TRADINNOVA-FUZ: FUZZY PORTFOLIO INVESTMENT
Dynamic Stock Portfolio Decision-making Assistance Model based on a Fuzzy Inference System

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Abstract: This paper describes a decision system based on rules for the management of a stock portfolio using a fuzzy inference system to select the stocks to be incorporated. This system simulates the intelligent behavior of an investor, carrying out the buying and selling of stocks, such that during each day the best stocks will be selected to be incorporated in the portfolio with the use of technical indicators using a fuzzy logic based approach. The proposed novel fuzzy system only has a simple strict set of rules to decide if a share is bought or not, unlike other systems that also include rules for the sale and have a lot of complicated rules.

The system has been tested in 3 time periods (1 year, 3 years and 5 years), simulating the purchase/sale of stocks in the Spanish continuous market and the results have been compared with the revaluations obtained by the best investment funds operating in Spain.

1 INTRODUCTION

Investment management consists of *strategic asset allocation, tactical asset allocation, and stock picking* three phases (Amenc and Sourd, 2003). Our study is focus on *tactical asset allocation* and *stock picking*.

To perform the *tactical asset allocation* an *intelligent system based on rules* that will dynamically invest in shares for a certain period of time is proposed. This system will simulate the behavior of any rational investor, so that each day would look if there is any investment opportunity, buying or not depending on how much money is available.

The last phase, *stock picking*, is one of the most important phases, since it must decide for each day what shares should be bought to add to the portfolio. A simple rule that could be used to implement this step could be to invest in the stock that has been revalued at least a certain percentage in the last days. Although we could use more sophisticated rules using technical analysis, like Relative Strength Index (RSI) or Moving Average, (Murphy, 1999).

Each of the indicators used in technical analysis has some limitation, and in most cases, the answer by each of them is not a definite "yes" or "no". The best result could be achieved when combining many indicators at the same time and evaluating their output collectively.

Our proposal is to use technical indicators with fuzzy logic in order to create a strict fuzzy indicator that only recommends "buy", when most of the technical indicators recommend it.

The proposed system will be applied to the Spanish stock market, in particular, the IBEX 35[1], keeping in mind that we invest all the available money in stocks without regard to which sector they belong, and supporting a maximum of 4 % loss per share. We will compare our investment performance with the index itself and with the results that have obtained the better equity funds that invest in the Spanish continuous market in the time limit of 1, 3 and 5 years.

In Section 2, we introduce the concepts of investment portfolio and how fuzzy inference systems have been introduced in order to help in financial market analysis. In Section 3, we present the system proposed with all the elements that compose it. In Section 4, we show different computational results that illustrate the behavior of the proposed hybrid intelligent system. Finally, we present the conclusions in Section 5.

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[1]The IBEX 35 (an acronym of Iberia Index) is the benchmark stock market index of the Bolsa de Madrid, Spain's principal stock exchange.

2 LITERATURE REVIEW

2.1 Investment Portfolio

The concept behind investment portfolio is to combine different investment targets to avoid concentrating too much risk on any one target with the aim of dispersing overall investment risk. Any combination of two or more securities or assets can be termed an investment portfolio.

On the other hand *efficient-market hypothesis* is an idea partly developed in the 1960s by Eugene Fama and defended by Burton G. Malkiel (Malkiel, 1973) which asserts that financial markets are "informationally efficient", or that prices on traded assets (e.g., stocks, bonds, or property) already reflect all known information, and instantly change to reflect new information. Therefore, according to theory, it is impossible to consistently outperform the market by using any information that the market already knows, except through luck.

2.2 Fuzzy Inference Systems

A fuzzy inference system is a computer paradigm based on fuzzy set theory, fuzzy if-then-rules and fuzzy reasoning.

Fuzzy inference systems have been successfully applied in fields such as automatic control, data classification, decision analysis, expert systems, and computer vision. Because of its multidisciplinary nature, fuzzy inference systems are associated with a number of names, such as fuzzy-rule-based systems, fuzzy expert systems, fuzzy modeling, fuzzy associative memory, fuzzy logic controllers, and simply (and ambiguously) fuzzy systems.

In the field of financial market analysis, we have for example, (Dourra and Siy, 2002), which uses three technical indicators (rate of change, stochastic momentum and resistance indicator) in a fuzzy control system with the following modules: convergence (maps the technical indicators into new inputs), fuzzification, fuzzy processing and defuzzification (using the center of area method to map the output universe with four membership functions -low, medium, big and large- into a nonfuzzy action). Also in (Cheung and Kaymak, 2007) the fuzzy trading system is based on four technical indicators (Moving Average Convergence/Divergence, Commodity Channel Index, Relative Strength Index and Bollinger Bands) and the output of the fuzzy system is a signal on a normalized domain, on which four different fuzzy sets (strong sell, sell, buy and strong buy) are defined.

On the other hand in (Atsalakis and Valavanis, 2009) use a neuro-fuzzy based methodology to forecast the next day's trend of chosen stocks. The forecasting is based on the rate of change of three-day stock price moving average.

3 RESEARCH FRAMEWORK

3.1 Intelligent System for Tactical Asset Allocation

The proposed system for decision making is based on a policy of buying and selling the stocks that make up any stock market over a period of time.

This policy of buying and selling is based on that if we assume a stock market quoted from a start date ($date_{start}$) until an end date ($date_{end}$), each one of those days, all their stocks have had an opening price (p_{open}), a maximum price (p_{max}), a minimum price (p_{min}) and a closing price (p_{close}).

If we take a day d between $date_{start}$ and $date_{end}$ in which you have no shares purchased (there is no order of sale or purchase pending), then we will be able to select a set of m stocks (S_b), using technical analysis or any other technique, which would be most recommendable to buy, because it expects them to give a good return.

The technique for selecting stocks should calculate a value for each one of the stocks that make up the market on that day d, quantifying if it would be advisable to buy the shares. Stocks are ordered from most to least according to this value, and the system will have to choose the best set of stocks, defining which minimum value is considered for a stock to belong to this set of the better stocks, existing the possibility that one day any stock is not recommended ($m = 0$), or that many are recommended because its analysis has been the sufficiently satisfactory for all them.

Once we have this set S_b with the selection of the best stocks for a day d, we might try to buy all or some of these stocks. To simplify the algorithm we will try to buy only one of the selected stocks every day, so that after several days, we could have a portfolio of n stocks (S_a). Thus, to choose the stock to buy we would have two possibilities, to select the best or well to select any randomly.

Once we have chosen a stock, we will have to give the purchase order for the next day. We will limit the purchase price to any of the prices that has had the stock during that same day ($[p_{min}, p_{max}]$), or choose the opening p_{open} or closing p_{close} price of the next day. If we use a low purchase price, there are fewer

possibilities to execute the purchase in the following days, but on the other hand, the stock will be bought more cheaply.

The next day $(d + 1)$, one sees if the purchase of the share can be executed, whenever the purchase price of this day is between p_{min} and p_{max}. If this purchase order does not manage to be executed in W days, then we would eliminate this purchase order, and would give a new purchase order, selecting a share among the best ones of that day. On this following day in which we can already have bought shares, it is necessary to calculate which of them should be kept in the portfolio and which should be sold, reason why a new analysis is performed to select p stocks (S_c) so that using again anyone of the previously commented techniques would tell us which stocks shall be maintained in the portfolio, since it is expected that they give a good profit value. The technique used must calculate a value for each of the stocks that make up the market on that day $d + 1$, quantifying if it would be advisable to maintain this share in the portfolio.

In this new day we would check if each one of the bought stocks continues being among the best that are recommended to maintain in the portfolio (S_c), in which case we would not do anything or otherwise we would leave spent M days without the stock among the best to give a sale order. A sale order is given when M days have passed without the stock is recommended to keep in the portfolio or when the stock has had a loss regarding the price of purchase higher than a certain percentage $P\%$. A sale order will also have a limited price, as the purchase order. The higher the sale price more difficult it is to execute the sell order. In case during V days it is not possible to sell at this price, probably because the stock is in a bearish period, then we would descend the price of this sale order.

3.2 Stock Picking based on a Fuzzy Inference System

For every day d along the investment period it is necessary to look for which are the best shares S_b to be able to introduce them in the decision support system dedicated to tactical asset allocation commented in the previous paragraph, and that this one decides how it is going to invest in them.

To select the best shares of one day d, we are going to use technical analysis indicators, although the most difficult part of technical analysis is to decide which indicator to use.

We have chosen the following four technical indicators to select stocks:

1. **Average Revaluation Period (ARP).** Average

revaluation that has had a stock in a given period of time.

2. **Relative Strength Index (RSI).** Relative Strength Index of a stock in a given period of time.

3. **Moving Average (MA).** Calculates the revaluation that reaches a stock with respect to the average value of price in a given period of time.

4. **Double Moving Average (DMA).** Known also as double crossover method, uses a combination of long-term and short-term moving averages. When the shorter moving average rises above the longer moving average from below, a buy signal is issued.

The results of applying these indicators to the stocks are going to be the input variables in the fuzzy inference system. We have chosen these indicators because they are basic in the world of technical analysis, and because they are very easy to understand.

Technical analysis deals with probability and therefore multiple indicators can be used to improve the result. In most cases, the answer by each indicator is not a definite yes or no answer.

We are going to use technical indicators with fuzzy logic to create a strict fuzzy indicator that only recommends to buy a stock when the set of indicators does it. We will only focus on the purchase recommendations, because the intelligent system discussed in the preceding section will be in charge of managing the portfolio, selling those shares no longer necessary.

Our plan can be summarized as follows:

- To create membership functions, where the inputs are each one of the financial indicators and the outputs are these indicators "fuzzified"

- To create fuzzy rules that indicate if it is highly recommendable to buy a share.

- To translate the fuzzy output into a crisp trading recommendation.

3.2.1 Fuzzification

The input variables in this fuzzy inference system are mapped by sets of membership functions, known as "fuzzy sets". The process of converting a crisp input value to a fuzzy real value between 0 and 1 is called "fuzzification". The fuzzification comprises the process of transforming crisp values into grades of membership for linguistic terms of fuzzy sets. The membership function is used to associate a grade to each linguistic term.

Our fuzzy system also have a "ON-OFF" type of switch for the **Double Moving Average** input variable, because this input will always have a truth value

equal to either 1 or 0, depending if the buy signal has been issued.

There is yet no fixed, unique, and universal rule or criterion for selecting a membership function for a particular "fuzzy subset" in general: a correct and good membership function is determined by the user based on his scientific knowledge, working experience, and actual need for the particular application in question.

The criteria followed in the fuzzification of profitability and RSI are explained below.

Fuzzification of Profitability. At the moment of carrying out the fuzzification of the daily profit values, the present fuzzy systems usually assign "low", "normal" or "high" profitability values according to subjective estimations carried out by the writer of the article.

In this research, in order to perform this fuzzyfication we need to keep in mind that, statistically (Figure 1), is considered normal profitability between 0% and 0.5%-1%, high profitability between 0.5%-1% and 1.5%-4%, and very high profitability from 1.5%-4%, (BME, 2009).
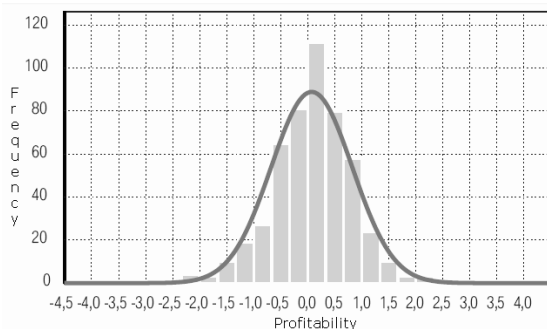

Figure 1: Histogram of daily returns on IBEX35 (2003-2005).

The membership grade functions defined on the profitability domain are based on trapezoid shapes (Figure 2). It can be seen that just positive returns have been considered, since only this type of profit values will be able to originate recommendations for purchase.

Fuzzification of RSI. The Relative Strength Index (RSI) method, which was developed by J. Welles Wilder, may be classified as a momentum oscillator, measuring the velocity and magnitude of directional price movements. Momentum is the rate of the rise or fall in price.

Wilder posited that when price moves up very rapidly, at some point it is considered overbought.

Likewise, when price falls very rapidly, at some point it is considered oversold. In either case, Wilder felt a reaction or reversal is imminent.

As a result, Wilder believed that tops and bottoms are indicated when RSI goes above 70 or drops below 30. Traditionally, RSI readings greater than the 70 level are considered to be in overbought territory, and RSI readings lower than the 30 level are considered to be in oversold territory. In between the 30 and 70 level is considered neutral.

The membership grade functions defined on the RSI domain (Figure 2) have the following fuzzy sets (overbought, oversold and neutral) based on a trapezoid shape.
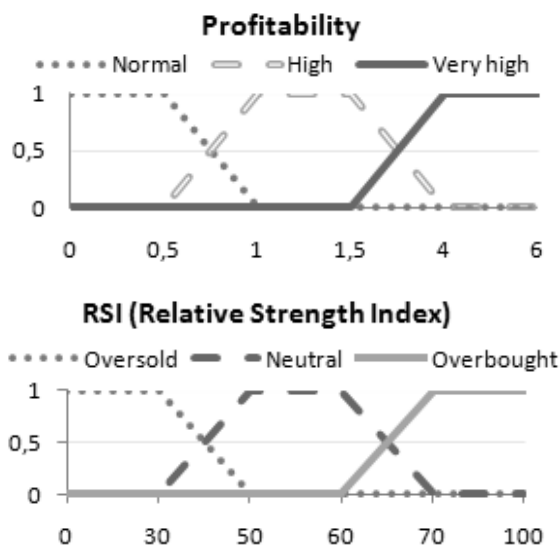

Figure 2: Membership functions.

3.2.2 Fuzzy Rule Base

Decisions are made based on fuzzy rules. These rules are characterized by a collection of fuzzy IF THEN rules in which the preconditions and post-conditions involve linguistic variables. This collection of fuzzy rules characterizes the behavior of the system in a linguistic form that is close to the way human think.

Designing a good fuzzy logic rule base is key to obtaining a satisfactory controller for a particular application. Therefore, when designing the rules, it has been taken into account that only those rules that define the purchase of a stock must be defined, and they should be the easiest possible rules so as to understand its application. Thus, only the linguistic term associated with the most representative membership function to make a purchase ("RSI oversold" or "profitability very_high") has been used when setting these rules, and not other membership functions such as

"profitability high" which are usually used in other researches.

For simplicity of design we have taken linear input-output relations (implications) in a SISO system. Generally, in multiple-input/multiple-output (MIMO) fuzzy inference systems, it is difficult to generate control rules.

Keeping the rules mentioned above in mind, the rules that we have defined are the following:

1. IF RSI is oversold THEN buy

2. IF DMA is buy_signal THEN buy

3. IF MA is very_high THEN buy

4. IF ARP is very_high THEN buy

5. OTHERWISE not_buy

The antecedent (the rules premise) describes to what degree the rule applies, while the conclusion (the rules consequent) assigns a membership function to the output variable.

The output variable "buy" is assigned a range between 0 and 1. A low value represents that is not a good idea to buy the stock and a high value represents an excellent opportunity to buy the stock. There is an inverse relationship between the output membership functions "buy" and "not_buy" so that: buy = 1 - not_buy.

The strength of the ith fuzzy rule is calculated by evaluating the strength of the precondition i (degree of truth) on the corresponding output membership. The final value of the output variable will correspond exactly with the value that reaches the membership function in the precondition.

3.2.3 Combining Rules and Defuzzification

As all the rules are activated every day resulting in different values for the output fuzzy set "buy", corresponding each output value with the value of the fuzzified input, we are going to perform a combination of rules additive, (Kosko, 1992), to obtain a unique final value assigning a weight to each rule.

The weight of the combiner can be thought of as providing degrees of belief to each rule, but we consider that all the rules have the same importance, so we set all the weights equal to unity.

Defuzzication is a mapping process from a fuzzy space defined over an output universe of discourse into a nonfuzzy (crisp) action. It is not a unique operation as different approaches are possible.

The final output of the system (a crisp control signal) is a value between 0 and 1. A strong buy signal is generated when the output is close to 1.0 and a strong not buy signal is generated when the output is close to zero.

The fuzzy logic and the fuzzy control rules are considered and are chosen so that the defuzzified output is always a linear function of the inputs to the fuzzy controller. According to (Sun and Liu, 2002) the output of multiple input single output fuzzy logic controller can be represented by the convex linear combination of the inputs of fuzzy logic controller.

Therefore, to calculate the final output of the system we calculate the average of the fuzzified values that have been returned by the selected membership functions.

4 EXPERIMENTS AND RESULTS

We are going to verify the operation of the system in 3 periods of time: 5 years (2005-2009), 3 years (2007-2009) and 1 year (2009). The market where we will operate will be the Spanish stock market, but restricted to the shares that conformed the IBEX35 in the year 2009. The historic prices have been corrected of dividends, splits and increases in capital. The short selling is not allowed. The cost of each trade has been taken into consideration, so that we assume that the financial intermediary charges a fee of 0.2% and we are going to consider the transaction fees published by the market of Madrid.

The portfolio will be formed by 14 shares as maximum and the rule which is responsible for defining whether a stock should remain in the portfolio has been defined so that the Relative Strength Index of the shares for 28 days must be worth at least 45 for not give a sale order if the stock has remained in the portfolio at least 14 days. The maximum loss allowed to give an immediate sale order is 4%.

In Table 1 are the results obtained in each one of the three periods and they are compared with the revaluation of IBEX 35 in that time. Also is the variance of the daily revaluation throughout each one of the periods. We can found that the system achieves a lower variance than produced by the IBEX 35 and therefore offers less risk. This is because the intelligent system for tactical asset allocation controls the behavior of the shares, selling for example those that have lost over 4%.

To evaluate the importance of the results obtained with this system, we are going to compare them with the results of a report elaborated by INVERCO (Spanish Association of Investment and Pension Funds). Table 2 shows the ranking (R) by annual equivalent return (APR) in periods of 1, 3 and 5 years of each one of the best Spanish equity funds (Foncaixa Bolsa España 150, BBVA Bolsa Ibex Quant, Bankinter Bolsa España 2, CC Borsa 11, Venture Bol.

Table 1: Result of the simulation vs. IBEX35 revaluation.

System	Period	Result	Variance
Simulation	2009	42.94%	1.65
IBEX35	2009	26.22%	2.45
Simulation	2007-2009	5.22%	1.42
IBEX35	2007-2009	-16.92%	3.34
Simulation	2005-2009	104.23%	1.12
IBEX35	2005-2009	30.85%	2.23

Española), until 31 December 2009.

Table 2: Ranking of funds from Spanish equity investment (INVERCO, 2009).

	2009		2007-2009		2005-2009	
Equity funds	APR	R	APR	R	APR	R
Foncaixa	51.9	1	−8.9	79	−	−
BBVA	48.7	2	−13.0	86	−	−
Bankinter	34.5	32	4.4	1	11.7	1
CC	19.1	88	2.4	2	4.1	72
Venture	34.9	24	0.0	6	10.6	2
Simulation	42.9	5	1.7	3	15.4	1

In this report elaborated by INVERCO we can see that the most profitable fund in 2009 was the *Foncaixa Bolsa España 150*, with a 51.9% of revaluation, although this fund was ranked in the position 79 by the return of -8.9% APR that obtained in the 3 previous years. Our system obtains in 2009 a yield of 42.9%, so that if we could participate in this ranking we would be included in the fifth position by yield to one year.

To three years view the stock market crisis continues nevertheless passing bill, since almost all the funds register red numbers, except the first funds, like *Bankinter Bolsa España 2*, that with a 4.4% APR would remain first inside this ranking of the better investment funds in Spain for 3 years. Our system achieves a revaluation of 1.7% APR in this period, therefore we would remain third in the ranking for profitability for 3 years.

With a horizon of five years, the situation is different: some funds, as the mentioned *Bankinter Bolsa España 2* (11.7% APR) or *Venture Bol. Española* (10.6% APR) obtain notable performances, although our system surpasses all of these funds with a profit value of 15.4% APR.

5 CONCLUSIONS

This article has proposed a hybrid intelligent system that solves quite successful investment in shares forming a portfolio. This system has two main parts: the first is responsible for buying and selling shares, managing a portfolio and monitoring the purchased shares and the second part is responsible for selecting which are the best shares to incorporate them into the portfolio.

The part entrusted to realize the tactical asset allocation, corresponds to a decision system based on rules and the part entrusted to select shares has been based on a fuzzy inference system.

In the obtained results the revaluation of the reference index is surpassed (IBEX35) in all the periods and even we can place the hybrid intelligent system in the first positions of the ranking by profit value if it is compared with commercial investment funds that invest in Spanish equities.

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REFERENCES

Amenc, N. and Sourd, V. L. (2003). *Portfolio Theory and Performance Analysis*. Wiley Finance.

Atsalakis, G. and Valavanis, K. (2009). Forecasting stock market short-term trends using a neuro-fuzzy based methodology. *Expert Systems with Applications*, 36(7):10696–10707.

BME (2009). *Bolsas y Mercados Españoles*. Normalidad de las series financieras. http://www.bolsasymercados.es.

Cheung, W. and Kaymak, U. (2007). A fuzzy logic based trading system. In *Proc. on the 3rd European Symposium on Nature-inspired Smart Information Systems*.

Dourra, H. and Siy, P. (2002). Investment using technical analysis and fuzzy logic. *Fuzzy Sets and Systems*, 127(2):221–240.

INVERCO (2009). *Asociación de Instituciones de Inversión Colectiva y Fondos de Pensiones*. Ranking Fondos Inversión a 31-12-2009. http://www.inverco.es.

Kosko, B. (1992). *Neural network and Fuzzy systems. A dynamical systems approach to machine intelligence*. Prentice Hall Press.

Malkiel, B. (1973). *A Random Walk Down Wall Street*. W. W. Norton & Company.

Murphy, J. (1999). *Technical Analysis of the Financial Markets: A Comprehensive Guide to Trading Methods and Applications*. Prentice Hall Press.

Sun, H. and Liu, L. (2002). A linear output structure for fuzzy logic controllers. *Fuzzy Sets and Systems*, 131(2):265–270.