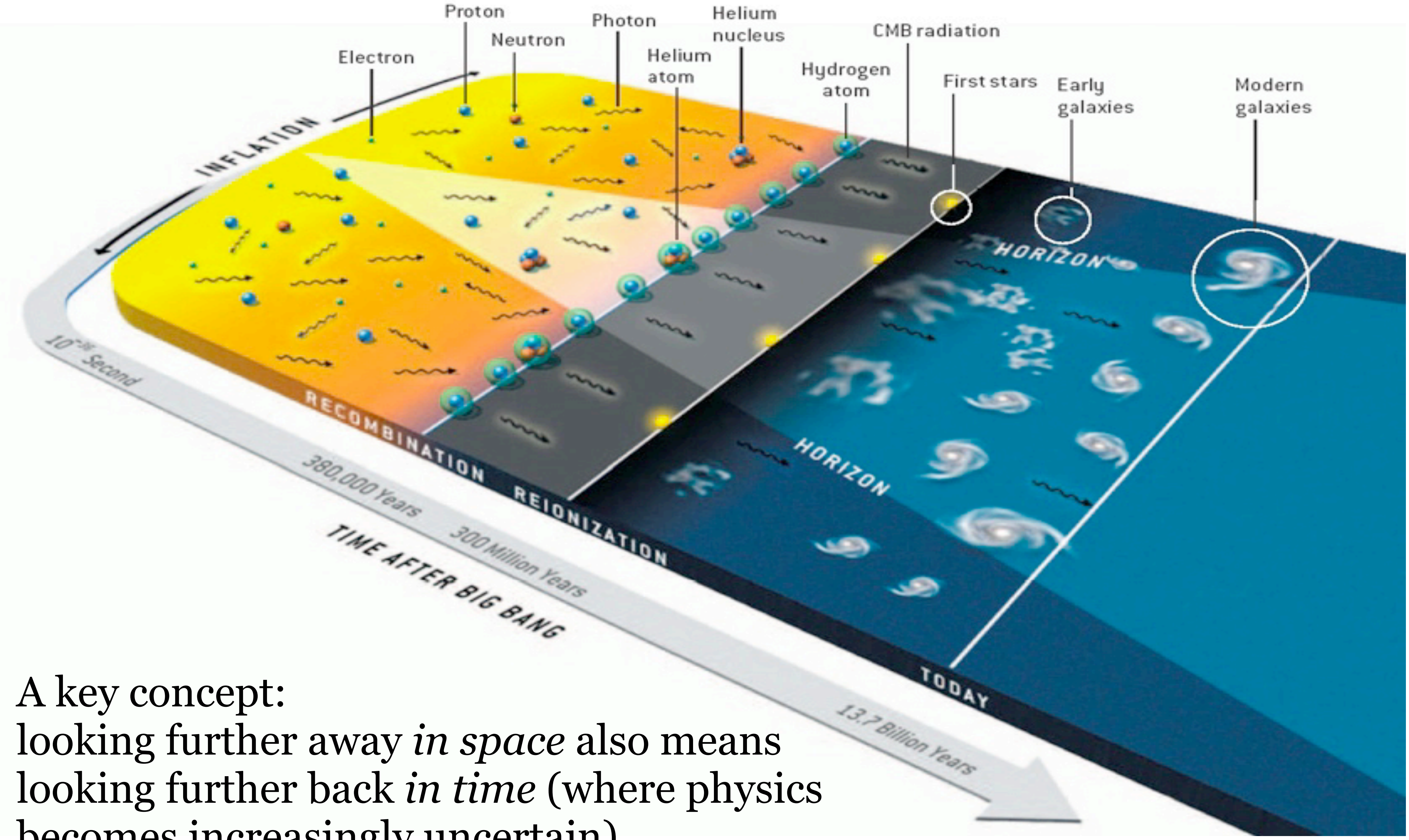


cosmic time

cosmic expansion

particle energy scale

Cosmic timeline



A key concept:
looking further away *in space* also means looking further back *in time* (where physics becomes increasingly uncertain)

Main predictions of the “Big Bang” model

- *Cosmic Expansion:*

  The model predicts large scale motion as a result of the expanding spacetime. Observationally, this is manifested as the Hubble recession of galaxies.

- *Nucleosynthesis (BBN)*

  It took place in the first 3-4 minutes, mostly creating hydrogen and helium. The *relative abundances* of the produced light elements can be calculated. The theoretical predictions are in good agreement with observations.

- *Cosmic radiation microwave background (CMB):*

  The primordial electromagnetic radiation (photons) decoupled from matter when the first hydrogen atoms were formed, about 400.000 years after the big bang. The thermalised photon gas cooled as a consequence of cosmic expansion.

Cosmology: some *solid* facts

- Recession of galaxies (= cosmic expansion)
- Linear (approximately) Hubble's law.
- Cosmic microwave background.
- Galactic & stellar structure and evolution.
- "Big bang" (= as we go back in time the universe becomes increasingly hotter, denser and geometrically more curved).

Cosmology: *plausible* physics

- Accelerating expansion (cosmological constant, dark energy ...).
- Dark matter (invisible galactic matter component).
- Inflation (rapid expansion in the early Universe).

The “redshift” factor z

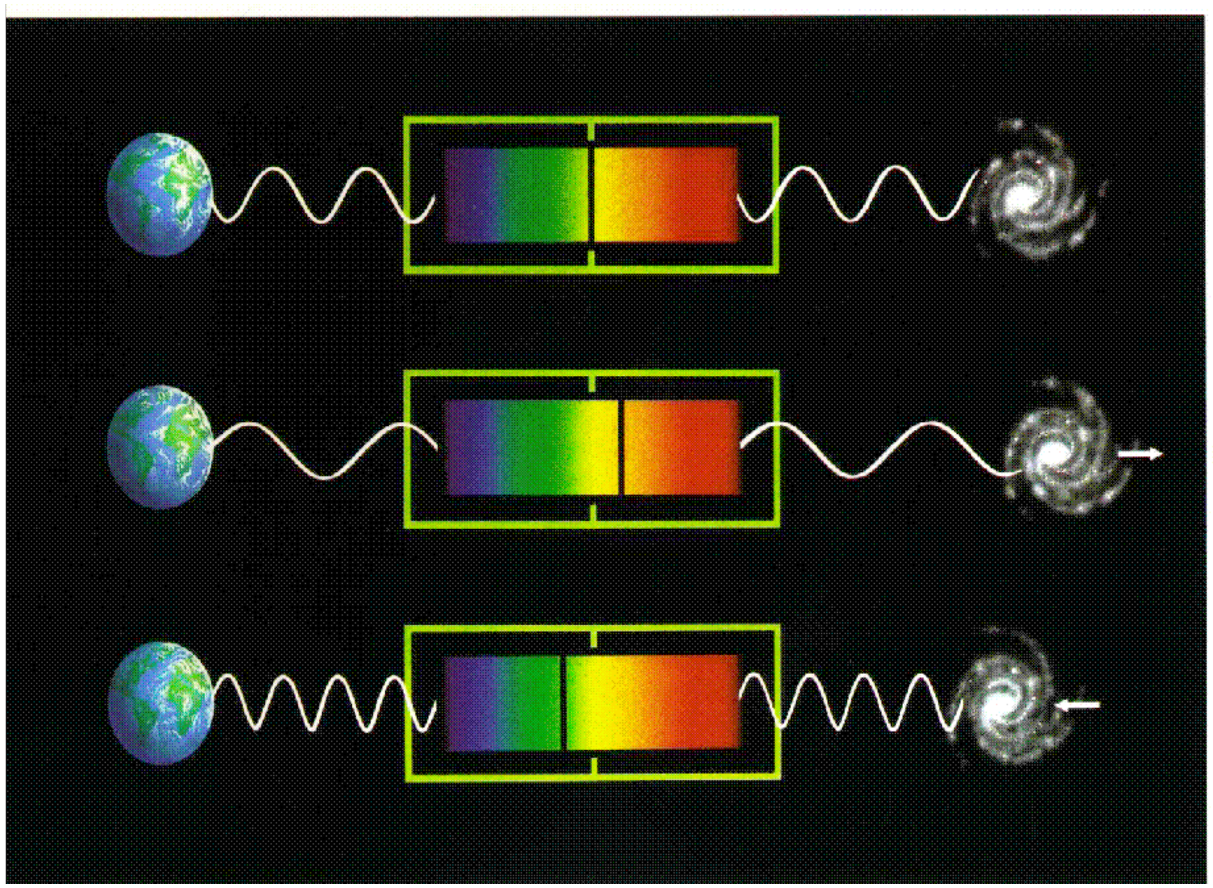
- Emitted radiation by a moving source (a galaxy) is *redshifted* (lower frequency) or *blueshifted* (higher frequency) with respect to the source’s rest frame.

- The “redshift factor” z is defined in terms of the *emitted & observed* wavelengths:

$$1 + z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm em}}$$

redshift: $z > 0$

blueshift: $z < 0$



- The source’s line-of-sight velocity is related with z by the Doppler formula:

$$v \approx cz$$

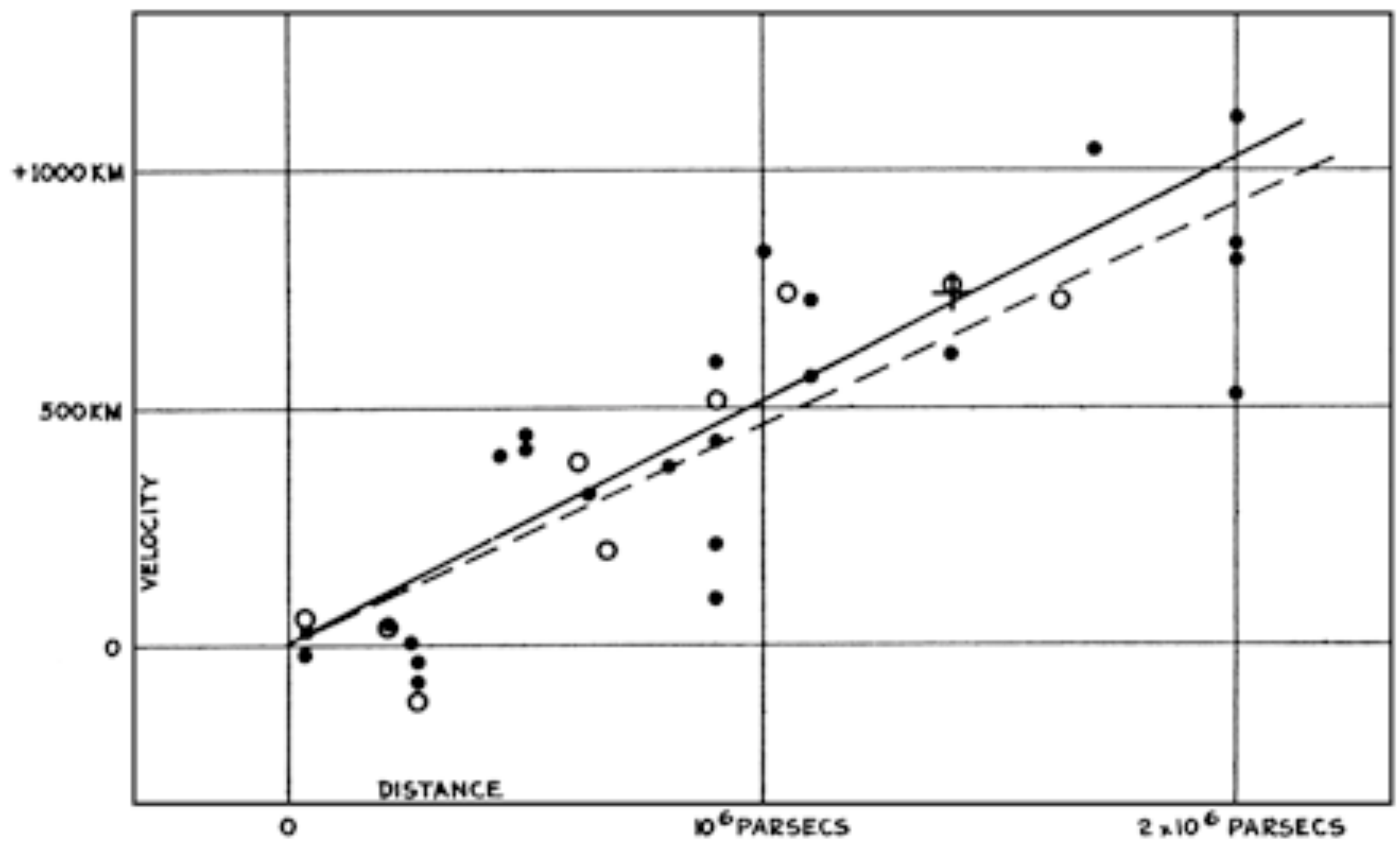
Cosmic expansion: Hubble's law

- 1920s: the first evidence of cosmic expansion provided by the receding motion of "extragalactic nebulae" (galaxies).

- Hubble's law: simple linear relation between recession velocity & distance.

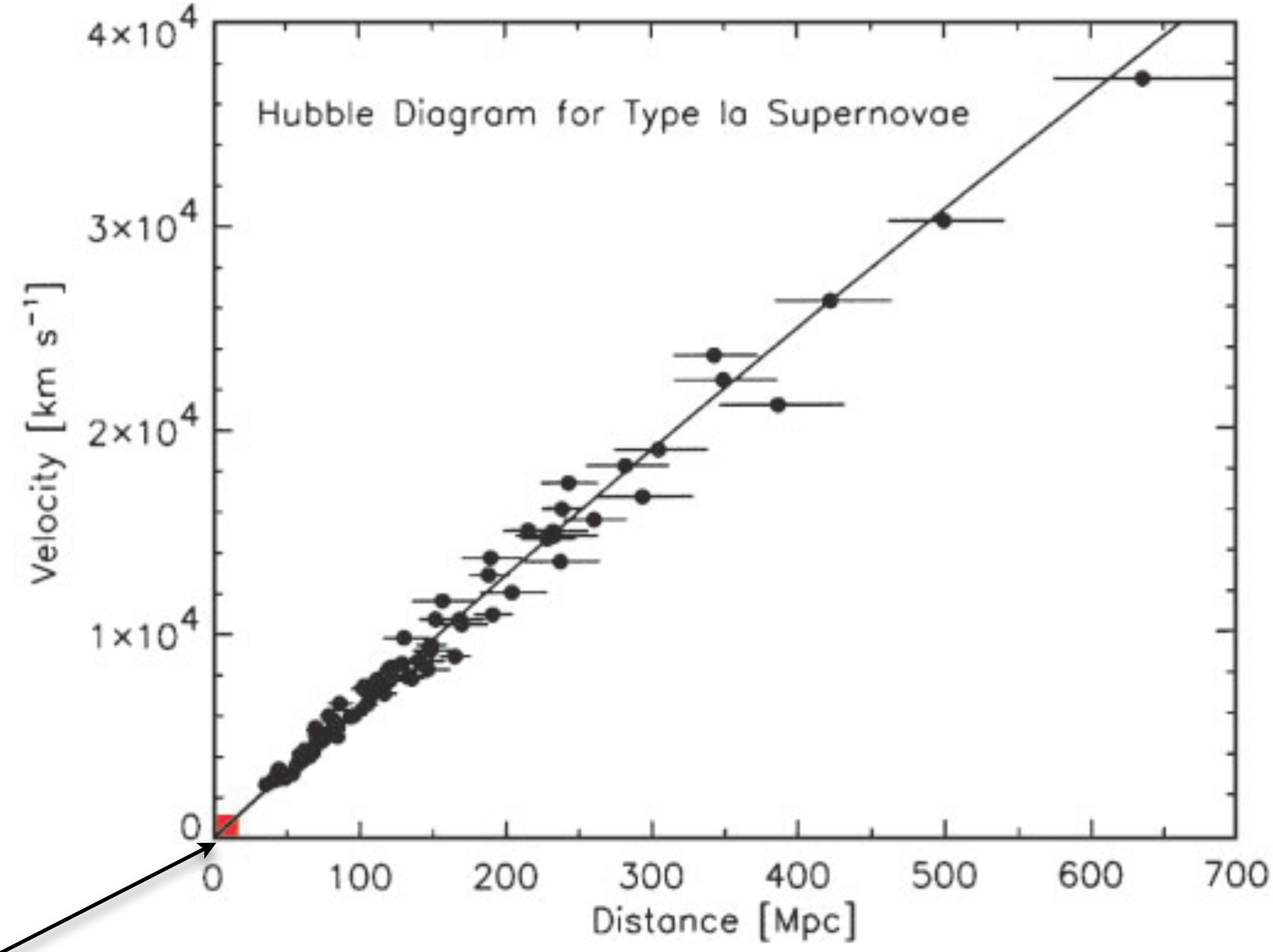
$$v = H_0 d \qquad H_0 = \text{Hubble's constant} \qquad d = \frac{cz}{H_0}$$

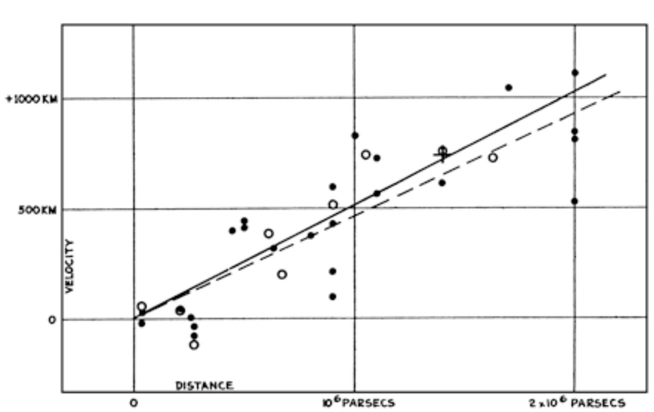
redshift is a measure of distance!



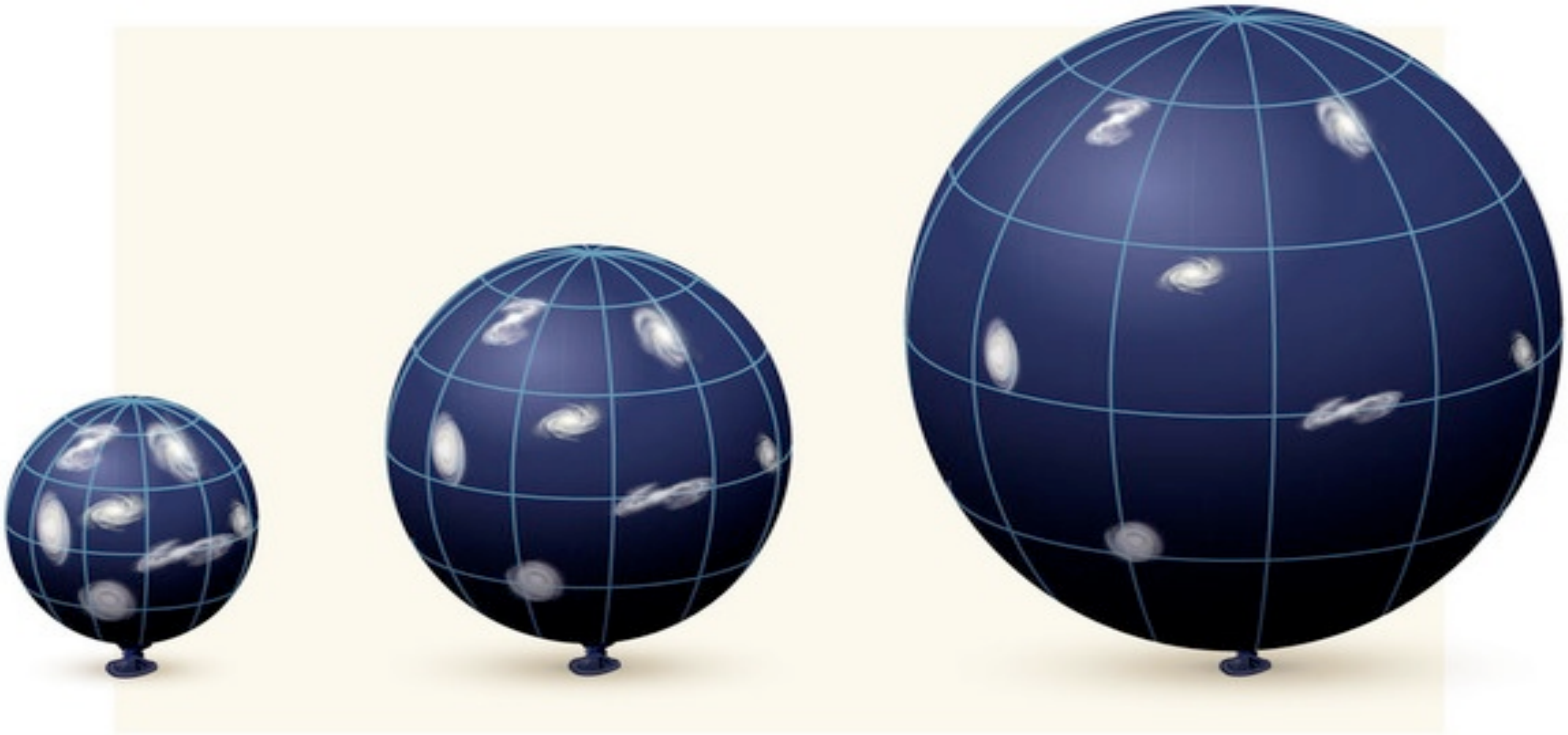
Hubble's original observations: The data points are galaxies with measured redshift and distance.

Hubble’s law: modern version





The expanding Universe



Each galactic observer sees all other
galaxies receding isotropically

The Hubble constant

- As shall see later, the Hubble constant roughly equals the age of the Universe:

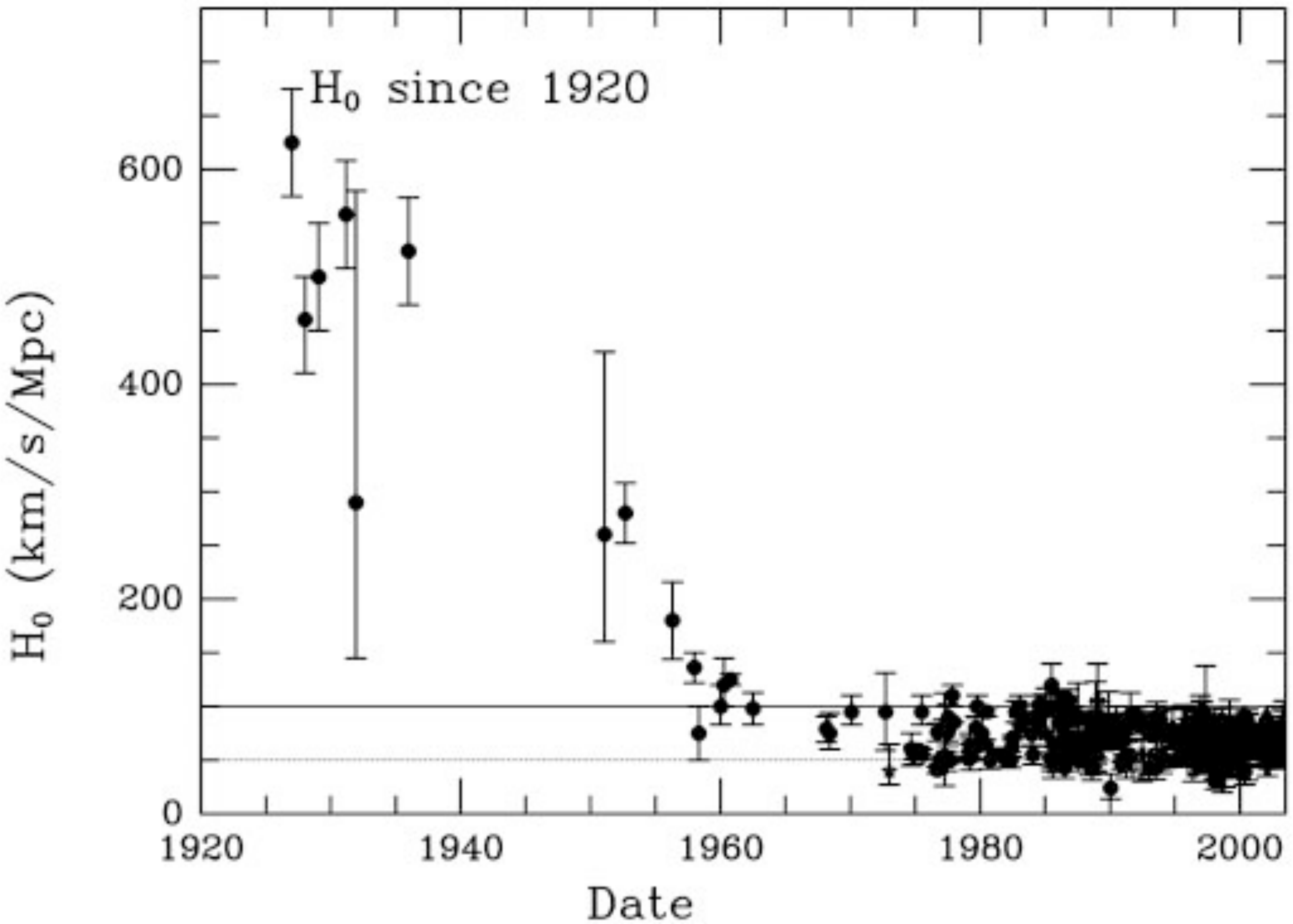
$$t_{\rm age} \sim \frac{1}{H_0}$$

- Earlier estimates led to an (erroneous) age below the age of old stars!

- Modern value (with a ~1% accuracy, measured by the Planck satellite):

$$H_0 \approx 68\,{\rm km\,s^{-1}\,Mpc^{-1}}$$

$$\longrightarrow \quad t_{\rm age} \approx 13.8\,{\rm Gyr}$$

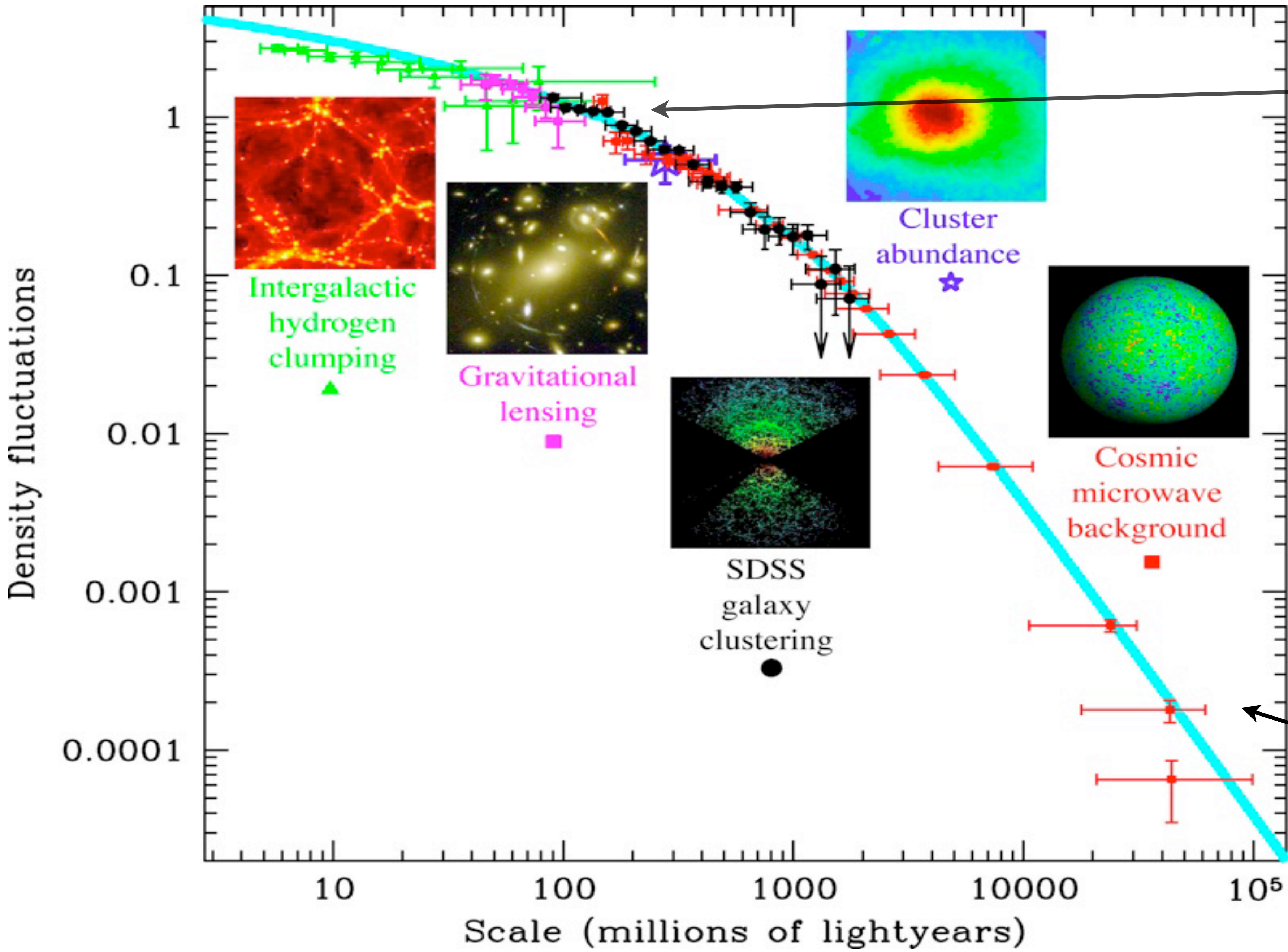


A reminder on observations

- The naked-eye night sky is made of our own galaxy's stars. Nearby galaxies can be barely seen (Andromeda, Magellanic Clouds).
- Seeing most galaxies requires large telescopes. But even with the most powerful telescopes almost all visible galaxies have redshift $z < 1$.
- Distant (high redshift) galaxies are extremely dim objects.
- Since large distances mean looking further back in time, eventually you get to a stage where the first galaxies were not formed yet.
- The furthest visible galaxies are at about $z \sim 8$. Theory suggests that the first stars were formed at $z \sim 10$.
- The CMB radiation originates from a time with $z{\sim}1100$.

Matter and radiation spatial distribution

General trend: the larger the observed spatial scale, the more homogeneous matter and radiation appear to be.



galaxies: z ~ 0 - 6, lumpy distribution

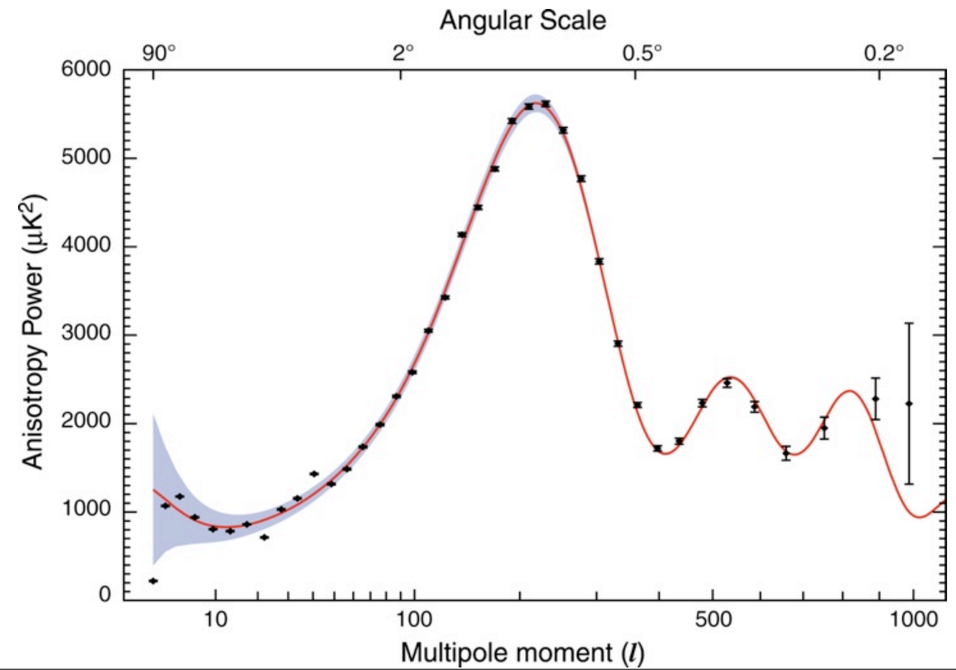
CMB: z ~ 1100, smooth distribution

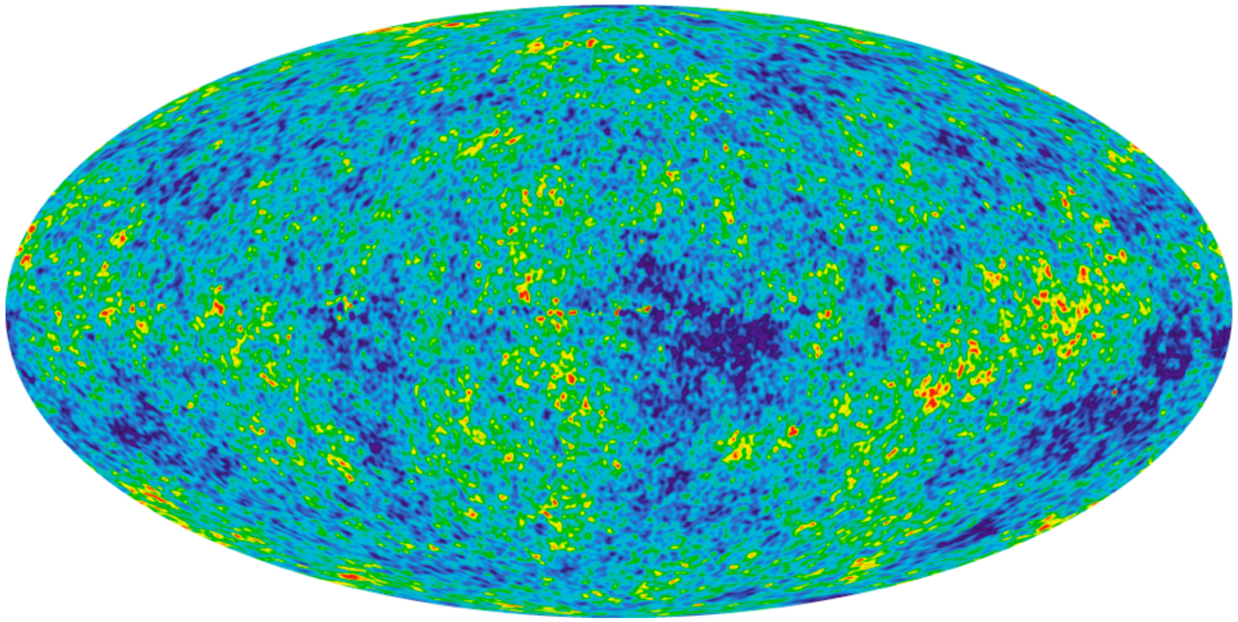
Cosmic Background Radiation

- This is the relic radiation from the time when matter and photons decoupled (recombination era ~ 400.000 years after the big bang). The radiation is thermal with a *black body temperature distribution*.

- The radiation cools with expansion, its present temperature is $T = 2.7$ K.

- The tiny deviations in temperature reflect the length-scale of structures in the early Universe. These are sensitive to the global cosmic dynamics and the geometry of the Universe.

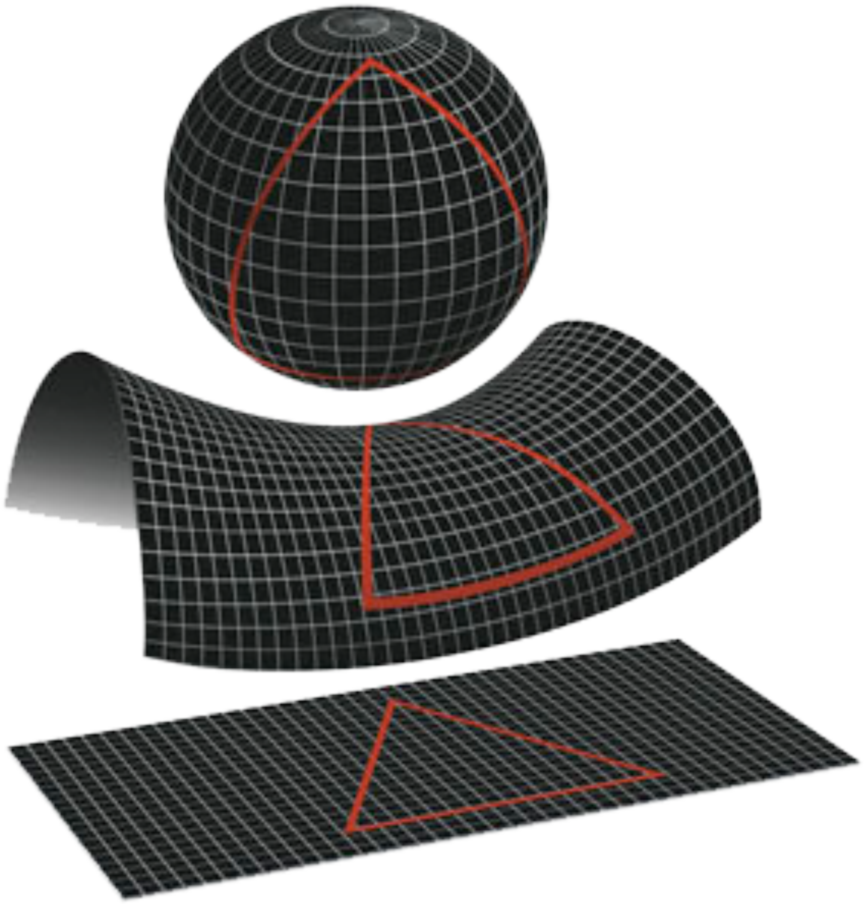




Eternal Universe? Olbers' paradox

- Before the era of modern cosmology the Universe was thought to be infinite, unchanging and uniformly filled with stars/galaxies.
- If that was true then at every direction in the sky one would observe some stars shining.
- The resulting night sky would be uniformly bright, instead of dark! This is the famous paradox pointed out by Olbers in 1826.
- The paradox is solved once we account for the *finite lifetime* of stars (and galaxies). The very existence of a dark night sky is evidence of an evolving Universe! (note: adding the effect of the cosmic expansion makes little difference to this argument).

Part II
The geometry of the universe



Beginning: the cosmological principle

- Having the theory of General Relativity at our disposal, how should we go on and build a "cosmology"?

- As it is always the case, we shall look for a "simple" solution of the field equations. *This is dictated by necessity and practicality*: the field equations are too hard to solve unless some simplification(s) is made. We look for a spacetime geometry endowed with certain symmetries.

- These are encapsulated in the "*cosmological principle*":

  Over large scales, the Universe is *isotropic* (same properties in every direction) and *homogeneous* (same properties at every point).

- This principle is "Copernicean" as it implies that our own location in the Universe is not special in any sense.

Cosmological spacetime: FLRW metric

- The most general spacetime metric compatible with the cosmological principle is the *Friedmann-Lemaitre-Robertson-Walker (FLRW)* metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 \left[d\theta^2 + \sin^2\theta d\varphi^2 \right] \right\}$$

- This metric depends only on the time coordinate t ("cosmic time") through the *scale factor* $a(t)$. The constant $k = 0, \pm 1$ specifies the geometry of the spatial, t=constant, 3-D hypersurfaces.

- The origin of the spatial spherical coordinates (r,θ,φ) can be any arbitrary point.

- An alternative common form of the FLRW metric is:

$$ds^2 = -c^2 dt^2 + a(t)^2 \{ d\chi^2 + F(\chi)[d\theta^2 + \sin^2\theta d\varphi^2] \}$$

where $F(\chi) = \{ \chi^2, \sin^2\chi, \sinh^2\chi \}$ for $k = \{0, \ +1, \ -1\}$

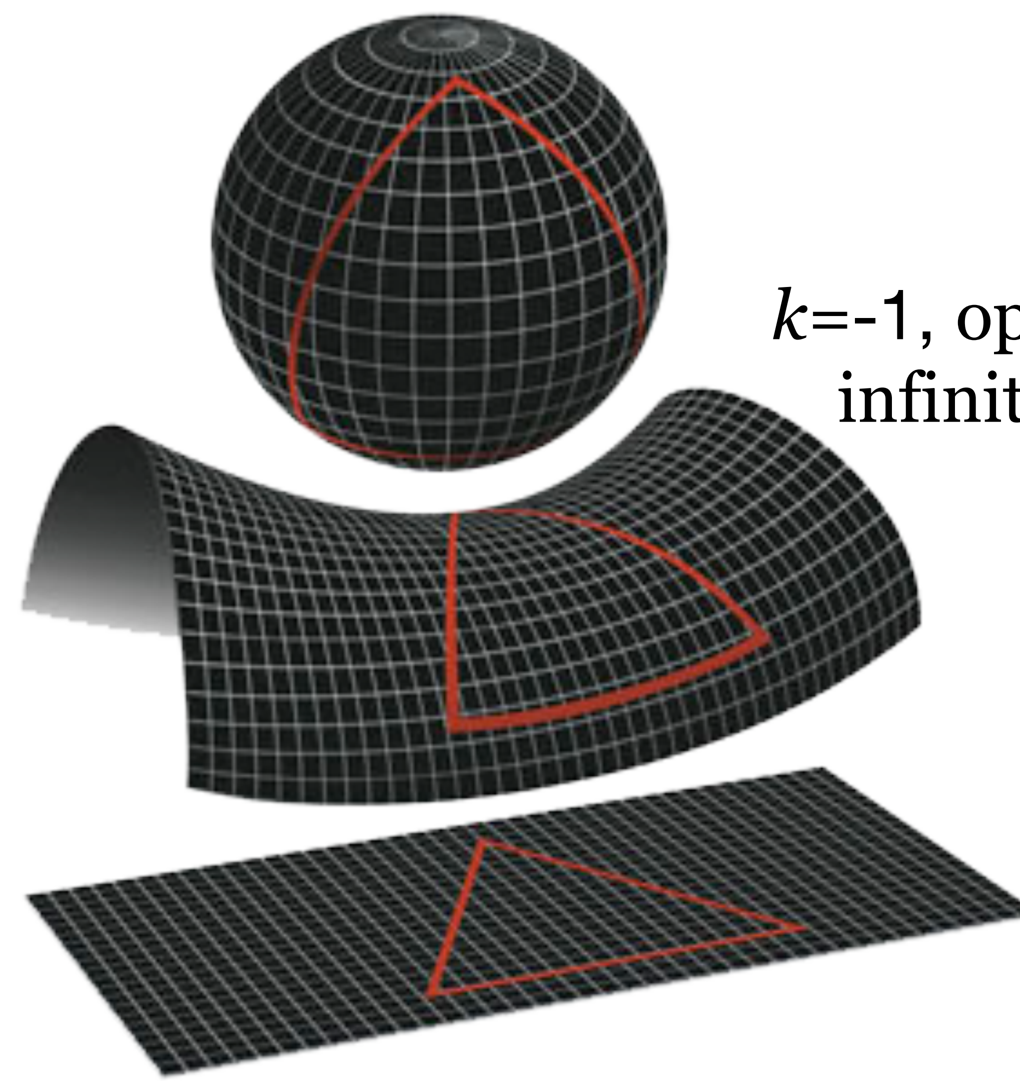
Topology of the FLRW spacetimes

- The FLRW geometry is general in the sense that it is the result of the imposed symmetries.

- The constant parameter k determines the geometry of 3-D space. k=+1represents a *closed* (finite) universe, k=0,-1 represent open (infinite) universes.

- The function $\alpha(t)$ is determined by the field equations of General Relativity. As we shall shortly see, its form depends on the assumed large-scale properties of matter and radiation (the “cosmic fluid”).

k=+1 closed, finite



k=-1, open, infinite

k=0, open, infinite

Fundamental observers

- The FLRW metric defines a family of "privileged" observers: their world-lines are described by constant spatial coordinates (χ,θ,φ). As a result, the cosmic time t can be identified with the proper time of these observers.

- For these observers, the FLRW 3-D space looks homogeneous and isotropic.

- The *proper distance* between two fundamental observers at any given time t can be written as (without loss of generality we can place one of the observers at the origin and the other at (χ,θ,φ)= (χ,0,0)):

$$D(t) = a(t)\chi$$

- From this it follows that the relative velocity between them is:

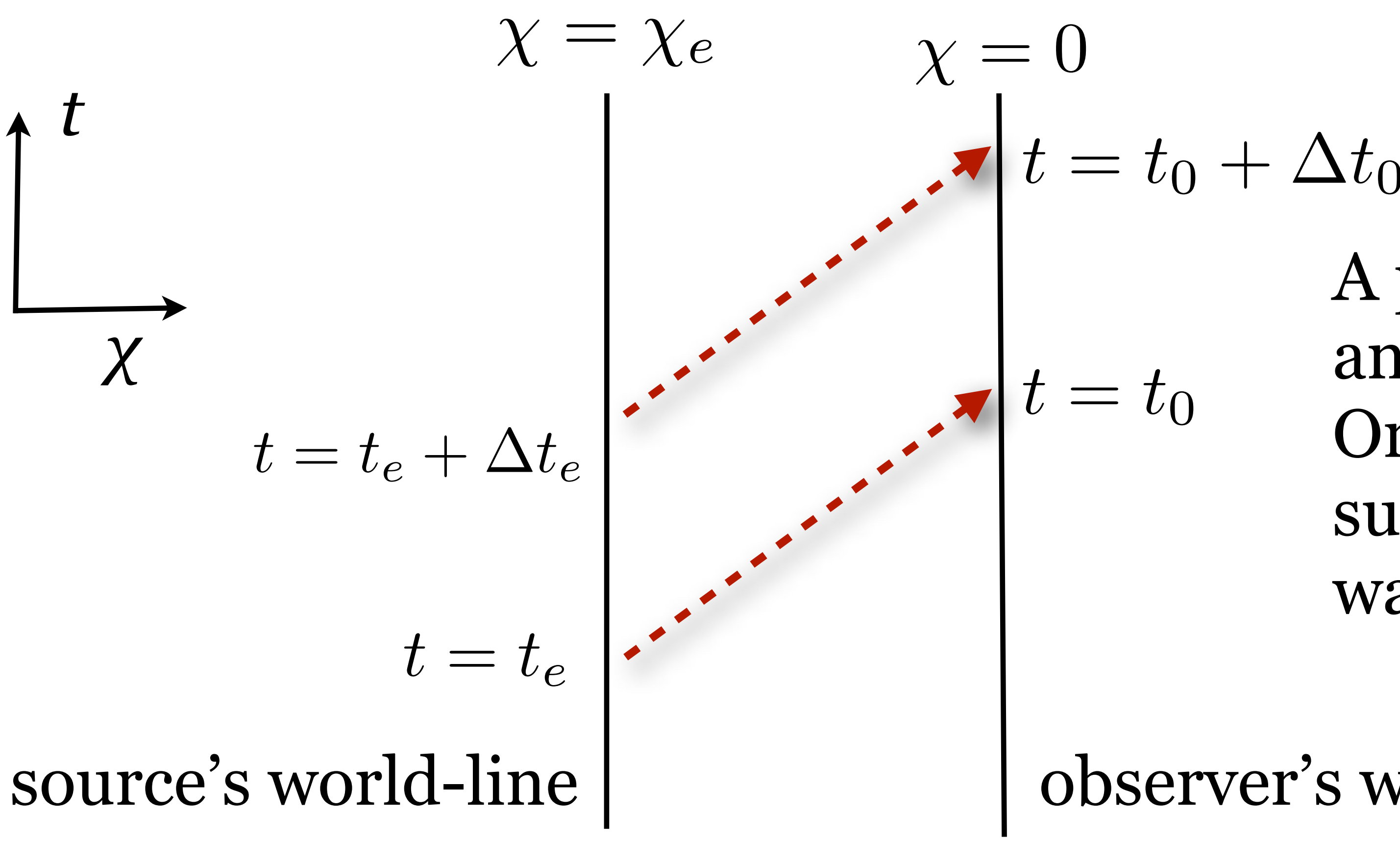
$$v = \frac{dD(t)}{dt} = \dot{a}(t)\chi \quad \rightarrow \quad v = H(t)D(t), \qquad H(t) \equiv \frac{\dot{a}}{a}$$

Hubble parameter

Light propagation in FLRW universes (I)

- We consider the emission of light from a source (e.g. a distant galaxy) and its reception by an observer (us). The emitted light can be thought to propagate radially outwards from the source to the observer.

- Both the source and the observer *have fixed FLRW coordinates,* $\chi = \chi_e$, $\chi = 0$ respectively, (this implies that the only relative motion is due to cosmic expansion).



A pulse of light is emitted at $t = t_e$ and a successive one at $t = t_e + \Delta t_e$. Or, these times could represent the successive “peaks” of a monochromatic wave.

Light propagation in FLRW universes (II)

- According to the FLRW metric, radial propagation of light is described by:

$$cdt = \pm a(t) d\chi$$

- Integrating this for the radially ingoing light of the previous figure:

$$\chi_e = \int_{t_e}^{t_0} \frac{cdt}{a(t)} = \int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{cdt}{a(t)} = \int_{t_e}^{t_0} \frac{cdt}{a(t)} + \frac{c\Delta t_0}{a_0} - \frac{c\Delta t_e}{a_e}$$

where $a_0 \equiv a(t_0)$ and $a_e \equiv a(t_e)$

- Therefore:

$$\frac{\Delta t_0}{\Delta t_e} = \frac{a_0}{a_e}$$

we can write this in terms of wavelength & frequency:

$$\frac{\lambda_0}{\lambda_e} = \frac{a_0}{a_e}, \qquad \frac{\nu_e}{\nu_0} = \frac{a_0}{a_e}$$

- The redshift is: $1 + z \equiv \dfrac{\lambda_0}{\lambda_e} = \dfrac{a_0}{a_e}$

The redshift of the source depends only on a_e, a_0 not $a(t)$ in between.

Deriving Hubble's law (I)

- The idea is to approximate the redshift formula "near" the present time t_0.

$$1 + z(t) = \frac{a_0}{a(t)}$$

- The Taylor-expansion of the scale factor leads to:

$$a(t) = a_0 + \dot{a}_0(t - t_0) + \frac{1}{2}\ddot{a}_0(t - t_0)^2 + \ldots = a_0\left[1 + H_0(t - t_0) - \frac{1}{2}H_0^2 q_0(t - t_0)^2 + \ldots\right]$$

where $\Delta t = t_0 - t$ and we have defined:

$$\underbrace{H_0 \equiv \frac{\dot{a}_0}{a_0}}_{\text{Hubble constant}}, \qquad \underbrace{q_0 \equiv -\frac{\ddot{a}_0}{H_0^2 a_0}}_{\text{deceleration parameter}}$$

- Note: the above expansion assumes $H_0 \Delta t \ll 1$, that is the light emission took place in the "recent" past.

Deriving Hubble's law (II)

- Then,

$$\frac{a_0}{a(t)} = 1 + H_0 \Delta t + \left(1 + \frac{q_0}{2}\right) H_0^2 (\Delta t)^2 + \ldots$$

- The desired expansion for the redshift is:

$$z(t) = H_0 \Delta t + \left(1 + \frac{q_0}{2}\right) H_0^2 (\Delta t)^2 + \ldots$$

This expression clearly demonstrates that *z* can be used for labeling/measuring the cosmic past.

- Using the above expansion for $a(t)$ for the emission of light from a source at $\chi = \chi_e$ we find:

$$\chi_e = \int_{t_e}^{t_0} \frac{cdt}{a(t)} = \frac{c\Delta t}{a_0} \left[1 + \frac{1}{2} H_0 \Delta t + \ldots\right] \qquad \text{where here } \Delta t = t_0 - t_e$$

Deriving Hubble's law (III)

- Therefore, for a source (e.g. galaxy) not too far away (=not too far in the past)

$$z_e \approx H_0 \Delta t, \qquad d_e \equiv a_0 \chi_e \approx c\Delta t \qquad \text{where } \Delta t = t_0 - t_e$$

redshift of the source

present epoch *proper* distance of the source

distance-redshift relation: $d_e \approx \dfrac{cz_e}{H_0}$

- Finally we need to relate these to the source's radial velocity at $t = t_e$:

$$v_e = \dot{a}_e \chi_e$$

From the previous Taylor-expansion:

$$\dot{a}(t) = a_0 \left[H_0 - H_0^2 q_0 (t - t_0) + \ldots \right] \quad \to \quad \dot{a}_e \approx a_0 H_0$$

$$\longrightarrow \quad v_e \approx H_0 d_e \approx cz_e$$

Hubble's law!

More on Hubble's law

- For the derivation of the Hubble we have only resorted to the cosmological principle and the FLRW metric that comes with it. The field equations of General Relativity have not been used at all.

- Hence, Hubble's law is a *purely kinematical effect*.

- In the derivation we have assumed $H_0 \Delta t \ll 1$ which implies low redshift sources, "slowly" receding:

$$z_e \approx H_0 \Delta t \ll 1 \quad \longrightarrow \quad \frac{v_e}{c} \approx z_e \ll 1$$

- The Hubble law may look like the usual Doppler-shift formula but it is *not* a Doppler effect since the observer and the source are not in the same inertial frame. The Hubble law is truly a consequence of the FLRW spacetime geometry.

Cosmological particle horizon (I)

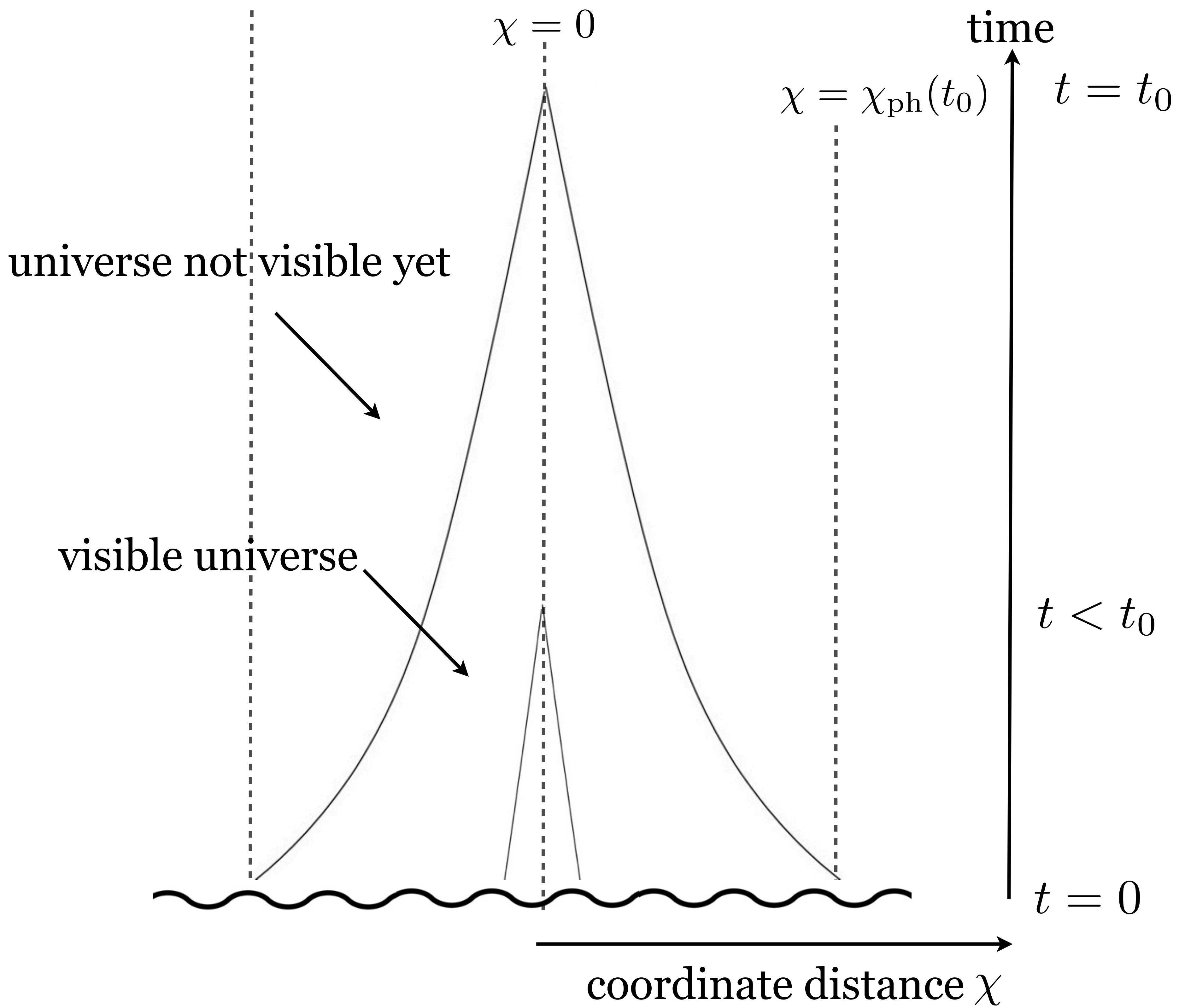
- Recall that the *proper* radial distance in the FLRW metric is: $D(t) = a(t)\chi$
- Light propagation is described by: $cdt = \pm a(t)d\chi$
- This means that the speed of light in the FLRW spacetime is:

$$v_{\rm light} = \left|\frac{dD}{dt}\right| = a(t)\left|\frac{d\chi}{dt}\right| = c$$

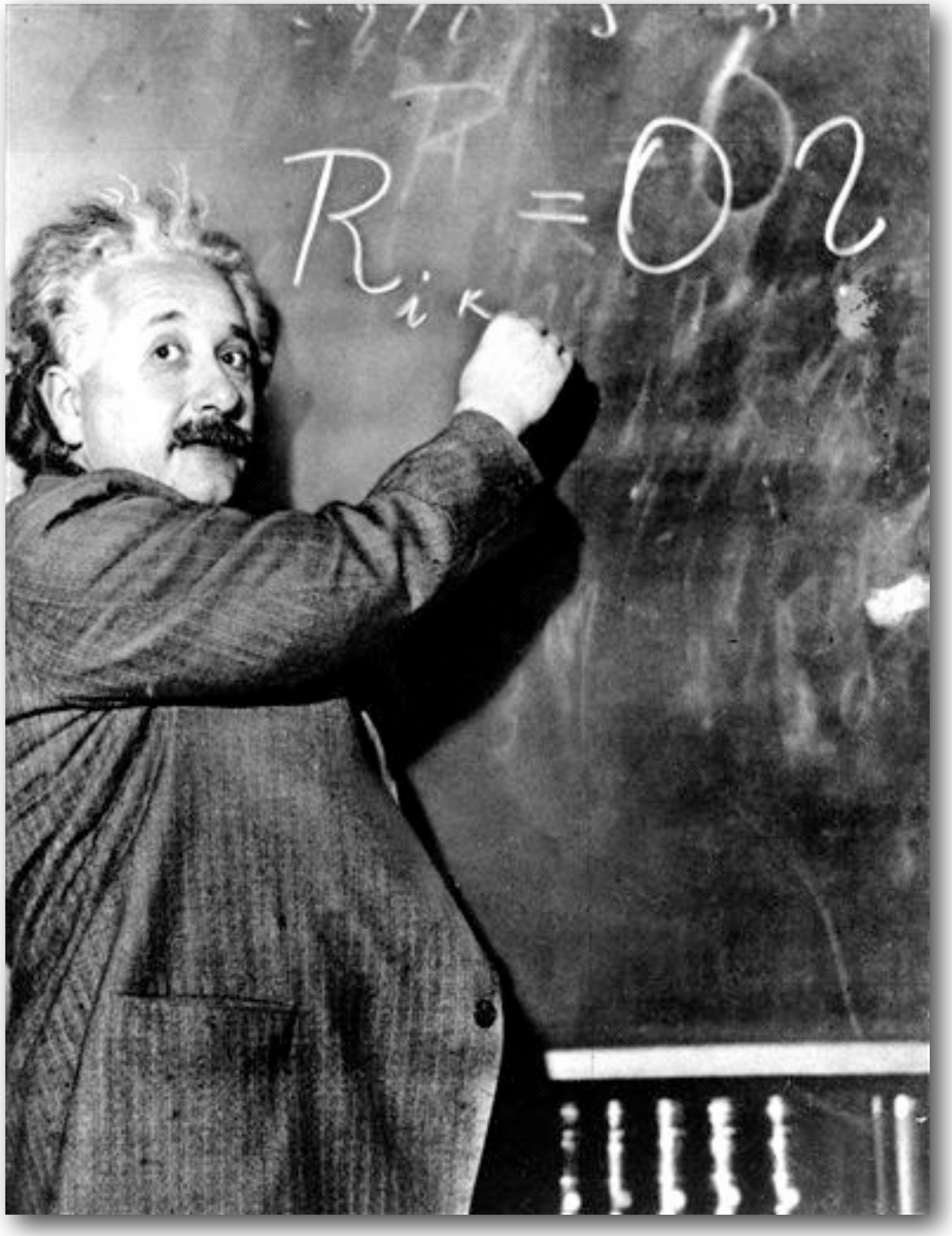
- The finite speed of light means that at any given time $t = t_0$ an observer can see only a *finite part* of the universe. This is the part inside his/her past light cone.
- The size of this "observable Universe" defines the *particle horizon*:

$$\chi_{\rm ph} = \int_0^{t_0} \frac{cdt}{a(t)} \qquad\qquad D_{\rm ph} = a_0\chi_{\rm ph}$$

Cosmological particle horizon (II)



Part III
Cosmic dynamics



Matter → geometry

- The most general form of the General Relativistic field equations is:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu}$$

These equations feature *three constants*: *G*, *c*, Λ. This latter parameter is called the "cosmological constant" and for a long time was thought to be =0.

- These equations are solved from "right to left": a physically motivated energy-momentum tensor $T^{\mu\nu}$ is constructed and then the geometry $G^{\mu\nu}$ (and the scale factor $a(t)$) is computed.

- The large-scale matter/radiation is assumed to be a *perfect fluid*:

$$T^{\mu\nu} = \left(\rho + \frac{p}{c^2}\right) u^\mu u^\nu + p g^{\mu\nu}$$

energy density (ρ), pressure (p)

four-velocity:

$$u^\mu = (u^t, u^r, u^\theta, u^\varphi) = (1, 0, 0, 0)$$

spatial velocity = 0, as there is no preferred direction.

Dynamics: The Friedmann equations (I)

- Once $T^{\mu\nu}$ has been specified, the GR field equations lead to the following Friedmann equations for the evolution of the scale factor:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2$$

Hubble parameter:

$$H(t) \equiv \frac{\dot{a}}{a}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{1}{3}\Lambda c^2$$

- These two equations can be combined to give:

$$\frac{d}{dt}(\rho c^2 a^3) + p\frac{da^3}{dt} = 0$$

this is 1st law of thermodynamics
(conservation of energy)

$$dE = -pdV$$

Friedmann eqns: a "Newtonian" derivation

- Consider the dynamics of an expanding, self-gravitating sphere of radius

$$R(t) = a(t)r \qquad r = \text{constant}$$

- 1st law of thermodynamics: change in energy = work done

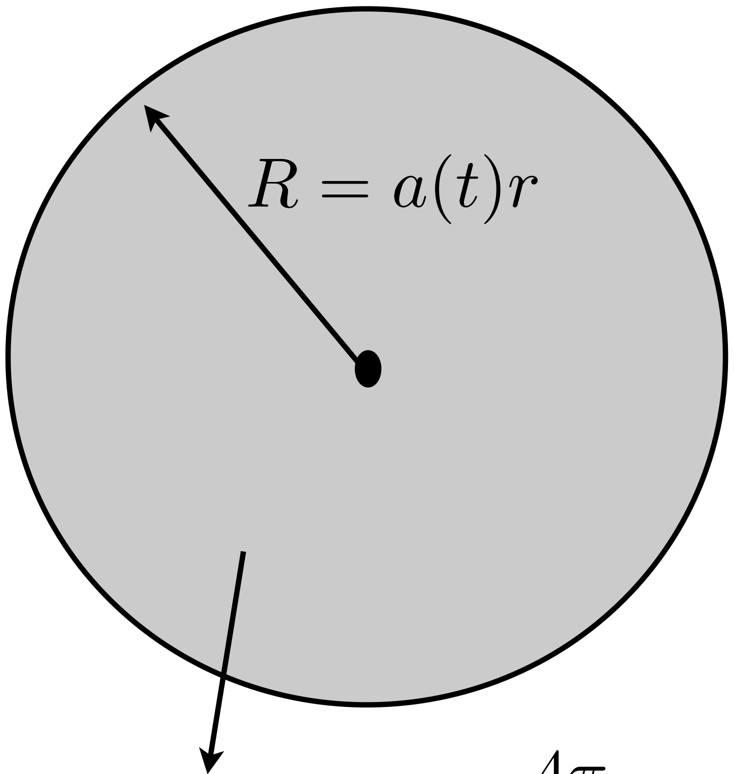
$$dE = -pdV \quad \rightarrow \quad d(\rho c^2 r^3 a^3) = -pd(r^3 a^3)$$

- Newton's second law:

$$\ddot{R} = r\ddot{a} = -\frac{G}{r^2 a^2}\left(\frac{4\pi}{3}\rho r^3 a^3\right)$$

$$\longrightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)$$

GR correction



$$M = \rho V = \rho \frac{4\pi}{3} R^3$$

Matter-dominated Friedmann model

- As it is customary, we solve the Friedmann equations for two special but physically relevant cases, representing *two different stages of cosmic evolution*. For simplicity we assume k=Λ=0 (flat models with no cosmological constant).

- *Matter-dominated era*:
  on large scales the cosmic fluid (galaxies) is pressureless "dust", p=0

we have:

$$\rho_{\mathrm{mat}} = \frac{\mathrm{constant}}{a^3}$$

$$\dot{a}^2 = \frac{C}{a}, \qquad C = \frac{8\pi G}{3}\rho_{\mathrm{mat}} a^3 = \mathrm{constant}$$

$$\longrightarrow \quad a(t) = \left(\frac{9C}{4}\right)^{1/3} t^{2/3}$$

$$\longrightarrow \quad \rho_{\mathrm{mat}} = \frac{\mathrm{constant}}{t^2}$$

Spacetime expands,
density of matter decreases

Radiation-dominated Friedmann model

- *Radiaton-dominated era*:
  the large scales photon gas (including any relativistic particles) has a pressure

$$p_{\rm rad} = \frac{1}{3}\rho_{\rm rad} c^2$$

- From the Friedmann equations we have:

$$\rho_{\rm rad} = \frac{\rm constant}{a^4}$$

$$\dot{a}^2 = \frac{D}{a^2}, \qquad D = \frac{8\pi G}{3}\rho_{\rm rad} a^4 = \rm constant$$

$$\longrightarrow \quad a(t) = (4D)^{1/3}\, t^{1/2}$$

$$\longrightarrow \quad \rho_{\rm rad} = \frac{\rm constant}{t^2}$$

Spacetime expands,
density of radiation decreases

Why Big Bang?

- The early Universe was dominated by matter and radiation.

- For the more general pressure-density relation $p = wc^2\rho$ (w=constant) we obtain:

$$\rho \propto a^{-3(1+w)} \quad \text{and} \quad \dot{a}^2 = \frac{B}{a^{1+3w}}, \qquad B = \frac{8\pi G}{3}\rho a^{3(1+w)} = \text{constant}$$

$$\longrightarrow \quad a(t) \propto t^{2/3(1+w)}$$

- *The essence of the "Big Bang"*: There is an "origin" in time $t=0$, where the density, the spacetime curvature (Riemann tensor), and other physical quantities diverge.

- *In reality GR itself breaks down at $t \to 0$ and we should wait for a more advanced theory (quantum gravity) to explain the birth of our Universe.*

More on the decay of density

- There is a very intuitive way to understand the evolution of the energy density with time.

- The volume of a spherical region of coordinate radius χ is: $V \propto a(t)^3$

- Let N be the number of particles (they could be photons) inside V, then the number density n (=number of particles per unit volume) is:

$$n \propto a^{-3}$$

- For non-relativistic particles of rest mass m: $\rho \approx mn \propto a^{-3}$

- For photons (or other highly relativistic particles): $\rho_{\rm rad} = \frac{h\nu}{c^2} n \propto a^{-4}$

recall that: $\nu \propto a^{-1}$

- For a photon gas in *thermal equilibrium*:

$$\rho_{\rm rad} \propto T^4 \quad \rightarrow \quad T \propto a^{-1}$$

Cooling of the CMB gas

Friedmann models with *k*, keeping Λ=0

- We define the "critical" density:

$$\rho_{\text{crit}} \equiv \frac{3H_0^2}{8\pi G} \qquad H_0 = H(t_{\text{now}})$$ = Hubble parameter at present epoch

- We also introduce:

$$\rho_0 = \rho(t_{\text{now}})$$ = total matter+radiation density at present epoch

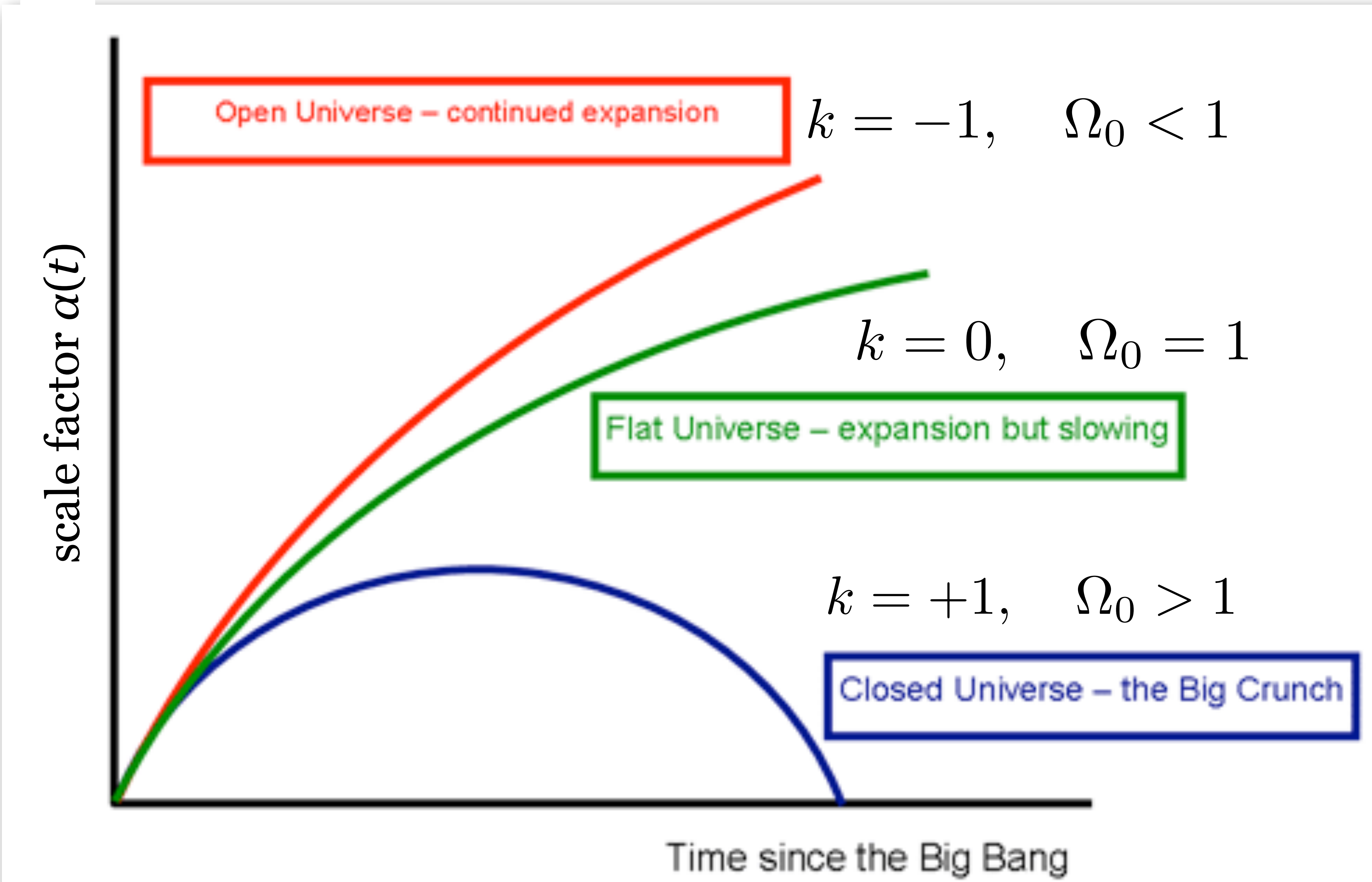
- The first Friedmann equation can be written as:

$$\Omega_0 \equiv \frac{\rho_0}{\rho_{\text{crit}}} = 1 + \frac{c^2 k}{a_0^2 H_0^2}$$

- We can see that the *topology k*=(-1, 0,+ 1) of each cosmological model directly related to the density ratio $\rho_0/\rho_{\text{crit}}$.

$$\rho_{\text{crit}} \sim \text{few protons}/m^3$$

Friedmann models with *k*, keeping Λ=0



Λ-dominated Friedmann model

- Now we assume that the cosmological constant term dominates in the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2 \approx \frac{1}{3}\Lambda c^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{1}{3}\Lambda c^2 \approx \frac{1}{3}\Lambda c^2$$

$\Rightarrow$

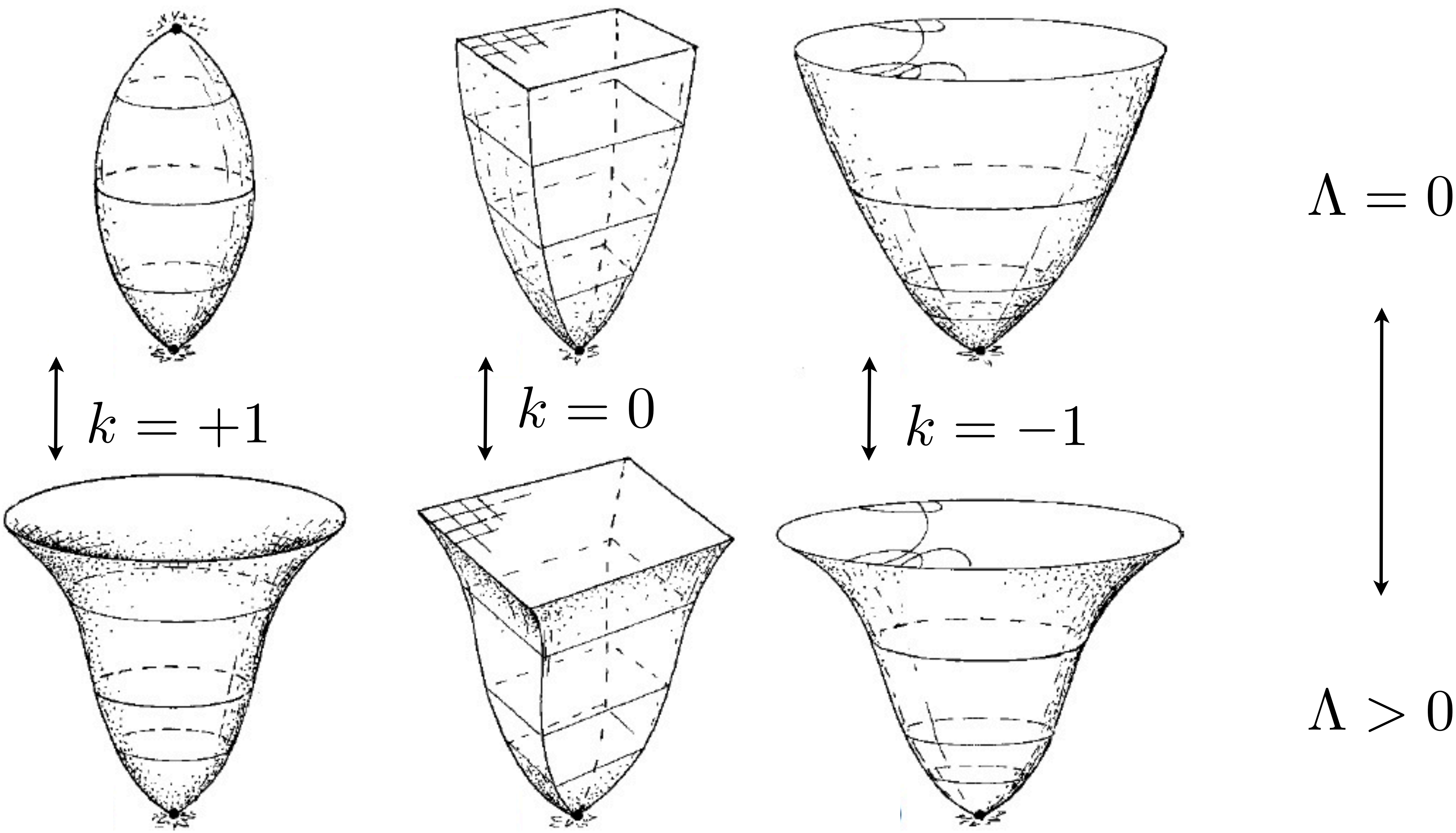
$$a(t) \approx \exp[(\Lambda c^2/3)^{1/2} t]$$

$$\ddot{a}(t) > 0$$

- That is, a $\Lambda > 0$ constant can drive *accelerated*, *exponential* expansion. This is known as the *de Sitter* universe.

- Since the density decays as $\rho \sim a^{-n}$ it is clear that all $\Lambda > 0$, expanding universes will asymptotically tend to the de Sitter universe!

Summary of the Friedmann universes



The present epoch Friedmann equation

- We return to the general Friedmann equations, and we consider the present cosmic epoch $t = t_0$:

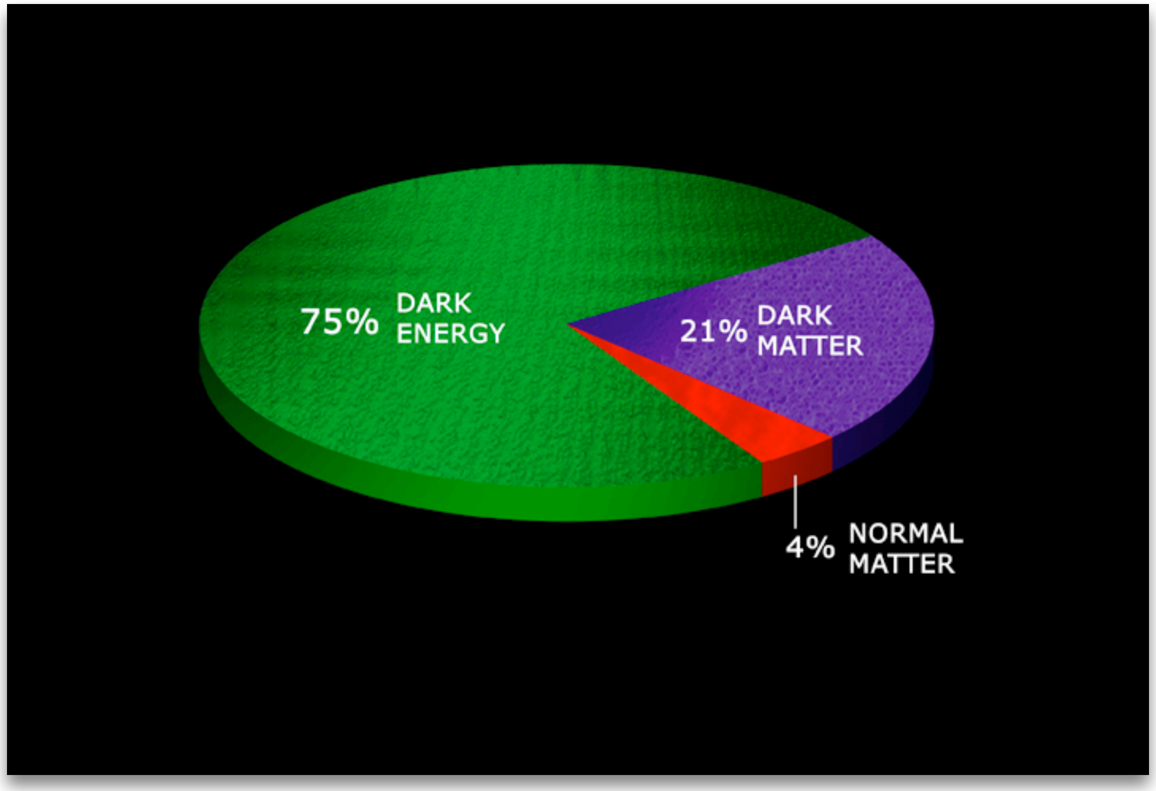
$$H_0^2 = \frac{8\pi G}{3}\rho_0 - \frac{kc^2}{a_0^2} + \frac{\Lambda c^2}{3} \longrightarrow \Omega_0 + \Omega_\Lambda + \Omega_k = 1$$

we have defined: $$\Omega_0 \equiv \frac{\rho_0}{\rho_{\mathrm{crit}}} = \Omega_{\mathrm{mat}} + \Omega_{\mathrm{rad}} \approx \Omega_{\mathrm{mat}}$$

$$\Omega_k \equiv -\frac{kc^2}{H_0^2 a_0^2}$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}$$

Part IV
The accelerating Universe



Accelerating expansion?

- Up until the mid-1990s, the cosmological constant Λ was systematically ignored.

- The Friedmann equations without the Λ terms predict:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) < 0$$

decelerating expansion since normal matter/radiation has $\rho > 0,\ p > 0$.

- Accelerating expansion requires some exotic type of matter (“dark energy”) with negative pressure:

$$\ddot{a} > 0 \quad \longleftrightarrow \quad p < -\frac{1}{3}\rho c^2$$

- What observations have to say about the sign of $\ddot{a}$?

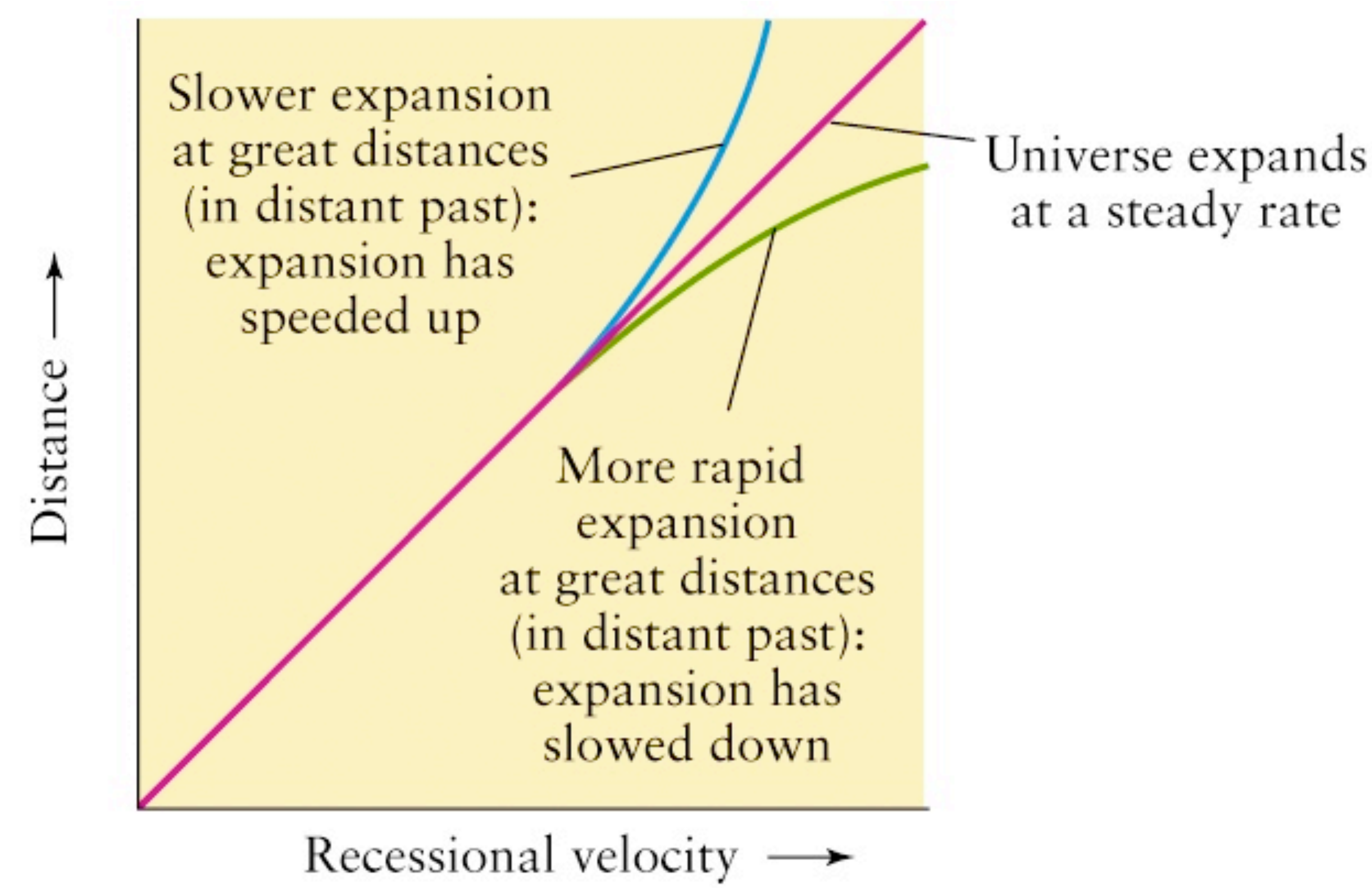
Beyond Hubble's law

- Hubble's linear law is accurate for low redshifts ($z << 1$), that is, for "nearby" galaxies.

- The theory generally predicts deviations from the linear relation:

$$d = \frac{cz}{H_0} + \mathcal{O}(z^2)$$

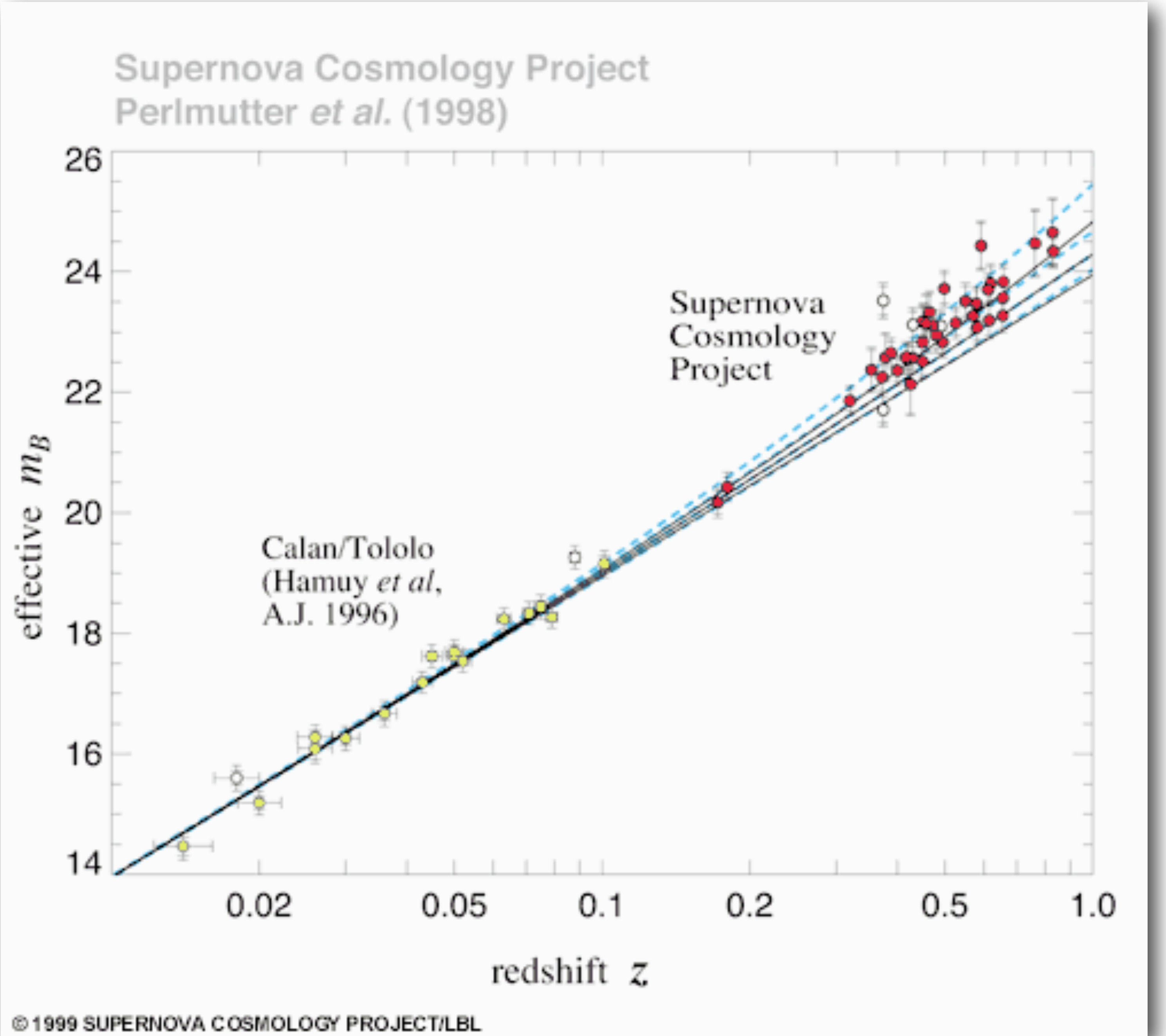
depends on $q_0 = -\frac{\ddot{a}_0}{H_0^2 a_0}$

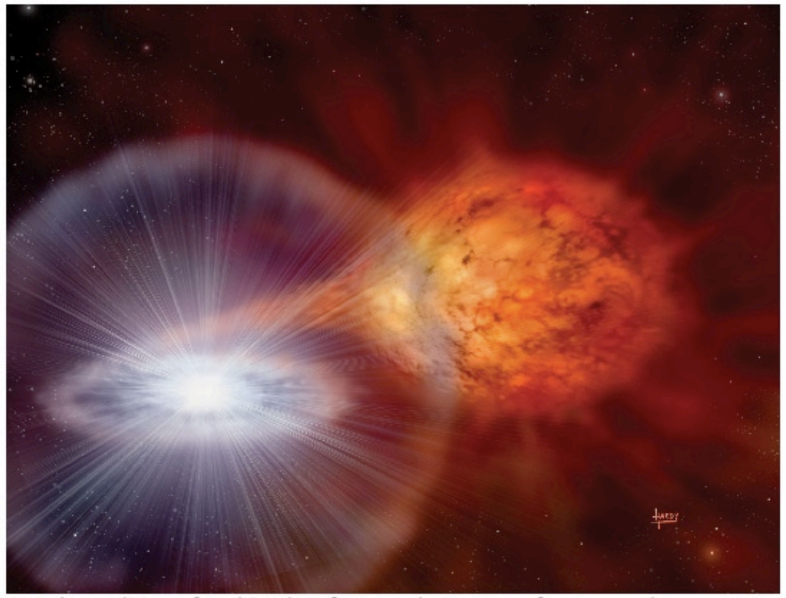


- Reasonable expectation: $\ddot{a} < 0$ the expansion slows down, after all it does work against gravity.

Evidence for accelerating expansion!

Using distant Type Ia Supernovae as “standard candles” observations seem to suggest an accelerating expansion, i.e. $\ddot{a}_0 > 0$





Artist's rendition of a white dwarf accumulating mass from a nearby companion star. This type of progenitor system would be considered singly-degenerate.

Image courtesy of David A. Hardy, © David A. Hardy/www.astroart.org.

This discovery won the 2011 Nobel prize in Physics.

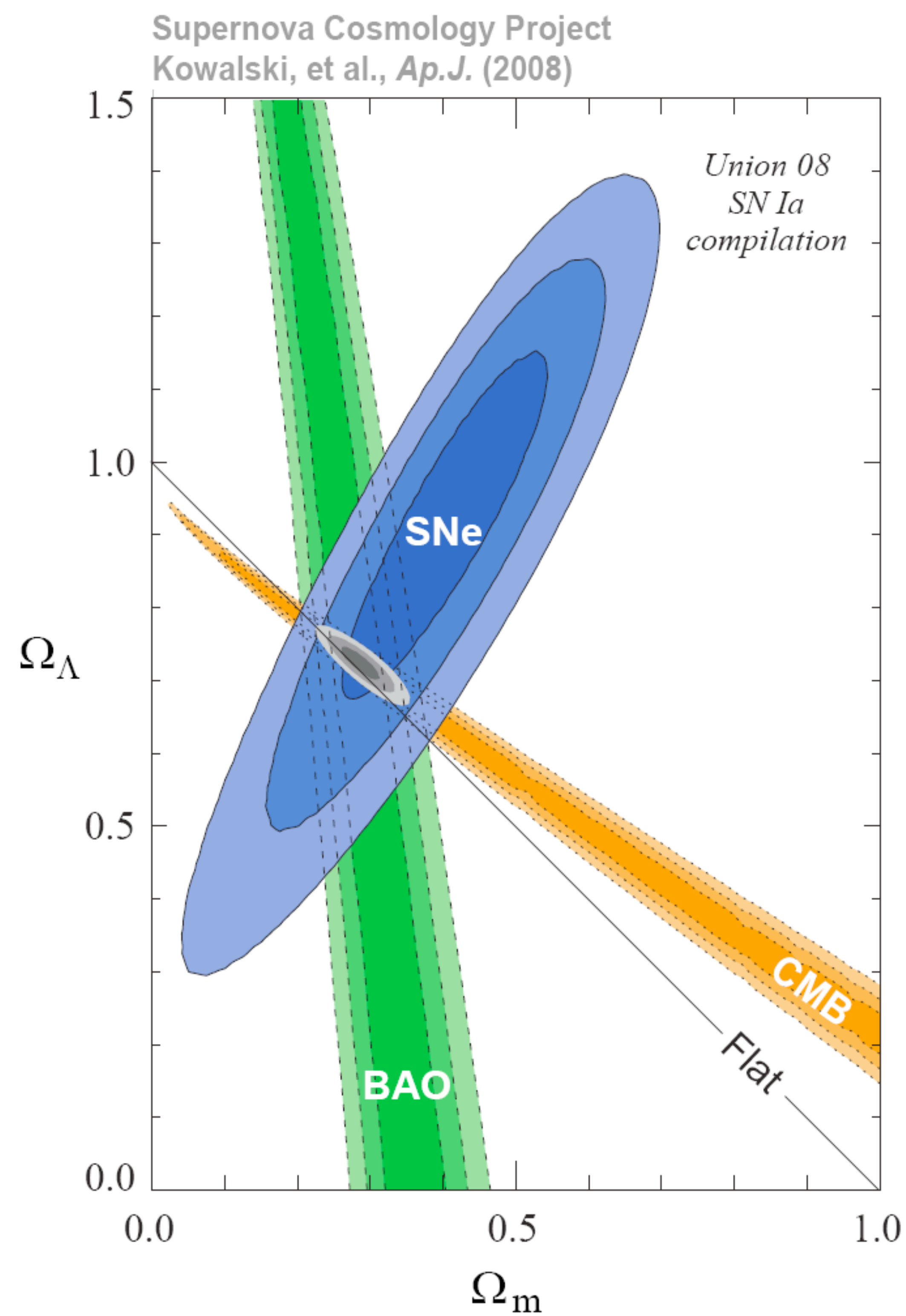
Further evidence for acceleration

- *Three independent observations*, modelled with the Friedmann cosmological models: Hubble's law, CMB and light element abuncances.

- Key findings:

✓ Large scale acceleration. The cosmic dynamics is presently dominated by a "*dark energy*" (a "fluid" with negative pressure).

✓ The local Universe is almost flat, $\Omega_k \ll 1$ but k is unspecified.



A Universe with a nonzero Λ

- We have seen that $\ddot{a} > 0$ requires a negative pressure fluid or a universe dominated by the cosmological constant.

- In fact, the cosmological constant can be viewed as an effective matter contribution:

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu} \longrightarrow G^{\mu\nu} = \frac{8\pi G}{c^4}(T^{\mu\nu} + T^{\mu\nu}_{\text{eff}})$$

- Seen as a "fluid", the cosmological constant has *negative* pressure and can therefore drive accelerated expansion:

$$p_\Lambda = -\rho_\Lambda c^2$$

$$\rho_\Lambda = \frac{c^2 \Lambda}{8\pi G}$$

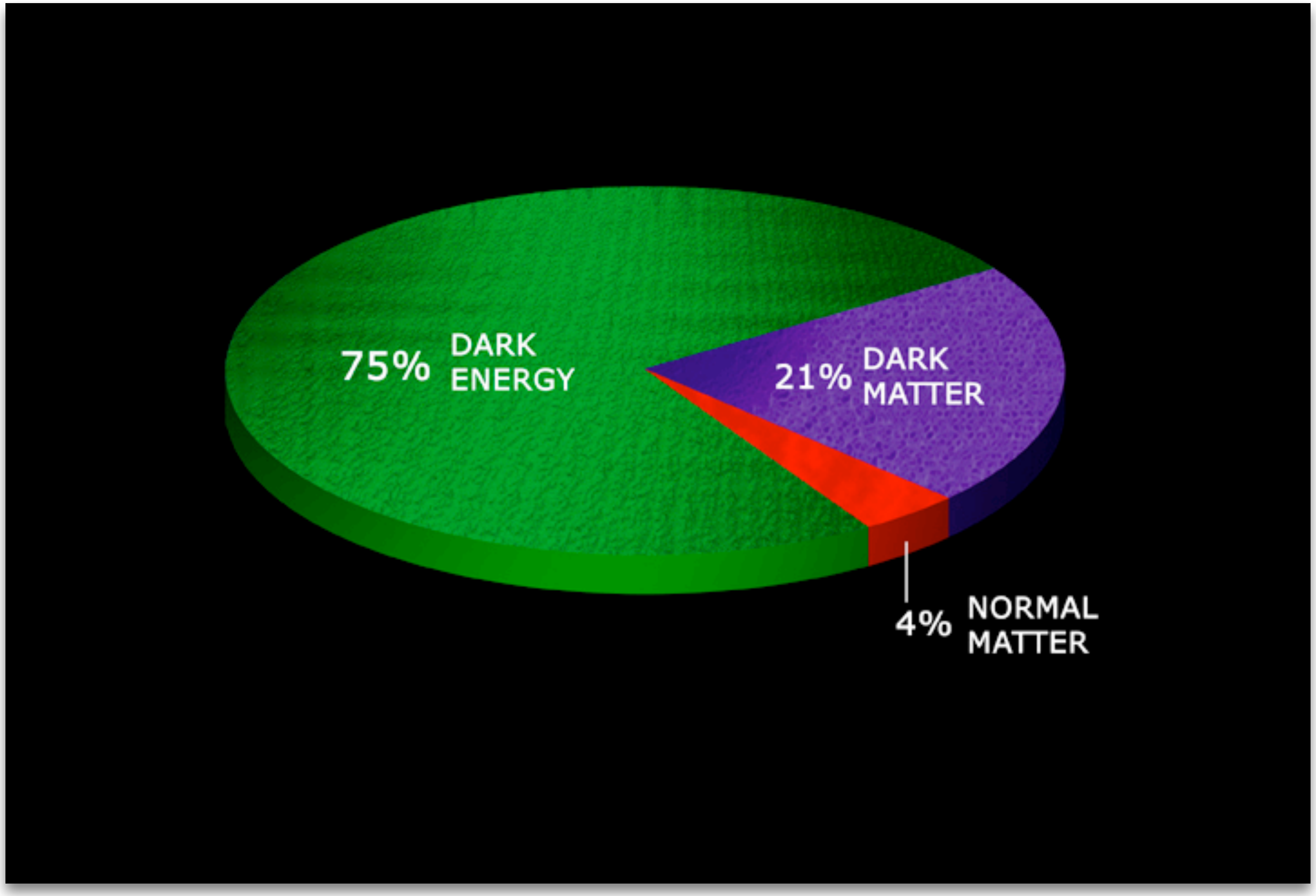
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + \rho_\Lambda + \frac{3}{c^2}(p + p_\Lambda)\right]$$

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{kc^2}{a^2}$$

The content of the present day Universe

Our present understanding can be summarized in this figure

$$\Omega_{\rm mat} + \Omega_\Lambda \approx 1$$



Cosmic acceleration: explanations

- Cosmological constant.

✓ simplest explanation, so far agrees with observations. Attempts to produce an effective cosmological constant using quantum field theory leads to an enormous discrepancy with the observed Λ.

- Dynamical dark energy.

✓ it implies the existence of some unknown "substance" with negative pressure.

- Modified gravity.

✓ it implies that General Relativity is not the correct theory of gravity.

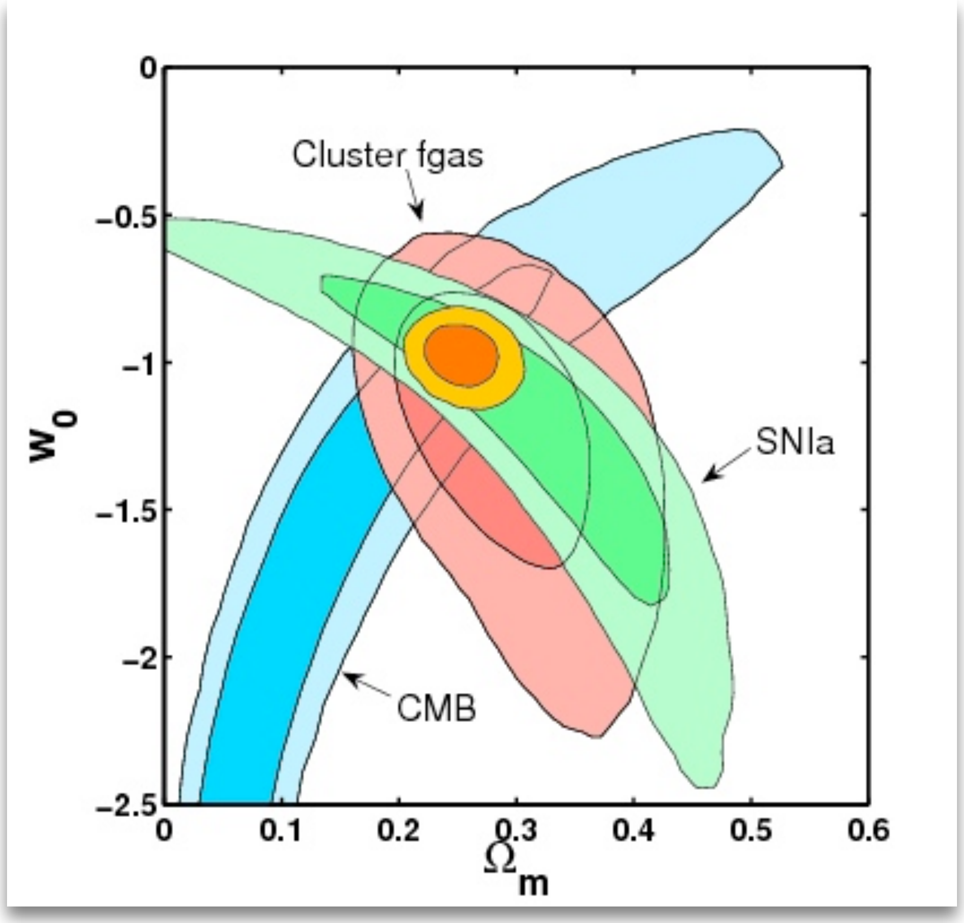
- Deviations from the homogeneity of the FLRW spacetime.

✓ no new physics required, seems unlikely though.

The theory challenge

- Is really Λ one of Nature's fundamental constant (like G) ?
- Or is Λ a form of "dark energy", a kind of exotic matter, with uniform distribution and negative pressure, driving the cosmic expansion ?
- Answering these questions poses one of the biggest theoretical challenges of modern physics.
- Future observations should be able to test with higher precision Λ's pressure-density relation. If it turns out that:

$$\frac{p}{\rho c^2} \neq -1 \longrightarrow$$ no cosmological constant, dark energy instead



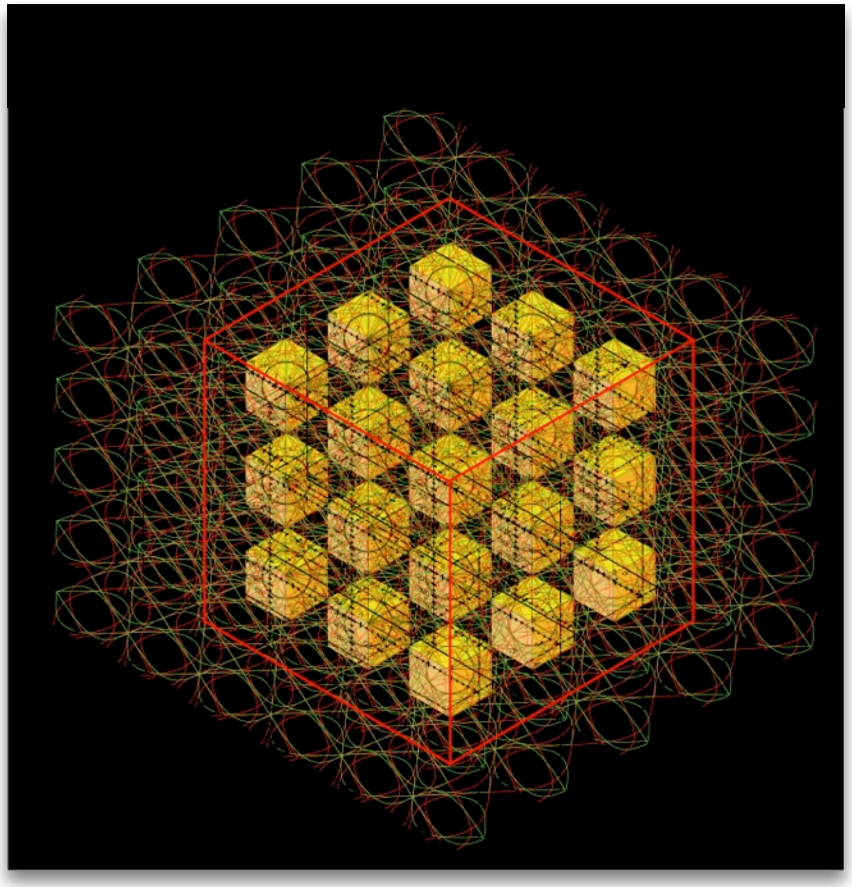
Does the vacuum gravitate?

- Heisenberg's uncertainty principle describes a fundamental property of Nature: particles and fields are never at rest.

- Even at zero temperature, atoms in a crystal have a "zero point energy" and oscillate around some average position.

- Similarly, the vacuum is filled by the zero-point energy of fields. The existence of this quantum vacuum has been verified experimentally (the Casimir effect).

- Early on, there were attempts to "explain" Λ as the gravitational contribution of the quantum vacuum. *Unfortunately, there is an enormous disagreement between the predicted value and the observed one.*



Part V
More unknown territory:
Dark Matter & Inflation

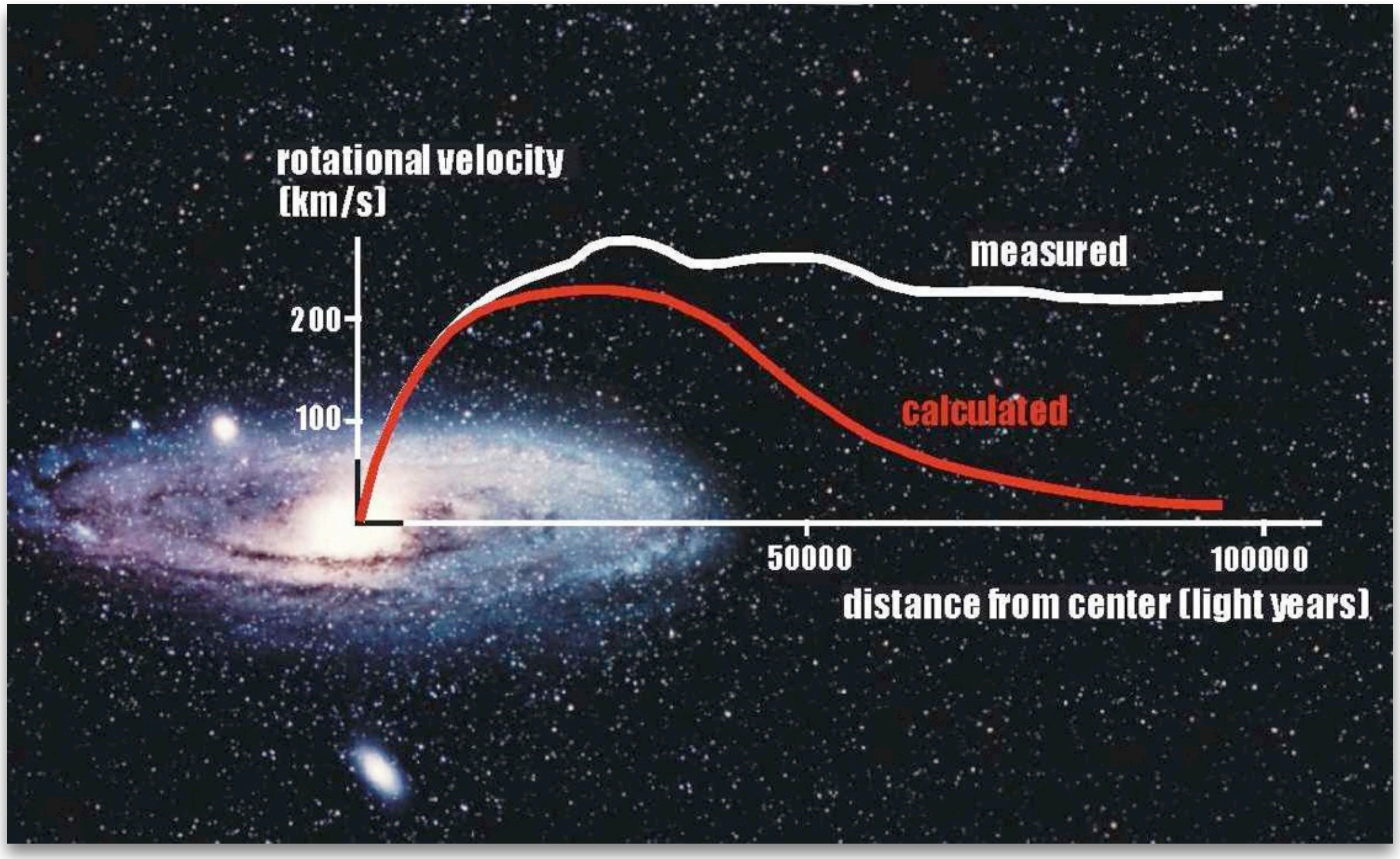
The elusive "dark matter" (I)

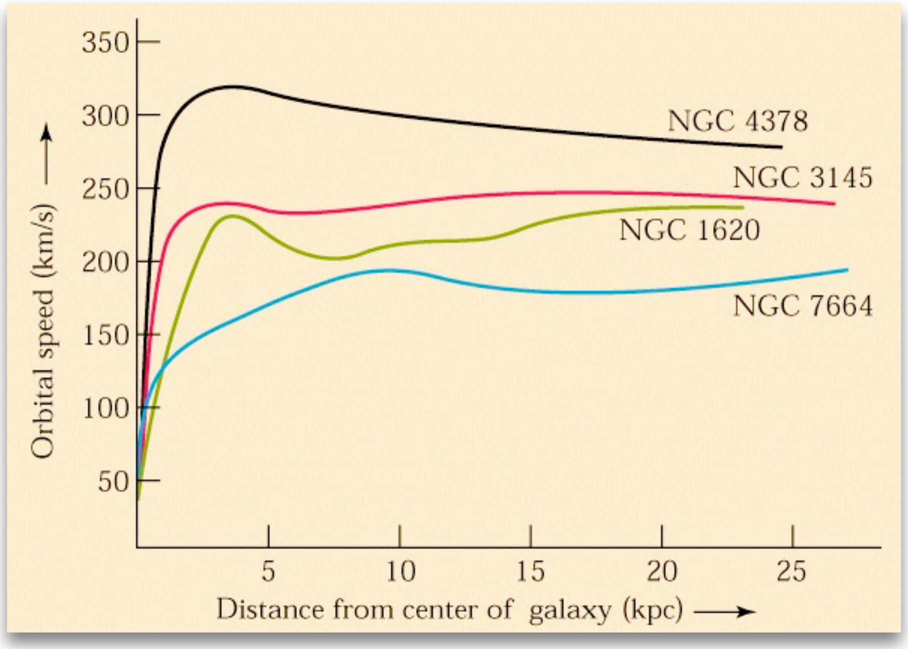
- The study of the rotational profile of galactic disks revealed something unexpected: the existence of some kind of "dark matter"!

- Dark matter does not emit any light or any other electromagnetic signal, it only interacts gravitationally. *Its exact nature remains unknown*.

Predicted profile: $v \sim 1/r$

Observed profile: $v \sim \text{constant}$





The elusive “dark matter” (II)

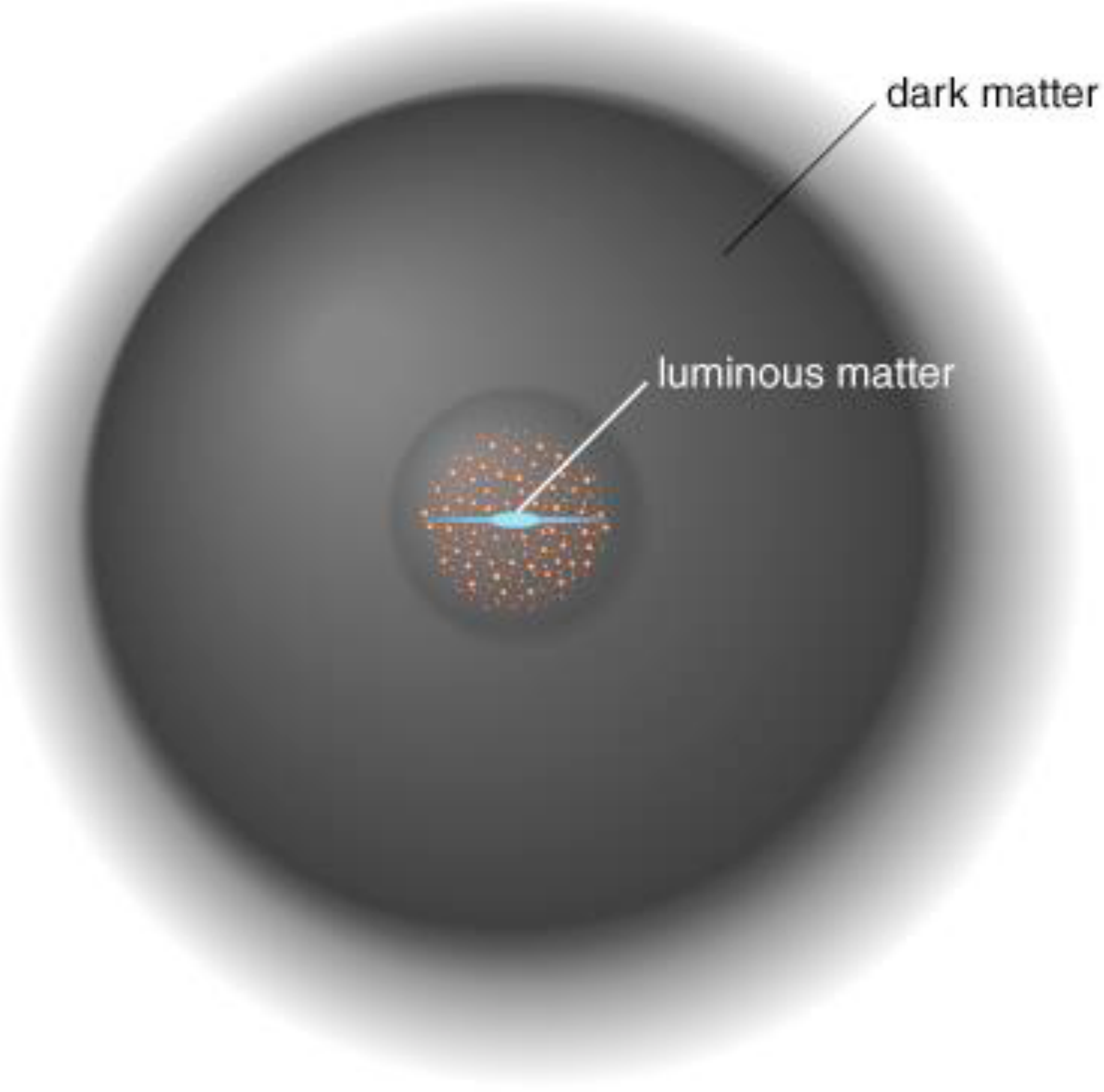
- We now know that most of the mass in a galaxy (about 90%) exists in the form of a dark matter halo.

- Additional evidence for the existence of dark matter is provided by observations of colliding galaxies and by gravitational lensing.

- The alternative interpretation, based on a modification of Newtonian gravity, is now out of fashion.

- *Dark matter dominates over normal baryonic matter:*

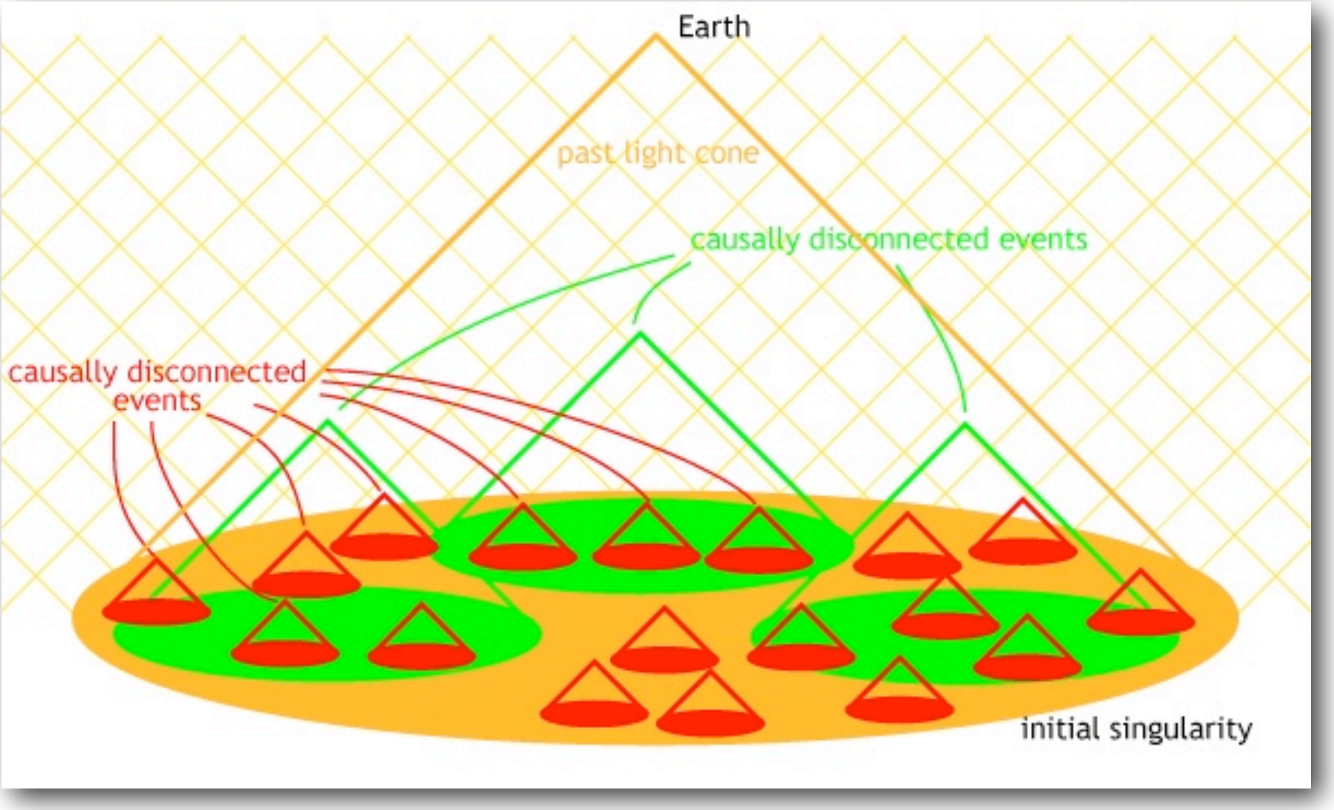
$$\Omega_{\rm mat} \approx \Omega_{\rm DM}$$



The horizon problem

- There two main problems attributed to the standard FLRW cosmological model we have discussed in this lecture:

✓ *The homogeneity problem, also known as the "horizon problem"*: spacetime regions that would not have been in causal contact in a normal radiation/matter dominated universe appear to have the same properties (e.g. the same CMB temperature).



The flatness problem

✓ *The flatness problem:*
the local Universe appears to be almost flat, that is "almost k=0".

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\underbrace{\rho}_{\sim a^{-3},\, a^{-4}} - \frac{kc^2}{a^2} + \frac{1}{3}\Lambda c^2$$

Unless we truly have k=0, it seems difficult to explain why the k-term is so much smaller than the matter/radiation term, given its slower decay during matter/radiation-dominated eras.

Inflation?

- There theory of inflation provides a resolution of the horizon and flatness problems by invoking a very brief phase of *exponential expansion* in the very early universe ($t \sim 10^{-35}\,\mathrm{s}$).

- During the inflationary phase: $a(t) \propto \exp(Ht), \quad H(t) = \frac{\dot{a}}{a} = \mathrm{constant}$

- *Solving the horizon problem*: an initially small, causally connected region, can grow and encompass the entire observable Universe.

- *Solving the flatness problem*: inflation makes the *k*-term tiny because it makes $\alpha(t)$.

- The model of inflation relies on the phase transition of a "scalar field". Its predictions are consistent with existing cosmological observations (like the CMB) but it remains a highly uncertain theory.

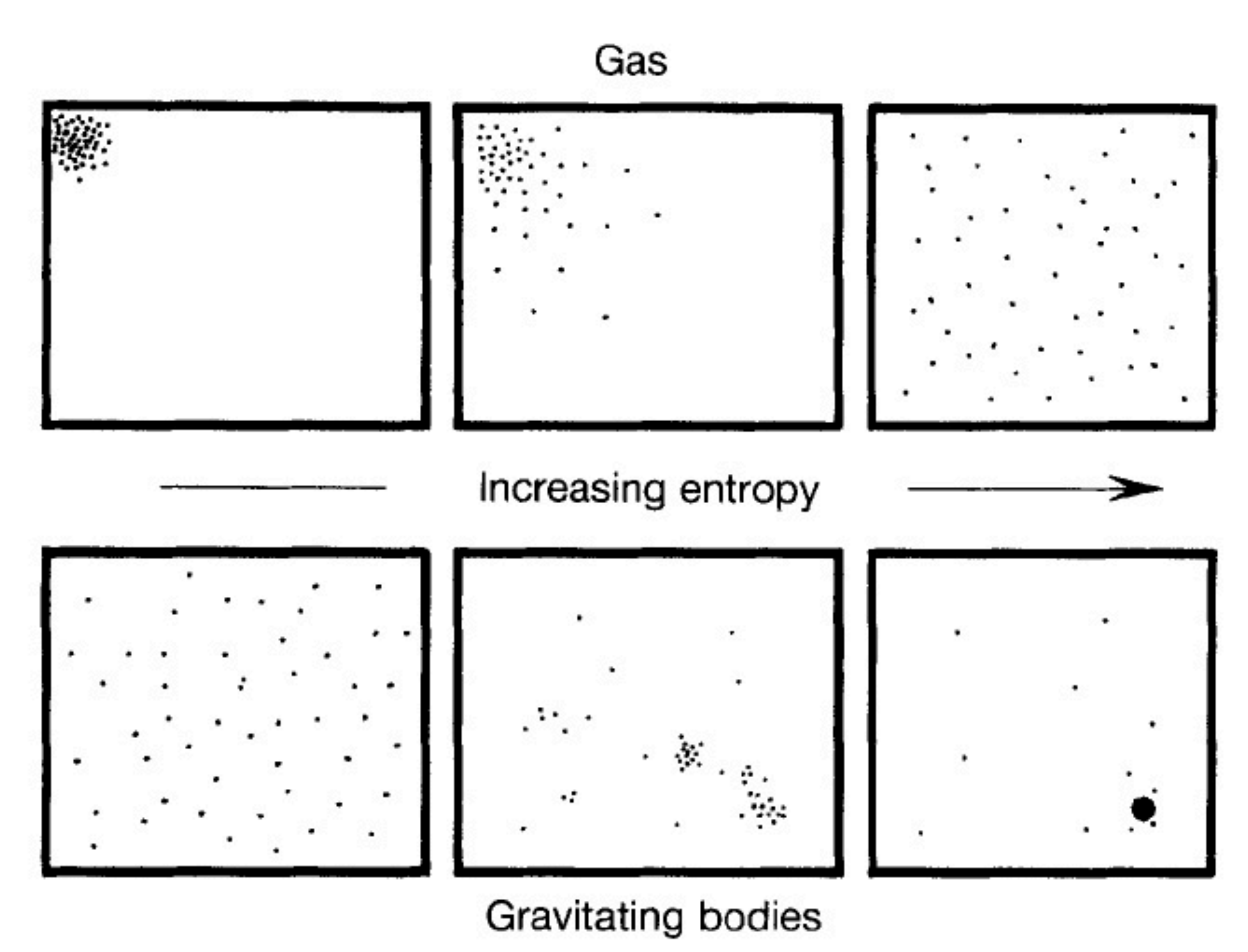
Cosmic fine tuning? Gravity & Entropy

- *Gravity imparts a fundamental change in the high entropy state of a system.* It promotes non-uniformity rather than uniformity.

- Our Universe, being homogeneous on large scales, is in fact in a state of extremely low entropy!

- In this sense, our Universe appears to be *fine-tuned.*

- *Nobody knows why is that*, it could be the result of quantum gravity physics at the moment of the big bang (t=0).



The Big Question is still open …

Epilogue

The expanding Universe: a historical perspective

Who discovered the expanding Universe?

Alexander Friedmann

Friedmann was the first to discover (1922) the model of an expanding Universe filled with matter.

However, he made no attempt to relate his mathematical model with the real world (and observations).

Who discovered the expanding Universe?

Edwin Hubble

Using the world's most powerful telescope, Hubble was able to show (1929) that "extragalactic nebalue" (galaxies) obey a linear velocity-distance law, what later became known as Hubble's law.

However, Hubble himself never claimed that he discovered the "expanding Universe". He actually never associated his empirical law with any theoretical models for the Universe.

Who discovered the expanding Universe?

George Lemaitre

After rediscovering the Friedmann models, Lemaitre was the first to make the connection between theory and actual observations, suggesting the notion of an expanding Universe.

Unfortunately, his original paper (1927) was written in French (and published in a Belgian journal) and was largely ignored by the scientific community. Only later, in the 1930s, when an english translation was available, Lemaitre's work became widely known.

Homework assignments

Assignment 1

There are two apparent "paradoxes" associated with the cosmic expansion:

(i) The large-scale Universe really expands. Does the same happen to the room you are in?

Does a self-gravitating system like our Galaxy (or our solar system) "stretch" as a result of the expanding spacetime?

(ii) We have seen that the relative velocity between two fundametnal Friedmann observers (i.e. two galaxies) $v = H(t)\, D(t)$.

This velocity can easily become superluminal for a sufficiently long proper distance D. How do you interpret this result? Is it possible?

Assignment 2

In the early 20th century it was a common belief (including Einstein himself) that the Universe was *static,* i.e

$$a(t) = a_{\rm stat} = \text{constant}$$

(i) Show that this is a valid solution of the Friedmann equations.

(ii) Show that the static universe is dynamically *unstable*.

Hint: once you find a static equilibrium try a perturbation in the scale factor:

$$a(t) = a_{\rm stat} + \delta a(t)$$

This exercise summarizes Einstein's famous "greatest blunder".

Assignment 3

You are asked to estimate the cosmic time at which matter decoupled from radiation (recombination era, formation of first hydrogen atoms).

You can assume a matter-dominated universe $13.8\ \mathrm{Gyr}$ old, and a photon gas obeying a black body thermal distribution.

You are reminded of Wien's law for the radiation wavelength at the peak energy of the black body spectrum:

$$\lambda_{\mathrm{max}} = \frac{0.3}{T}\,\mathrm{cm} \qquad (\,T \text{ measured in K}\,)$$

and you are also given that the photon energy at λ_{max} is roughly 10 times lower than the ionization energy (13.6 eV) of the hydrogen atom (this accounts for the high energy "tail" $(\lambda < \lambda_{\mathrm{max}})$ of the Planckian black body spectrum.