DESVIACIÓN DE LA LUZ

otro test clásico de la R.G.
eclipse 1919 (Eddington, Davidson, Dyson)

$\frac{\tilde{L}}{\tilde{E}} = b > \sqrt{27}\,M$ porque hay "rebote" $\Rightarrow \tilde{L} \neq 0$

$r=\infty$) $\quad \varphi_\infty \equiv 0$

$\tilde{E} = \left[1-\frac{2M}{\infty}\right]\frac{dt}{d\lambda} = 1$ tomando $\frac{dt}{d\lambda} \equiv 1$ en $r=\infty$

$d\phi \simeq 0 \xrightarrow[\vec{u}\cdot\vec{u}=0]{\text{métrica}} -[\,]\frac{dt^2}{\cancel{d\lambda^2}} + \frac{dr^2}{[\,]} = 0$

$\Rightarrow \frac{dr}{dt} = \pm\left[1-\frac{2M}{\infty}\right] = -1 \equiv V_\infty$ a $r=\infty$

si elijo tomar como parámetro de trayectoria el tiempo del obs. en infinito, entonces $\tilde{E}=1$

$\forall r$) $\quad \tilde{E} = [\,]\frac{dt}{d\lambda}$ con $\tilde{E}=1 \Rightarrow \left(\frac{dt}{d\lambda}\right)^2 = \frac{\tilde{E}^2}{[\,]^2} = \frac{1}{[\,]^2}$

$\tilde{L} = r^2\frac{d\phi}{d\lambda}$

Métrica o $\vec{u}\cdot\vec{u}=0 \Rightarrow -[\,]\left(\frac{dt}{d\lambda}\right)^2 + \frac{\left(\frac{dr}{d\lambda}\right)^2}{[\,]} + r^2\left(\frac{d\phi}{d\lambda}\right)^2 = 0 \Rightarrow \frac{\left(\frac{dr}{d\lambda}\right)^2}{[\,]} + \frac{\tilde{L}^2}{r^2} - \frac{1}{[\,]} = 0 \quad (*)$

$\frac{\tilde{L}^2}{r^2}$

$\Rightarrow \frac{dr}{d\lambda} = -\sqrt{\left(\frac{1}{[\,]} - \frac{\tilde{L}^2}{r^2}\right)[\,]} = -\sqrt{1-[\,]\frac{\tilde{L}^2}{r^2}} = -\sqrt{\frac{r^2-[\,]\tilde{L}^2}{r^2}} \Rightarrow \frac{d\lambda}{dr} = -\frac{r}{\sqrt{r^2-[\,]\tilde{L}^2}}$

Buscamos $\frac{d\phi}{dr} = \frac{d\phi}{d\lambda}\frac{d\lambda}{dr} = -\frac{\tilde{L}}{r^2}\frac{r}{\sqrt{r^2-[\,]\tilde{L}^2}} = \frac{-1}{r\sqrt{\frac{r^2}{\tilde{L}^2}-[\,]}} = \frac{-1}{\sqrt{\frac{r^4}{\tilde{L}^2}-[\,]r^2}}$

Integramos: $\varphi(r) - \overset{0}{\varphi(\infty)} = \varphi(r) = \int_\infty^r -\frac{dr}{\sqrt{\ }} = \int_r^\infty \frac{dr}{\sqrt{\frac{r^4}{\tilde{L}^2}-[\,]r^2}}$

El ángulo de salida será $\varphi_s = 2\varphi(r_0)$ y la desviación $\delta \equiv 2\varphi(r_0) - \pi$

r_0) Calculemos $\varphi(r_0) = \int_{r_0}^\infty \frac{dr}{\sqrt{\ }} = \int_{u_0}^0 -\frac{du}{u^2\sqrt{\frac{1}{u^4\tilde{L}^2} - \frac{[1-2Mu]}{u^2}}} = \int_0^{u_0}\frac{du}{\sqrt{\frac{1}{\tilde{L}^2} - u^2[1-2Mu]}}$

c.v. $u \equiv \frac{1}{r}$; $du = -u^2 dr$

Busquemos $\tilde{L}^2 = f(M, r_0)$ como en "precesión perihelio"

$\left.\frac{dr}{d\varphi}\right]_{r_0} = 0$ sólo hay tangencial $\overset{(*)}{\Rightarrow} \frac{\left(\cancel{\frac{dr}{d\varphi}}^{\,0}\frac{d\varphi}{d\lambda}\right)^2}{[\,]} + \frac{\tilde{L}^2}{r^2} - \frac{1}{[\,]} = 0 \Rightarrow \tilde{L}^2 = \frac{r_0^2}{\left[1-\frac{2M}{r_0}\right]}$

Sigo con $\varphi(r_0) = \int_0^{u_0}\frac{du}{\sqrt{[1-2Mu_0]u_0^2 - u^2[1-2Mu]}} = \int_0^{u_0}\frac{du}{\sqrt{u_0^2-u^2}}\left[1+M\left(u+\frac{u_0^2}{u+u_0}\right)\right]$

Caso del Sol ($2M_\odot \simeq 3$ Km) $\Rightarrow \frac{2M_\odot}{r_0} << 1$, $\frac{2M_\odot}{r} << 1$

$u_0^2 - 2Mu_0^3 - u^2 + 2Mu^3 = (u_0^2-u^2)\left(1-2M\frac{u_0^3-u^3}{u_0^2-u^2}\right)$

$\left\{1-2M\frac{u_0^3-u^3}{u_0^2-u^2}\right\}^{-1/2} \simeq 1 + M\frac{u_0^3-u^3}{u_0^2-u^2}$

$\frac{u_0^3-u^3}{u_0^2-u^2} = \frac{(u_0^2+u^2+u_0u)(u_0-u)}{(u_0+u)(u_0-u)} = u + \frac{u_0^2}{u_0+u}$

$\int_0^{u_0}\frac{du}{\sqrt{u_0^2-u^2}} = \arcsin\left.\frac{u}{u_0}\right]_0^{u_0} = \frac{\pi}{2}$

$\int_0^{u_0}\frac{du\cdot u}{\sqrt{u_0^2-u^2}} = -\left.\sqrt{u_0^2-u^2}\right]_0^{u_0} = u_0$

$\int_0^{u_0}\frac{du}{\sqrt{u_0^2-u^2}}\frac{1}{u+u_0} = \left.\frac{2\sqrt{-u+u_0}}{(-u_0-u_0)\sqrt{u+u_0}}\right]_0^{u_0} = \frac{1}{u_0}$

$\Rightarrow \boxed{\delta = 4Mu_0 = \frac{4M}{r_0} = \frac{4GM}{c^2 r_0}}$

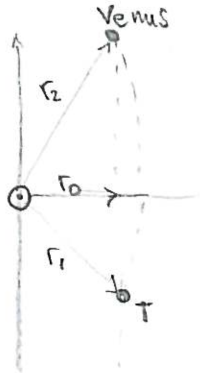
Rayo que pase rozando la sup. del Sol $\delta_\odot = 1.75''$

Medición eclipse 1919: $\delta_\odot = 1.98 \pm 0.16$

b parámetro de impacto (en realidad $b^2 = \frac{r_0^2}{\left[1-\frac{2M}{r_0}\right]}$)

QQ

QQ



$r >$ ~ 100 μs, verificado Marina, Viking

trayecto curvado ⇒ longitud mayor ⇒ retraso respecto a caso clásico

$\tilde{L}^2/r^2$ $\quad \frac{[\ ]^2}{\tilde{E}^2} = [\ ]^2$

Buscamos $r(t)$ ó $r(\tau)$:

métrica o $\vec{u}\cdot\vec{u}=0 \Rightarrow \frac{0}{dt^2} = -[\ ]\frac{dt^2}{dt^2} + \frac{dr^2}{[\ ]dt^2} + r^2\frac{d\varphi^2}{d\lambda^2}\frac{d\lambda^2}{dt^2}$

$$\Rightarrow \left(\frac{dr}{dt}\right)^2 + [\ ]^3\frac{\tilde{L}^2}{r^2} - [\ ]^2 = 0$$

Tiempo entre r_0 y r: $t(r_0,r) = \int_{r_0}^{r} \frac{dr}{[\ ]\sqrt{1-\left(\frac{r_0}{r}\right)^2\frac{\left[1-\frac{2M}{r}\right]}{\left[1-\frac{2M}{r_0}\right]}}}$

Desarrollando en $\frac{2M}{r}$ se llega a $t(r_0,r) = \sqrt{r^2-r_0^2} + 2M\ln\frac{r+\sqrt{r^2-r_0^2}}{r_0} + M\sqrt{\frac{r-r_0}{r+r_0}}$

No olvidar que los relojes que miden el retraso están sometidos a campo gravit.:

$$\Delta\tau_{\oplus} = \sqrt{[\ ]}\ \Delta t_{\infty}$$

⇒ predicciones de Einstein confirmadas con precisión del 0.1 %

gyr 9.11

QQ Partimos de $\frac{1}{b^2} = \frac{\tilde{E}^2}{\tilde{L}^2} = \frac{1}{\tilde{L}^2}\left(\frac{dr}{d\lambda}\right)^2 + W^{FOT} \Rightarrow \frac{dr}{d\lambda} = \tilde{L}\sqrt{\frac{1}{b^2}-W^{FOT}}$; $\frac{d\phi}{dr} = \frac{d\phi}{d\lambda}\frac{d\lambda}{dr} = \frac{\tilde{L}}{r^2}\left(\pm\frac{1}{\tilde{L}\sqrt{\frac{1}{b^2}-W^{FOT}}}\right)$

con $W^{FOT} \equiv \frac{[\ ]}{r^2}$; $\qquad \Delta\phi = 2\int_{r_0}^{\infty}\frac{dr}{r^2}\frac{1}{\sqrt{\frac{1}{b^2}-\frac{[\ ]}{r^2}}}$

r_0) $\frac{1}{b^2} = \frac{\left[1-\frac{2M}{r_0}\right]}{r_0^2}$ Comparar con anverso $\tilde{L}^2 = \frac{r_0^2}{\left[1-\frac{2M}{r_0}\right]}$ $\Longrightarrow b^2 = \tilde{L}^2 \Rightarrow \tilde{E} = 1$ que habíamos elegido gracias a la libertad en parámetro afín λ