UNIDADES GEOMETRIZADAS Y ANÁLISIS DIMENSIONAL

unidades $c \equiv G \equiv h \equiv 1$

$G = 6.67\times10^{-11}$ S.I.
$c = 299\,792\,458$
$h = 6.63\times10^{-34}$

$$[G] = \frac{L^3}{MT^2} \qquad [c] = \frac{L}{T} \qquad [h] = \frac{ML^2}{T}$$

$$\begin{array}{c|ccc} & G & c & h \\ \hline M & & & \\ L & & & \\ T & & & \end{array} \quad \underbrace{\begin{pmatrix} -1 & 0 & 1 \\ 3 & 1 & 2 \\ -2 & -1 & -1 \end{pmatrix}}_{\equiv A} \cdot \underbrace{\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}}_{\equiv C} = X$$

matriz de la magnitud que buscamos descomponer en $G^\alpha c^\beta h^\gamma$

* Ejemplo longitud de Planck: $X = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Podemos resolver matricialmente $A\cdot C = X \Rightarrow C = A^{-1}\cdot X = A\backslash X = \begin{pmatrix} 1/2 \\ -3/2 \\ 1/2 \end{pmatrix}$

O el sistema de ecuaciones:

$$\left.\begin{array}{l} -\alpha+\gamma=0 \Longrightarrow \alpha=\gamma \\ 3\alpha+\beta+2\gamma=1;\quad 5\alpha+\beta=1 \Rightarrow \boxed{\beta = 1-5\alpha = -\tfrac{3}{2}} \\ -2\alpha-\beta-\gamma=0;\quad -3\alpha-\beta=0 \\ \qquad 2\alpha = 1 \Rightarrow \boxed{\alpha = 1/2 = \gamma} \end{array}\right\} G^{1/2}c^{-3/2}h^{1/2}$$

Por lo que $\boxed{l_P = \sqrt{\dfrac{Gh}{c^3}}}$

* Ejemplo $\omega = \sqrt{\dfrac{M}{r^3}}$ en rad/s con $M = 5M_\odot = 10^{31}$ kg ; $r = 9.4\times10^6$ m :

$\left[\dfrac{M_{kg}}{r_m{}^3}\right] = \dfrac{M}{L^3}$; $[\omega] = T^{-1} \Rightarrow \left[\omega^2 = \dfrac{M}{r^3}\right] = T^{-2}$; $\Rightarrow$ Busco X tal que $\dfrac{M}{L^3}\cdot[x] = T^{-2} \Rightarrow [x] = \dfrac{L^3}{MT^2}$

que son justamente las dimensiones de G.

Podemos hacer $[x] = \dfrac{L^3}{MT^2} \equiv [G]^\alpha\cdot[c]^\beta = \left(\dfrac{L^3}{MT^2}\right)^\alpha\cdot\left(\dfrac{L}{T}\right)^\beta = L^{3\alpha+\beta}\cdot M^{-\alpha}\cdot T^{-2\alpha-\beta}$

(no hay factores cuánticos)

$$\left.\begin{array}{l} 3\alpha+\beta=3 \\ -\alpha=-1 \\ -2\alpha-\beta=-2 \end{array}\right\} \begin{array}{l}\alpha=1\\ \beta=0\end{array} \Rightarrow \boxed{x = G} \Rightarrow \boxed{\omega = \sqrt{\frac{GM}{r^3}} = 0.90 \text{ rad/s}}$$

* Ejemplo $\tilde{W} = \dfrac{-M\tilde{L}^2}{r^3}$, donde $\tilde{W}$ y $\tilde{L}$ son energía potencial y momento angular ambos por unidad de masa, respectivamente.

$$\left.\begin{array}{l} [\tilde{W}] = \dfrac{L^2}{T^2} \\ [\tilde{L}] = \dfrac{L^2}{T} \Rightarrow \left[\dfrac{M\tilde{L}^2}{r^3}\right] = \dfrac{ML}{T^2} \end{array}\right\} [x]\cdot\frac{ML}{T^2} = \frac{L^2}{T^2} \Rightarrow [x] = \frac{L}{M} \quad \text{con } x = G^\alpha\cdot c^\beta$$

$$[x] = \left(\frac{L^3}{MT^2}\right)^\alpha\left(\frac{L}{T}\right)^\beta = L^{3\alpha+\beta}\cdot M^{-\alpha}\cdot T^{-2\alpha-\beta} = \frac{L}{M}$$

$$\left.\begin{array}{l} 3\alpha+\beta=1 \\ -\alpha=-1 \\ -2\alpha-\beta=0 \end{array}\right\} \begin{array}{l}\alpha=1\\ \beta=-2\end{array} \Rightarrow \boxed{x = \frac{G}{c^2}} \Rightarrow \boxed{\tilde{W}(r) = -\frac{GM\tilde{L}^2}{c^2r^3}}$$

Si ahora queremos hacerlo adimensional: $\tilde{W}_{adim}(r) = -\dfrac{1}{c^2}\dfrac{GM\tilde{L}^2}{c^2r^3} = -\dfrac{GM\tilde{L}^2}{c^4r^3}$