FÍSICA DEL COSMOS 2014/15

PROBLEM SHEET FC5

10. Write down the dynamical equation for the flat FLRW models in terms of $(\Omega_r, \Omega_m, \Omega_\Lambda, \Omega_k)$ in the case when there is no radiation $\Omega_r = 0$ but both Ω_Λ and Ω_m are present. How large would Ω_Λ have to be for the universe to be accelerating at the present time and how small would then Ω_m have to be?

11. Assume that you are monitoring the shape of a cube of cosmological dimensions (consisting of test particles) under the influence of cosmic expansion. You observe that with time the cube does not retain its initial shape, becoming elongated along the z-axis, while each $z = \text{const}$ slice retains its square x − y shape. Write down a spatially homogeneous spacetime line element consistent with this observation.

12. You are asked to estimate the cosmic time at which matter decoupled from radiation (recombination era, formation of first hydrogen atoms). You can assume a matter-dominated universe 13.8 Gyr old and a photon gas obeying a black body thermal distribution.
    You are reminded of Wien's law for the radiation wavelength at the peak energy of the black body spectrum: $\lambda_{\max} = \frac{0.3}{T(\mathrm{K})}\,\mathrm{cm}$, and you are also given that the photon energy at $\lambda_{\max}$ is roughly 10 times lower than the ionization energy (13.6 eV) of the hydrogen atom (this accounts for the high energy "tail" $\lambda < \lambda_{\max}$ of the Planckian black body spectrum.