

# On ships, trains, and the equation of time

Anna Sajina

Because of Earth's elliptical orbit, tilted axis, and standardized time zones, high noon as measured by a clock does not coincide with the Sun's apex in the sky.

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**Early astronomers** studied the motion of the Sun and other celestial bodies to predict the changes of the seasons and determine when seeds needed to be sown. Subsequently, the equinoxes and solstices that mark seasonal transitions acquired religious connotations, and their accurate prediction became essential. Inspired by that need, the Babylonians discovered that the solar day—the time it takes Earth to complete a  $360^\circ$  rotation with respect to the Sun—varies throughout the year relative to the average, or mean, solar day, which can be determined by observations of the fixed stars. The equation of time is defined as the cumulative difference between time based on the solar day, for which noon corresponds to the Sun's highest point in the sky, and time based on the mean solar day. In other words, it is the difference between time as measured by a sundial and time as measured by a standard clock whose zero is set so that the difference between solar time and mean solar time vanishes when averaged over a year.

## Natural causes

What is responsible for the variations first recorded by the Babylonians? The length of a solar day depends primarily on Earth's rotation about its axis. A full revolution relative to the fixed stars takes approximately 23 hours, 56 minutes, as measured by a mean-solar clock. During that time Earth travels about  $1^\circ$  in its orbit around the Sun. Thus to complete the  $360^\circ$  revolution with respect to the Sun requires an extra four minutes or so. The details of the "or so" are what leads to the equation of time.

In the 17th century, astronomer Johannes Kepler discovered that each planet travels in an elliptical orbit around the Sun, sweeping out an equal area in equal time. As a consequence, Earth travels along its orbit fastest at the perihelion, when it is closest to the Sun. And that means Earth must rotate a bit more than average around its axis to complete the  $360^\circ$  rotation relative to the Sun: The solar day is longest at the perihelion, which will next arrive on 4 January 2009, Greenwich Mean Time. In the vicinity of the perihelion, the sundial runs slowly compared with a clock, and the slope of the equation of time is negative, as indicated in the figure. Conversely, the equation of time has a positive slope near the aphelion, when Earth is farthest from the Sun. (The next aphelion will be 4 July 2009.) If the elliptical orbit were the only contributor to the equation of time, the equation would be a single oscillation with zero points at the perihelion and aphelion. But there is a second key contributor: the  $23.5^\circ$  tilt of Earth's axis.

Because of that tilt, which is responsible for the seasons, the ecliptic on which the Sun appears to move is angled with respect to the celestial equator, the projection of Earth's equator onto the celestial sphere. The relation between the solar day and mean solar day is determined by the projection of the apparent solar motion onto the celestial equator. At the

equinoxes, the apparent solar motion has its steepest north-south component; the solar day is shortest as measured by a mean-solar watch, and—at least as far as the tilt effect is concerned—the slope of the equation of time is most positive. At the solstices, the projection of the apparent solar motion sweeps relatively rapidly across the equator, and the equation of time has its most negative slope. If the tilt were the only contributor to the equation of time, the zeros would be at the equinoxes and solstices.

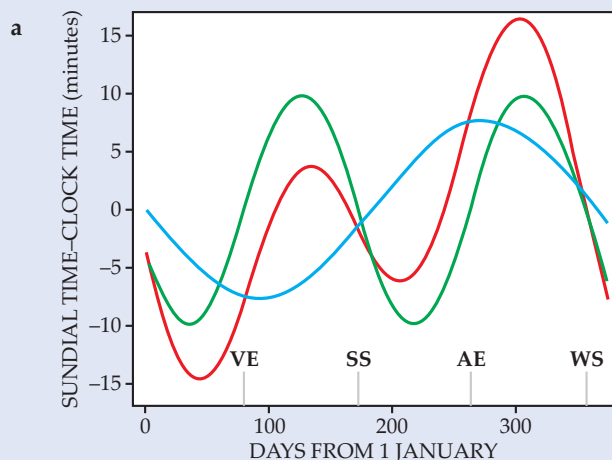
The equation of time combines the eccentricity and tilt effects. As the figure shows, sometimes the two reinforce each other, and sometimes they are counteracting. The minimum and maximum values for the equation occur in February, when the sundial will lag a mean-time watch by 14 minutes, and in November, when the sundial will be ahead by 16 minutes.

A dramatic manifestation of the equation of time, also shown in the figure, is the analemma—the shape made by the Sun if it is photographed several times through the year, but always at the same mean solar time. The up-down motion of the Sun reflects the tilt of Earth's axis. The east-west motion reflects tilt and the effect of the elliptical orbit. If Earth's orbit were circular, the analemma would be a figure eight shape; because the orbit is an ellipse, the top of the figure eight is pinched.

## A need for standardization

Through the centuries, sundials, sand clocks, water clocks, and other devices allowed fairly accurate timekeeping, but they relied on good weather or functioned only for a limited time. Eventually, more accurate and reliable timekeeping was necessary; increasing urbanization meant a more hectic lifestyle, which was hampered if workers didn't show up on time or if merchants couldn't predict the tides and plan the loading and unloading of their ships accordingly. The rise of the city led to an explosion in mechanical clock building, which quickly became synonymous with craftsmanship and the power of technology in the new industrial age. Towns typically built clocks in a central square to strike the hours as determined by local mean solar time. European powers were beginning to build their naval empires but were constrained by a single problem—how to determine longitude while on a ship in the middle of the ocean.

In 1665 the Dutch natural philosopher and clockmaker Christiaan Huygens wrote his treatise "A Study of the Use of Clocks to Determine the Longitude East and West." In essence, he argued that due to Earth's rotation, the Sun reaches its highest point later as one moves west and earlier as one moves east. All that is needed to determine one's longitude, therefore, is to observe the solar time, translate it to local clock time, and compare that time with the reading from



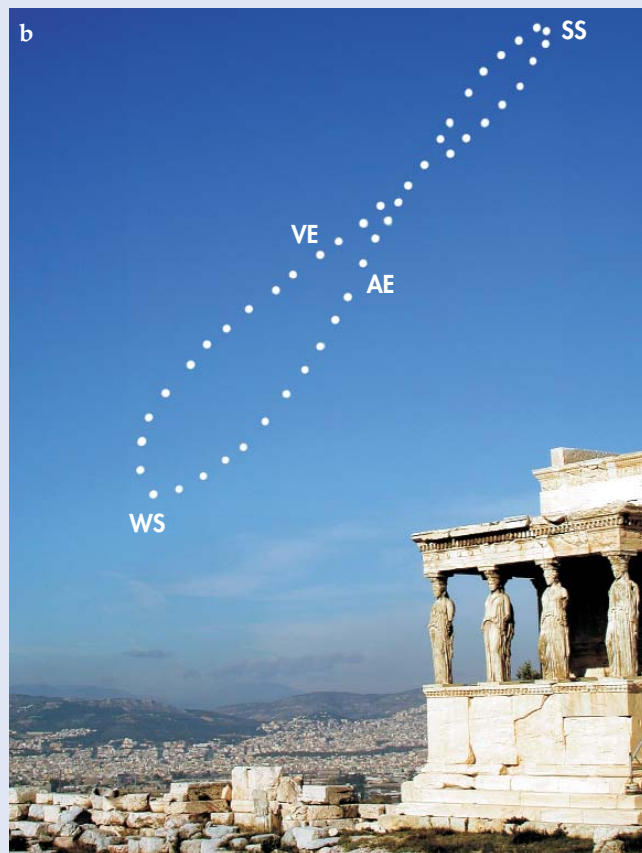
**The equation of time and the analemma. (a)** The equation of time (red) is the difference in time as measured by a sundial and a clock. Its two main contributions arise from the eccentricity of Earth's orbit (blue) and the tilt of Earth's axis (green). **(b)** The analemma is the shape traced out by the Sun when it is photographed several times over the course of the year, but always at the same clock time of day. Indicated on the panels are the vernal and autumnal equinoxes (VE, AE) and the summer and winter solstices (SS, WS) for the Northern Hemisphere. The perihelion is in early January, the aphelion in early July. (Panel b courtesy of Anthony Ayiomamitis.)

a clock set to give the time at a reference of known longitude. That reference would become the Greenwich observatory, founded by Charles II in 1675 with the aim of helping to solve the longitude problem. Huygens's treatise contains what may be the earliest modern table of the equation of time. Such tables would become staples in the nautical almanacs used by navigators for centuries to come. But that would have to wait for more than 100 years, since the best clocks of the time, pendulum clocks, were useless when confronted with the motion of the sea.

In 1714 the British Parliament offered a prize of £20 000 to the man who managed to devise a suitable clock and method of obtaining a ship's longitude with an accuracy of 30 nautical miles (about 60 km). The prize was eventually won in 1773 by John Harrison, who had labored on the problem for more than 50 years. James Cook undertook his second voyage guided by Harrison's chronometer and a table of the equation of time; that expedition heralded the golden age of ocean exploration, trade, and emigration.

Until the late 19th century, cities still set their clocks to the local mean solar time. Meanwhile, railways were becoming the principal mode of transporting goods and people. In the 1860s the railroad joined California with the eastern US. Trains provided a convenient means of long-distance travel, but the numerous local time standards greatly complicated train schedules. In 1870 Charles Dowd was the first to propose time zones that would allow for simplified timetables. Nine years later Canadian railway engineer Sandford Fleming proposed dividing Earth into 24 time zones, each covering 1 hour out of the 24-hour mean solar day.

Time zones were first used in 1883 by the US railroads. Modern time zones cover roughly one hour of solar transit, but there is considerable variation. Depending on where you live within a time zone, the time read by your watch may differ from the mean solar time by 30 minutes or more. Thus the discrepancy between watch time and solar time introduced



with human-constructed time zones could well exceed the maximum discrepancy accounted for by the equation of time. If you use daylight-saving time, your watch can be an additional hour off from the solar time.

The equation of time is not fixed. Over long time scales, Earth's orbit is subject to periodic changes known as Milankovitch cycles. Earth's eccentricity varies between 0 and 0.05 on a 100 000-year cycle. Its axial tilt oscillates in the range of 21.5°–24.5° over a period of 41 000 years. The axis itself precesses with a 23 000-year period. As a result, the perihelion will not always be close to the northern winter solstice, as it is today. In about 11 000 years, it will be in summer. Then the northern summer will also see the longest solar days; the seasonal contrast will be even greater than it is at present.

Each planet in the solar system has its unique orbital parameters and its distinct equation of time. For example, on Mars, the difference between solar and clock time can be as great as 50 minutes; much of that number can be attributed to the eccentricity of the Martian orbit, which is larger than Earth's. The red planet's relatively large eccentricity leads to a second interesting consequence—the analemma is so pinched at the upper lobe that it looks like a teardrop.

### Additional resources

- To learn more about the equation of time, see <http://www.analemma.com/Pages/framesPage.html>.
- D. Sobel, *Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time*, Walker, New York (1995).
- T. S. Yeow, "The Analemma for Latitudinally-Challenged People" (2002), <http://www.math.nus.edu.sg/aslaksen/projects/tsy.pdf>.

The online version of this Quick Study includes a simulated photograph of the Martian analemma.