

Nombre: .....

SOLUCIONES

10

1.- Resuelve las siguientes integrales

a)  $\int \frac{x}{x^2+4} dx$

b)  $\int x e^{-x/2} dx$

c)  $\int \frac{4x^2+x-2}{x^2(x-1)}$

a)  $\int \frac{x}{x^2+4} dx = \frac{1}{2} \int \frac{2x}{x^2+4} dx = \boxed{\frac{1}{2} \ln|x^2+4| + C}$  ✓ (3)

b)  $\int x e^{-x/2} dx = -2x e^{-x/2} - \int -2e^{-x/2} dx = -2x e^{-x/2} + 2 \int e^{-x/2} dx =$   
 $= \boxed{-2x e^{-x/2} - 4e^{-x/2} + C} = -2e^{-x/2}(x+2) + C$  ✓ (3)

$u = x \Rightarrow du = dx$   
 $du = e^{-x/2} dx$   
 $v = -2e^{-x/2}$

c)  $\int \frac{4x^2+x-2}{x^2(x-1)} dx = \int \frac{1}{x} dx + \int \frac{2}{x^2} dx + \int \frac{3}{x-1} dx = \ln|x| + 2 \cdot \frac{x^{-1}}{-1} + 3 \ln|x-1| + C$

$\frac{4x^2+x-2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x-1}$

$4x^2+x-2 = Ax(x-1) + B(x-1) + Cx^2$

$x=0 \rightarrow -2 = -B \Rightarrow B=2$

$x=1 \rightarrow 3 = C$

$x=-1 \rightarrow 1 = 2A - 2B + C$  ✓

$1 = 2A - 4 + 3$

$A = 1$

$\int \frac{4x^2+x-2}{x^2(x-1)} = \boxed{\ln|x| - \frac{2}{x} + 3 \ln|x-1| + C}$  ✓

(4)