4.5 Measures of time

The study of celestial motions requires a definition of "time". It turns out that time keeping is not a simple process. There are many factors that contribute to this complexity. It was difficult in early times when clocks at different geographic locations could not be easily synchronized, and it is difficult in modern times as one pushes to obtain greater and greater accuracy, even with the aid of atomic clocks.

We humans instinctively conceptualize time as a smoothly running entity that can be agreed upon by all observers. This is the *Newtonian model* of time. That is, they can synchronize their watches (time and rate) and agree on the time of events at different locations. However, when high velocities or strong gravity are encountered, the comparisons are no longer so simple. General relativity (GR) gives a different model of time, taking into account the effects of speed and gravity. Special relativity takes into account the effect of speed.

In contrast to these models, there are *nature's clocks* which can be used to keep track of time. We have already seen that Caesar's model of time (his calendar; Section 3) did not track well nature's clock, the passing of the seasons. Manufactured clocks make use of nature's clocks. Traditionally they have been based on the natural frequency of the pendulum or that of the spring/mass oscillator. More recently the quartz crystal is commonly used to produce much more stable electrical oscillations. Each oscillation can be thought of as a tick of time, and time in "seconds" is defined in terms of a number of ticks. The SI standard of time is now based on oscillations of the cesium atom in *atomic clocks*.

Nature's astronomical clocks are the daily rotation period of the earth, the earth's annual rotation about the sun, and a rapidly rotating neutron star, seen as a regularly (up to a point) pulsing *radio pulsar*. A few of these pulsars exhibit an extremely high degree of stability that begins to rival that of atomic clocks, but they have not yet been used as a time standard.

One must keep in mind the distinction between a model of time and a clock of nature. One can tell time no better than the most stable clock one has. It may drift in rate, but there is no way to know it until a more stable clock is invented or discovered. In contrast, the model of time specifies ideally how time behaves, as a function of location, speed, and gravity. For example, the Newtonian model describes a time that is infinitely stable and everywhere the same. The story of time is one of the development and discovery of increasingly more stable clocks together with the development of models that successfully explain the subtleties of time keeping these improved clocks reveal.

Time according to the stars and sun

The most fundamental time keeping is based on the motions of the sun and stars as they pass overhead with seasonal variations.

Sidereal time

As the earth rotates under the sky, the zenith of a given observatory (e.g., Palomar Mountain) moves along the celestial sphere from west to east (Fig. 3.1). Thus, as we have described, a typical celestial object will rise in the east and move westward *relative* to Palomar Mountain. At any given time, the meridian of right ascension directly over the observatory is by definition the *sidereal time*, e.g., 15 h 25 m 35 s. A star at right ascension, $\alpha = 15$ h 25 m 35 s, crosses over the longitude meridian of Mt. Palomar at sidereal time 15 h 25 m 35 s (by definition). At this sidereal time, stars at $\alpha = 16$ h will not yet have transited the observer's meridian, and stars at $\alpha = 15$ h 00 m will have already transited it.

Sidereal time is strictly a *local time*. To set his sidereal clock, the observer need only look overhead to see what meridian of the celestial sphere is passing overhead; its right ascension is the time to which he would set the clock. If, at some instant, the observer compares his sidereal time to that of another observer at a different longitude, by radio signal for example, they would report different sidereal times.

Mean solar time

Solar time is another local time; it indicates the location of the observer's meridian with respect to the sun. By definition, the sun is overhead (or due south or north of the zenith) at noon, solar time. Solar time varies in rate relative to a fixed-rate universal time because of the eccentricity of the earth's orbit and the tilt of the earth's axis relative to the normal to the earth's orbital plane. *Mean solar time* averages out these variations. The difference between solar time and mean solar time kept by clocks varies by as much as 16 min during the year; the difference is known as the *equation of time*. Clock makers used to put a table of the time difference for each day of the year on their clocks.

Mean solar time still lacks something in convenience; it will differ in two nearby towns if one is west or east of the other because the sun will pass over one meridian before it passes over that of the other. To solve this problem, everyone in a given region (time zone) agrees to keep the time of an observer at the central meridian of the region. This is called *zone time*, e.g., Eastern Standard Time. There are 24 such time zones, each 15° wide, more or less. The historic observatory at Greenwich England lies at the zero of geographic longitude. The zone time of this region has been called *Greenwich Mean Time* (GMT). This system of time zones dates back to 1884.

Astronomers find it convenient to use the modern equivalent of GMT (Universal Coordinated Time UTC, see below) during their observations. Frequently, observations at different observatories must be compared, and comparison of data from different time zones can lead to confusion. Astronomers tend to use the date of the beginning of an observing night to label that night's data even if the data were actually taken after midnight. This can add more confusion. In contrast, the UTC time *and* date (at Greenwich) of each specific observation during the night is unambiguous.

Sidereal and solar (or zone) times are not synchronized to one another because the former is referenced to the stars and the latter to the sun which moves through the stars as viewed from the earth. Since the sun moves along the ecliptic, the lengths of the two types of days must differ slightly, by about 1 part in 365, or by about 4 min. Which is the longer? Since the sun moves along the celestial sphere in the same direction that the earth spins, a given point on the earth must move more than 360° (relative to the stars) in order to catch the sun. The solar day is thus longer than a sidereal day.

The mean solar day is divided into $24 \times 60 \times 60 = 86\,400$ UT seconds which are slightly elastic(!) due to small irregular variations in the earth spin rate (see below). In civil life, we keep track of time with fixed atomic or SI seconds. Thus,

$$\begin{aligned} 1 \text{ Mean solar day} &= 24\text{ h } 00\text{ m } 00\text{ s (solar time)} \\ &= 86\,400.00\text{ s (solar time)} \\ &(\approx 86\,400\text{ s atomic SI time)} \end{aligned} \tag{4.12}$$

The shorter sidereal day is

$$\begin{aligned} 1 \text{ Sidereal day} &= 23\text{ h } 56\text{ m } 04.09\text{ s (mean solar time)} \\ &= 86\,164.09\text{ s (mean solar time)} \\ &(\approx 86\,164.09\text{ s, atomic SI time)} \end{aligned} \tag{4.13}$$

which tells us that the sidereal day is shorter than the solar day by 235.91 s $\approx$ 4 min.

The sidereal day is referenced to the vernal equinox which moves slowly through the sky in the westward direction (Fig. 4). The day relative to the fixed stars is thus about 0.01 s longer than that given in (13). The relation (13) is valid even as the earth rotation rate varies irregularly because the mean solar day lengthens proportionally with the sidereal day.

The mean solar day is now defined such that, in about 1820, it would have been exactly 86 400 SI seconds in duration. It is now somewhat greater, by about 2 ms, due to the decreasing (on average) rotation rate of the earth. See discussion of UTC time and leap seconds below.

Universal and atomic times

Universal time (UT) and earth spin

The modern equivalent of GMT is *universal time*. There are two principal versions of universal time, UT (= UT1), and UTC. Here we discuss the former; the latter will come soon. The UT time is obtained from the overhead passages of stars rather than the passage of the sun, but it is continually adjusted (in principle) to closely approximate mean solar time at Greenwich such that the mean sun is on the meridian at noon and one day equals precisely 86 400 s. This time standard thus uses the earth spin as the clock.

Formally, UT is defined in terms of the sidereal time at Greenwich. Specifically, 0 h UT (midnight in Greenwich) on Jan. 1 is defined to occur at Greenwich mean sidereal time (GMST) $\approx$ 6.7 h. Examine Fig. 3.1 and note that on Jan. 1, which is 285 d after Mar. 21, the sun is about 18.7 h east of the equinox [$(285\text{ d}/365\text{ d}) \times 24\text{ h} \approx 18.7$ h], that is, $\alpha_\odot \approx 18.7$ h. At midnight in Greenwich on this date, the zenith will be on the side of the celestial sphere exactly opposite (in RA) to the sun, i.e., at $18.7 - 12 = 6.7$ h. The sidereal time at Greenwich will thus be about 6.7 h at 0 h UT on Jan. 1, in accord with the definition.

Variations in the earth spin rate result in variations in the rate of time based on star transits, relative to an ideal Newtonian model. These variations can arise from tidal friction which slows the earth spin, varying angular momentum carried by oceans and the atmosphere which cause seasonal variations in the rotation rate and in the motion of the earth's spin axis relative to the earth's crust (*polar motion*). In large part these changes are unpredictable. UT has been corrected for polar motion (UT0 is the uncorrected version), but UT still slows and speeds up with the earth spin. The duration of a second or a minute actually grows and shrinks so that 86 400 s carries one from one mean solar transit to the next. For precise measurements, a variable second is not very useful. We use atomic time for that; see below. The precise relation between UT and GMST is presented in the next section.

Greenwich mean sidereal time (GMST) at 0 h UT

As stated above, one can set a UT clock by observing the stars passing over Greenwich at midnight on Jan. 1. When the appropriate meridian of the celestial sphere (sidereal time ~6 h 42 m = 6.7 h on Jan. 1) transits the zenith, it is exactly 0 h UT. The current relation used to set UT in terms of the star transits at Greenwich gives Greenwich mean sidereal time at 0 h UT in terms of the day of the year,

$$\begin{aligned} \text{GMST at 0 h UT} = {} & 6\text{ h } 41\text{ m } 50.548\,41\text{ s} \\ & + (8\,640\,184.812\,866\,T \qquad \text{(Definition of 0 h UT)} \\ & + 0.093\,104\,T^2 \\ & - 6.2 \times 10^{-6}\,T^3)\text{ s} \end{aligned} \tag{4.14}$$

where the parameter T specifies the day for which one desires to find the GMST at 0 h UT. Recall that the part of the sky that is overhead at midnight in Greenwich changes with the seasons. Here T is measured in Julian centuries of 36 525 d from epoch J2000.0 (Jan. 1, 12 h UT, 2000). Specifically, $T = d/(36\,525)$ where d is the number of days from epoch J2000.0. The day count d is constrained to the values of ± 0.5, ± 1.5, $\pm 2.5\ldots$ because it is applied only at 0 h UT. (We discuss epochs below.)

The first term on the right of (14) specifies the ~6.7 h sidereal time discussed above. The second term is linear with time; in one (Julian) day, it advances the GMST at 0 h UT by 236.55 sidereal seconds to take into account the difference between the sidereal and solar days. (The difference between (12) and (13) is somewhat less, 235.91 s, because it is in solar seconds. The two kinds of seconds differ by one part in 365.) The numerical coefficient of this (second) term is the number of seconds of sidereal time that must be added for this correction in a Julian century. As an example, at 2000 Jan. 1, 0 h UT, the date is $d = -0.5$ d and this term subtracts 118.28 s (ST) (=236.55/2) from the first term as expected for the solar-day correction in 12 h of elapsed time. Twenty-four hours later, the correction is +118.28 s, the next day +384.83 s, and so on until in 1 yr the correction would correspond to 24 h bringing the GMST at 0 h UT back to ~6.7 h ST. Finally, the two non-linear terms correct for polar motion; in one century ($T = 1$) the correction is ~0.1 s.

Ephemeris second

Beginning in the late 1920s, it was realized that there were better clocks than the variable earth spin period. Accordingly, the highly stable orbital motion of the earth about the sun was adopted as a reference. Thus in 1958, the *ephemeris second* was defined as a fixed fraction of the "tropical year 1900", namely $1/(31\,556\,925.9747)$. Recall that the tropical year is the time required for the sun to travel from vernal equinox to vernal equinox on the celestial sphere. The ephemeris second was free of the vagaries of earth spin.

Atomic time (TAI)

Atomic clocks introduced a new level of accuracy to time keeping in the late 1940s. Comparisons of different atomic clocks indicated that these uncertainties were about 1 second in 3000 yr, or 0.3 ms in 1 year, or better. This means that the time of a clock tick after one year would have this uncertainty. This is 1 part in 10^{11} since there are $\sim 1 \times 10^{11}$ seconds in 3000 yr. There are several types of atomic clocks such as cesium beams, hydrogen masers, rubidium vapor cells, and mercury ion frequency standards. They have differing accuracies ranging up to 1 part in 10^{14} in a day. The fractional accuracies can differ for different time intervals.

The accuracy of atomic clocks permitted the demonstration by 1972 that the earth's spin period (the traditional clock) is lengthening at a variable rate averaging ~1.7 ms per century, most of which is attributable to tidal action. This is not as small as it sounds because each day the small changes accumulate. After only one year at this slowdown rate, the earth clock would be ~3.1 ms retarded. (After six months, the day would be 1.7 ms/200 longer than at the beginning. Take this as the average excess period during the year and multiply by 365.25 d in a year to obtain 3.1 ms.) Thus the earth clock is good to 1 part in (1 yr/3.1 ms) = 1.0×10^{10}, or about 10 times worse than our rudimentary atomic clock with 1 part in 10^{11} accuracy. Thus atomic time became a better standard than the earth's spin.

In 1967, the *atomic second* (now called the *SI second*) was adopted as a fundamental unit of measurement,

$$1.0 \text{ atomic second} = 9\,192\,631\,770 \text{ cycles of } Cs^{133} \quad \text{(SI second)} \qquad (4.15)$$

where the cycle count refers to the radiation from a ground-state transition between two hyperfine levels of Cs^{133}. The number of cycles was chosen to agree with the ephemeris second. *Atomic time* was defined to agree with UT at 1958.0. Currently, the atomic time standard is TAI (*Temps Atomique International*). It is based on the average of ~150 atomic clocks in 30 countries. Note that UT and TAI run independently of one another; the one tied to the transits of the stars (i.e., earth spin) and the other to an atomic standard. TAI is stable to about 30 μs in a century, or 1 part in 10^{14}. Expected advances, e.g., with cold cesium in space, could yield stabilities 100 times better.

Universal coordinated time (UTC) and leap seconds

In 1972, *Universal Time Coordinated* (UTC) was adopted to bring together the UT and TAI systems. It is based on the atomic second but it is occasionally adjusted by the addition of an extra second, a *leap second*, to maintain it within 0.9 s of UT. In principle, the leap second could be subtracted, if necessary to maintain the 0.9 s difference.

Leap seconds are inserted into UTC when needed at midnight on June 30 or Dec. 31. Sometimes a Dec. 31 will last 86 401 s! Compare to the insertion of 3 months by Caesar in 46 BCE. This method allows UTC to reflect the changing spin period of the earth while maintaining the unit of 1 s as a very stable atomic unit of time.

One might ask why one adds an entire leap second every year or two when we argued above that the earth clock is retarded by only ~3.1 ms in a year. Recall that this was the retardation relative to the tick rate at the beginning of the year. The leap-second corrections refer, though, to the date when the day was exactly 86 400 s long, which occurred in ~1820, or ~180 yr ago. Since that time, the day has lengthened by ~1.7 ms/century × 1.8 centuries ≈ 3.1 ms. (Do not confuse this "length of day" with the 1-yr "clock retardation" value of 3.1 ms.) This would suggest that the day is now about 86 400.003 s in duration. In one year, after 365 rotations at this period, the discrepancy relative to the 86 400-s day would build up to 3.1 ms × 365.25 ≈ 1.1 s. Hence, even if the earth spin were to remain constant at a period of 86 400.003 s, the daily accumulation of 3-ms intervals would require a leap second every year or so to keep our watches in step with the sun's overhead transits.

The irregular variations of the earth clock are huge; annual fluctuations alone change the earth period by ~1 ms or more. You can find a plot of the length of day excess over 86 400 SI seconds (LOD) at the US Naval Observatory "Earth Orientation" web site, http://maia.usno.navy.mil; click on "What is Earth Orientation?" The excess was 2 to 2.5 ms in ~1993, but now as I write this in January 2003, it is fluctuating around 0.6 ms. The earth spin has been speeding up (!) over the past decade, but now is relatively steady. At the current 86 400.0006 s period, the earth clock would lose only 0.6 ms × 365 d = 0.22 s in a year compared to UTC. No wonder there have been no leap second insertions since 1999 Jan. 1 (through 2002).

Terrestrial time (TT)

Astronomers and others have a need for a continuously running time standard that is never adjusted for irregularities of the earth's rotation. A natural choice for this would be atomic time TAI. In fact, since 1972 this has been the standard for such purposes, except for a constant offset of 32.184 s required to match it to the previous (discarded) free-running standard, ephemeris time (ET) described above. This corrected time is known as *terrestrial time* (TT),

$$\mathrm{TT} = \mathrm{TAI} + 32.184\,\mathrm{s} \qquad \text{(Terrestrial time)} \qquad (4.16)$$

For most purposes, the two time standards, ET and TT can be considered a single seamless time standard: ET before 1958 and TT from 1958 onward. It runs at the same rate as TAI. The TT day contains exactly 86 400 SI seconds.

An earlier name for TT was *Terrestrial Dynamic Time* (TDT). A dynamic time is one that is used as the independent variable in physical equations such as Newton's laws. Thus, in principle, an ideal TT could be obtained only from extended observations of the planetary motions in the solar system, and this time could in principle diverge from atomic time. Divergence could also arise from small inaccuracies in the rates of atomic clocks. At present, nevertheless, TT is defined to have the same rate as TAI, but with the offset given in (16). The nominal atomic clock is taken to be on the surface of the earth because experiments show that clock rates depend on their location in the earth's gravitational potential in accord with the GR model of time.

In practice, TT is a method of keeping track of the leap seconds inserted into UTC. This is important if one wants to calculate time intervals over several years as in the timing of pulsar pulses. The *Astronomical Almanac* published each year gives

Table 4.1. *TT and TAI offsets relative to UTC*[a]

Start (UTC)	End (UTC)	TAI – UTC (s)	TT – UTC (s)
1902 Jan. 1 0 h			~0[b]
1958 Jan. 1 0 h		~0	32.18[b]
1977 Jan. 1 0 h	1977 Dec. 31 24 h	16	48.184
1988 Jan. 1 0 h	1989 Dec. 31 24 h	24	56.184
1990 Jan. 1 0 h	1990 Dec. 31 24 h	25	57.184
1991 Jan. 1 0 h	1992 Jun. 30 24 h	26	58.184
1992 Jul. 1 0 h	1993 Jun. 30 24 h	27	59.184
1993 Jul. 1 0 h	1994 Jun. 30 24 h	28	60.184
1994 Jul. 1 0 h	1995 Dec. 31 24 h	29	61.184
1996 Jan. 1 0 h	1997 Jun. 30 24 h	30	62.184
1997 Jul. 1 0 h	1998 Dec. 31 24 h	31	63.184
1999 Jan. 1 0 h	>2002	32	64.184

[a] *Astronomical Almanac for 2002*, US Naval Obs. and Royal Greenwich Obs. (US Govt. Printing Office). All offsets from 1988–2002 are listed.
[b] ET – UT

a history of the changing offsets of TT and TAI from UTC. For example, during the 1989–1993 mission of the Hipparcos satellite, the TT offsets were 56–59 s and at the end of the century it was, and is at this writing, 64.184 s (Table 1). The irregular variation of the earth spin is evident from the irregular intervals between inserted leap seconds in Table 1. Table 1 shows that TAI was set to agree with UTC in 1958 and TT (actually ET) was set to agree with UTC in 1902. This is the origin of the 32.184 s offset in (16).

Keep in mind that an added leap second in UTC amounts to holding back the clock for one second before it becomes the next day. Thus TT times are increasingly greater than are UTC times, insofar as the leap seconds continue to be positive. One could imagine that someone, someplace, has a 24-h clock that reads TT time; i.e., it is never held back by the insertion of a leap second. It would be running ahead (fast) of a UTC clock by about 1 minute. If I were waiting to celebrate my birthday on 1990 Dec. 7, the TT clock would allow me to start whooping it up 57.184 s before the UTC clock strikes midnight.

If I wanted to know how many SI seconds (atomic time) I had lived, I would find the number of days since my date of birth, taking into account leap years (possibly by using Julian days; see discussion below). I would adjust this for partial days at each end of the interval and then multiply by 86 400 s/d. Finally I would add the difference in the two TT – UTC offsets at the ends of the interval, $(\mathrm{TT}-\mathrm{UTC})_{\mathrm{now}} - (\mathrm{TT}-\mathrm{UTC})_{\mathrm{birth}}$. The latter step takes into account the leap seconds that were inserted between the two dates.

TT is a *terrestrial* time scale. It is used for timing events occurring at the earth or distant events observed at earth and timed with clocks located on the earth's surface (mean sea level). For example, it is used for expressing the locations of earth orbiting satellites as a function of time.

Barycentric times

According to the GR model of time, a clock deep in a potential well will run more slowly than one less deep. Also, a clock moving at high speed relative to some stationary clocks runs more slowly than the "at rest" clocks it passes. The latter is the *time dilation* effect also encountered in special relativity. A clock on the earth, with its elliptical orbit, experiences both effects relative to a stationary clock far from the solar system. Each effect has a fixed and a cyclic component. The cyclic terms arise from the eccentricity of the earth's orbit; on an annual basis, both the speed and the depth in the solar potential change.

Barycentric Dynamical Time (TDB) is a *coordinate time* in general relativity; it marks the progress of "time" in the GR model, but is not necessarily the time kept by any particular real clock. It is defined in a coordinate system with spatial origin at the solar-system barycenter. It may be used as the independent variable in the equations of motion that represent the positions and velocities of solar-system bodies in general relativity. One may think of it as the time kept by a clock that is on the surface of a hypothetical earth that orbits the barycenter in a strictly circular orbit at constant velocity, at an orbital radius and velocity typical of the actual earth. It runs at the same rate as TT except that it is free of the cyclical effects of the earth's elliptical orbit, to which TT is subject. The annual periodic term in the difference TT – TDB is of amplitude only 1.7 ms, while other periodic terms due to the planets and moon contribute up to 20 μs.

In 1991 the International Astronomical Union defined *Barycentric Coordinate Time* (TCB), a coordinate time for another system with spatial origin at the barycenter of the solar system. This too can be the independent variable in the equations of motion. TCB is the time kept by a series of synchronized clocks that are at rest relative to the barycenter and far removed from it. They are in flat space where gravitational redshift and velocity effects are null. TCB can be called "far away time". At present, the TCB clock runs faster than TDB by a constant 49 s per century. This time is appropriate for keeping track of events taking place throughout the solar system.

TCB differs from TDB by the constant rate offsets of the two relativistic effects mentioned above, namely time dilation and solar gravitational potential. TCB time runs faster than the TDB clock for each effect. Recall that our hypothetical TDB clock is in an earth-like but circular orbit. It therefore runs slower than the "far away" TCB clocks by (*i*) 33 s per century because it is in the gravitational potential

(or space distortion) of the sun's gravity, and (*ii*) 16 s per century because its orbital velocity is 29.8 km s^{-1}, or 10^{-4} times the speed of light, relative to the solar system barycenter. The latter effect is the relativistic time dilation, or equivalently, the transverse Doppler effect. The two effects together yield the above mentioned total rate difference of 49 s per century.

Julian date (JD)

The comparison of observations over many years is not simple because of the varying numbers of days in a year (due to leap years) and the different numbers of days in the several months. Accordingly, a continuously running counting system for days is used in astronomy; these are known as Julian dates (JD). The Julian date 0.0 was set prior to most dates one would encounter in astronomy, namely at noon on Jan. 1 in 4713 BC. It was defined by Justus Scaliger in 1583, the year after the initiation of the Gregorian calendar, and was named after the Julian year. We discussed the several calendars in Section 3 above.

Julian days are counted as continuously running numbers which are now approaching 2.5 million. The Julian day beginning at noon on 2000 Jan. 1 is JD 2 451 545, while for Jan. 2 it is JD 2 451 546, etc. The Astronomical Almanac gives equivalences between Gregorian dates and Julian dates (JD). The beginning of each Julian day is now defined to be at noon at Greenwich, either 12 h UTC or 12 h TT. More precise times within a Julian day are indicated with decimal figures, not h m s. If one uses JD with precision of 1 minute or better, it is important to specify the units one is using, such as JD(TT), JD(TDB), or JD(UTC), because, for example, JD 2 451 545.000 00 (TT) and JD 2 451 545.000 00 (TDB) both occur about one minute before JD 2 451 545.000 00 (UTC).

Julian days can be grouped conveniently into centuries of 100 yr. Now the number of days in a Gregorian century will vary depending on the number of leap years in that given century. This motivates reference to the (Roman) calendar wherein each year has exactly 365.25 d (i.e., a leap year every 4 yr without fail) so that a *Julian century* always has 36 525 d. Thus J2100.0 will be exactly 36 525 d later than J2000.0. If further, the Julian day is defined with TT time wherein every day has exactly 86 400 s (no leap seconds), we have a system wherein every Julian century has 86 400 s/d × 36 525 d/C = 3 155 760 000 s, and where each second is the atomic (or TT) second, also known as the *SI second*.

Epochs for coordinate systems

The equatorial coordinate system used for celestial measurements depends on the orientation of the earth, and this is a continuously changing function of time

(Section 3.2). The time chosen during some period (usually decades) for the specification of celestial coordinates in catalogs and communications between astronomers is called the *standard epoch*, traditionally expressed in years.

The standard epochs in use in the last century, B1900.0 and B1950.0 were based on the Besselian year which begins when the mean sun is at $\alpha = 18$ h 40 m. As noted in Section 3.2, B1950.0 occurred about 2 hours before the New Year of 1950.

The standard epoch in use today is J2000.0 (TDB); it is based on the Julian century/day/second system just described. The epoch J2000.000 was set to occur exactly at 2000 Jan. 1, 12 h (TDB), i.e., at JD 2 451 545.0 (TDB). All other epochs E are defined relative to this,

$$\begin{aligned} \text{JD(TDB)} = {} & 2\,451\,545.0 \text{ (TDB)} \\ & + [(E - 2000.00)\ 365.25] \qquad \text{(Defines epoch)} \end{aligned} \tag{4.17}$$

where E is the epoch in years, e.g., $E = 1991.25$ is the approximate mean epoch of the observations made by the Hipparcos satellite. From (17) one finds

$$\begin{aligned} \text{J1900.0} &= \text{JD } 2\,415\,020.0 \text{ (TDB)} = 1899 \text{ Dec. } 31,\ 12 \text{ h TDB} \\ \text{J1991.25} &= \text{JD } 2\,448\,349.0625 \text{ (TDB)} = 1991 \text{ Apr. } 2,\ 13 \text{ h } 30 \text{ m TDB} \\ \text{J2000.0} &\equiv \text{JD } 2\,451\,545.0 \text{ (TDB)} = 2000 \text{ Jan. } 1,\ 12 \text{ h TDB} \\ \text{J2100.0} &= \text{JD } 2\,488\,070.0 \text{ (TDB)} = 2100 \text{ Jan. } 1,\ 12 \text{ h TDB} \\ \text{J2200.0} &= \text{JD } 2\,524\,595.0 \text{ (TDB)} = 2200 \text{ Jan. } 2,\ 12 \text{ h TDB} \end{aligned} \tag{4.18}$$

Each of the century epochs in (17) will occur somewhat before UTC noon according to the TT – UTC offsets; see Table 1. Recall that TDB differs from TT by at most 1.7 ms. Although the definition of epoch is based on TDB time, one would do well to eliminate the possibility of confusion by writing J2000.0 (TDB). Sometimes one sees J2000.0 (TT) which is effectively the same thing, within 1.7 ms. One often sees simply J2000.

The two epochs J1900 and J2000 are separated by a Julian century, exactly 36 525 d; see (17). This leads to a one-day date shift in the Gregorian calendar date because it has no leap day added in 1900 February whereas the older Julian calendar does. (See Section 3 above, "Calendar".) Thus this Gregorian century was one day shorter than the Julian century, namely 36 524 d (to better match the tropical year). The next century yielded identical dates because 2000 was a leap year in both calendars, and the following century again differs because 2100 will not be a (Gregorian) leap year.

This system of Julian centuries thus gradually gets out of step with the Gregorian calendar just as Caesar's calendar got out of step with the seasons, by 3 d every 400 yr or 15 d in 2000 yr. Does this mean that astronomers have adopted again Caesar's calendar for the epoch definition? Not really, because they do not give month names to it, nor do they live by it. Although they do use the length of the Julian century, they chose not to set the J2000.0 epoch to the extrapolation of the New Year from Caesar's Julian calendar, but rather defined J2000.0 to be on 2000 Jan. 1 of the *Gregorian* calendar, and set it at noon rather than at midnight, following Scaliger's convention for the Julian date.

In another 2000 yr, J4000.0 will occur on Jan. 16 12 h (TDB). On this date, at a rate of insertion of leap seconds into UTC of somewhat less than one per year, TT time will be advanced over UTC by, say, 1500 s more than today's ~1 min offset. Thus J4000.0 should occur roughly 26 minutes before UTC noon on 4000 Jan. 16.

A *modified Julian date* (MJD) defined as MJD = JD – 2 400 000.5 is sometimes used. It starts at midnight in Greenwich rather than at noon. MJD is a smaller number than JD and thus is less cumbersome to use in plots and text. Again, if precision is required, one should specify MJD(UTC), MJD(TT) or MJD(TDB).

Signals from pulsars

The timing of signals from outer space has long been a fundamental part of astronomy. The discovery of radio pulsars extended this aspect of astronomy to very short time scales, to seconds and milliseconds. These rotating neutron stars emit a pulse of radio noise once each rotation. The pulse is probably due to acceleration of electrons along the magnetic field lines emerging from the pole of the star. The rotation rate of these stars can be very stable if they have minimal energy loss from magnetic dipole radiation. The pulsar PSR 1937 + 21 is particularly stable; it has a pulse period of 1.6 ms, and its stability rivals that of atomic time. The time standard does not now make use of the signals from such pulsars, but it may come to pass that certain pulsars will become an important time keeping standard.

Problems

4.2 *Gravity*

Problem 4.21. (a) If the entire mass of the sun were compressed into a sphere the size of the earth, what would be its density relative to that of the earth? (This is typical of a *white dwarf* star.) (b) If your scale indicates 80 kg when you weigh yourself on earth, what would it read if you weighed yourself with it on the surface of the white dwarf? (c) Could you pick up a penny? See Appendix for the masses and sizes of the earth and the sun. A penny weighs about 2 g. [Ans. (b) $\sim 10^7$ kg]

Introduction to Problems 4.22–25. The moon's interaction with the earth's oceans causes the earth to slow down and the moon to gain energy. Energy is dissipated by