Ecuaciones Diferenciales, 1º Biotecnología Test 3, 2/3/2015

Nombre: ....

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1.- De una reacción reversible $A + B \underset{k_2}{\overset{k_1}{\rightleftharpoons}} X$ se sabe que $k_1 = 3$ y se desconoce k_2. Experimentalmente observamos que cuando $A(0) = B(0) = 1$ y $X(0) = 0$, entonces a largo plazo se alcanza $X_{eq} = 2/3$.

a) Formula una ED para la cantidad de producto $X(t)$

b)Determina el valor de la constante k_2.

c) Determina A_{eq} y B_{eq} y esboza la gráfica de $A(t)$, $B(t)$ y $X(t)$.

d) Resuelve la ED y calcula $t_{0,90}$ para $\frac{dx}{dt} = \frac{1}{2}(3x-2)(2x-3)$.

a) $X'(t) = K_1\, A(t)\, B(t) - K_2\, X(t)$ $\qquad A(0) - A(t) = X(t) - \cancel{X(0)}^{0} \rightarrow A(t) = A(0) - X(t) = 1 - X(t)$

$X'(t) = 3(1 - X(t))(1 - X(t)) - K_2 X(t)$ $\qquad B(0) - B(t) = X(t) - \cancel{X(0)}^{0} \rightarrow B(t) = B(0) - X(t) = 1 - X(t)$

$$\boxed{X'(t) = 3(1 - X(t))^2 - \frac{1}{2} X(t)}$$

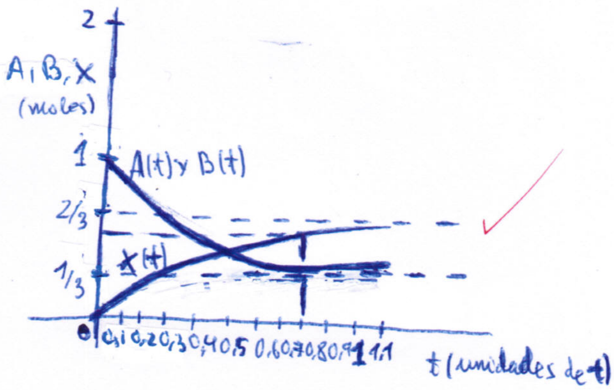
b) Busco $K_2 \,/\, X_{eq} = 2/3$

$0 = 3\left(1 - \frac{2}{3}\right)^2 - K_2 \cdot \frac{2}{3}$

$0 = \frac{1}{3} - \frac{2}{3} K_2 \rightarrow K_2 = \frac{1}{2} \left[\frac{1}{\text{unidades de } t}\right]$

c) $A_{eq} = 1 - X_{eq} = 1 - \frac{2}{3} = \frac{1}{3}$ mol

$B_{eq} = 1 - X_{eq} = 1 - \frac{2}{3} = \frac{1}{3}$ mol



d) $\underbrace{\int \frac{dx}{(3x-2)(2x-3)}}_{\circledast} = \int \frac{1}{2}\, dt \rightarrow \frac{1}{5}\ln\left|\frac{2x-3}{3x-2}\right| = \frac{1}{2}t + C \rightarrow \boxed{\frac{1}{5}\ln\left|\frac{2x-3}{3x-2}\right| = \frac{1}{2}t + \frac{1}{5}\ln\left(\frac{3}{2}\right)}$

$\int \frac{dx}{(3x-2)(2x-3)} = -\frac{3}{5}\int \frac{dx}{3x-2} + \frac{2}{5}\int \frac{dx}{2x-3} = -\frac{1}{5}\ln|3x-2| + \frac{1}{5}\ln|2x-3|$

$\frac{1}{(3x-2)(2x-3)} = \frac{A}{(3x-2)} + \frac{B}{(2x-3)} = \frac{A(2x-3) + B(3x-2)}{(3x-2)(2x-3)}$ $\left\langle \begin{array}{l} \text{si } x = \frac{3}{2} \rightarrow 1 = A\cdot 0 + B\left(\frac{9}{2} - 2\right) \rightarrow B = \frac{2}{5} \\ \text{si } x = \frac{2}{3} \rightarrow 1 = A\left(\frac{4}{3} - 3\right) + B \cdot 0 \rightarrow A = -\frac{3}{5} \end{array}\right.$

si $x = 0, t = 0$ $\rightarrow \frac{1}{5}\ln\left|\frac{2\cdot 0 - 3}{3\cdot 0 - 2}\right| = \frac{1}{2}\cdot 0 + C \rightarrow C = \frac{1}{5}\ln\left(\frac{3}{2}\right)$

Busco $t \,/\, x(t) = 0{,}9 \cdot X_{eq} = 0{,}9 \cdot \frac{2}{3} = 0{,}6$ mol

$$\boxed{t_{0,9} = 2\left(\frac{1}{5}\ln\left|\frac{2\cdot 0{,}6 - 3}{3\cdot 0{,}6 - 2}\right| - \frac{1}{5}\ln\left(\frac{3}{2}\right)\right) = 0{,}725}$$