A Pedagogical Approximation to the Problem of the Structure of the Asteroid Belt between Mars and Jupiter

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Abstract. *The study of the many body problem associated to the dynamics of the different elements of our Solar System is a complex subject for introductory level Physics students. But building and using computer simulations allows the students to be introduced to this subject. Here we present a study of the dynamical structure of the main asteroid belt between Mars and Jupiter. First we considered only the simpler problem of a fictitious negligible mass asteroid in the gravity field of a Star and a big Planet (Sun and Jupiter, for instance); then, the inclusion of inner and external planets (Mars and Saturn) allows the reproduction of the more realistic situation found in our Solar System. The orbits of several hundreds of asteroids are integrated numerically and the periods and other characteristics of stable orbits are obtained. Our results reproduce many of the Kirkwood gaps, observed in the main belt asteroids, associated to the different non linear resonances there found.*

Keywords. Kirkwood's gap, computer simulation, nonlinear resonance.

1. Introduction

The students of General Physical course (1st. course of Physics at UMU) are making a set of computer assisted simulations in which they study a high variety of physical problems.

We have observed that the problems related to astronomy, relativity, quantum or magnetic confinement arouse a great interest to the students. Usually, the more attracting problems are the ones with difficult solution due to their academic level or great amount of calculations involved. This is the situation which is presented in this paper: to simulate the observed asteroid distribution between the orbits of Mars and Jupiter.

D. Kirkwood determined experimentally in 1867 the radial distribution of asteroids and he found that at given distances from the Sun seemed that there were no asteroids. These zones of smaller density of asteroids are denominated Kirkwood´s gap, Fig. 1, [1]. These gaps are not a random distribution; they satisfy that the ratio between the orbital period of Jupiter (T_J) and the period of the missing asteroid (T_a), is a rational number.

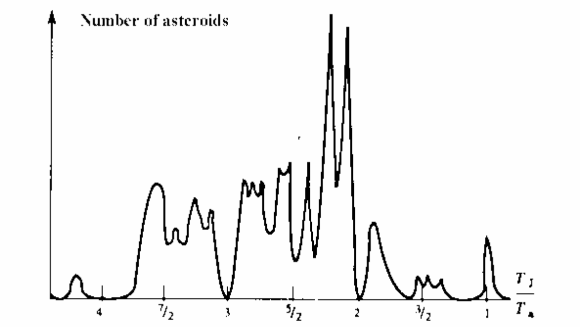

Figure 1. Kirkwood's gaps distribution

Asteroids motion, like for planets, is mainly due to the Sun attraction force. The gaps, nevertheless, are related to the gravitational Jupiter effect [2]; but for totally explaining the gaps distribution it is even necessary to include the Mars and Saturn gravitational effect [3]

From an academic perspective this is a simple problem: one considers the dynamics of a particle due to gravitational interaction with the planets and the Sun, so that the force on the asteroid depends on its position. From the didactic perspective, the cosmological exposition of the problem in the surroundings of the structure of the Solar System, makes our problem interesting for the student; in addition, if we connect this with the topics "chaos", "resonance", etc., it becomes a leading study, an open problem and the student, then, becomes a researcher.

In order to solve the motion equations, the numerical method of finite differences (FD) is used, [5]. This method has many nice features. First of all it is very intuitive and very easy to implement. Secondly, its application to solve the motion equations demands to know the initial conditions of the problem, therefore we need to know, the physical system state at the initial time. That idea: "I will guess the future knowing the initial state and the temporal evolution equations", is basic in the deterministic presentation of the Newtonian Mechanics; the FD method shows this.

2. Modelling our system

2.1 Model assumptions and initial conditions.

Firstly we have to talk about the simplifications made in our model and the resulting initial conditions of the system:

-The Sun is located at centre of the reference system.

-We consider the planets to describe circular coplanar orbits and, initially, we locate them in conjunction. Then, we work out the problem in 2 dimensions.

-We allocate a value to orbital Jupiter radius (R_J=10 length units) and we scale the other radii: Mars (R_M=3), Saturn (R_S=20) and the asteroids ($3.5<R_a<8$), Fig. 2.

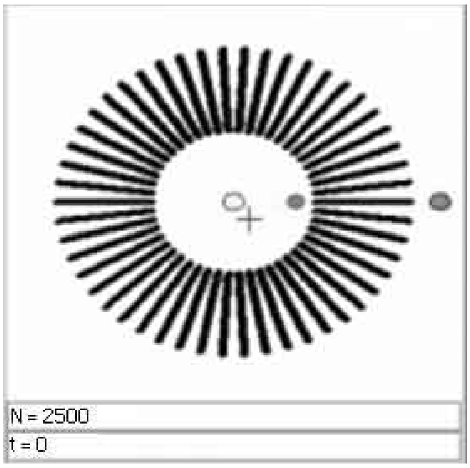

Figure 2. Initial distribution

-We fix the Jupiter period to T_J= 10 time units and this allows computing the value of $G.M_{SUN}$. In addition, by the Keppler's third law, the remaining periods (T_M, T_S) can be determined, [1].

-We assume the initial asteroids velocities as if they were in circular orbits due to just the gravitational action of the Sun. Logically, when, in their dynamical evolution, we consider planets gravitational fields, those orbits will not be circular anymore.

-The masses that we assigned to the planets, are, related to the Sun mass (M_{SUN}=10), much greater than the real ones; in this way, we improve the accuracy of our numerical computations (M_J=2, M_M=.01, M_S=.1)

2.2 Contents of the simulation.

Once the initial conditions were established, we go to the dynamics of the process. We impose the planets to describe a circular uniform motion

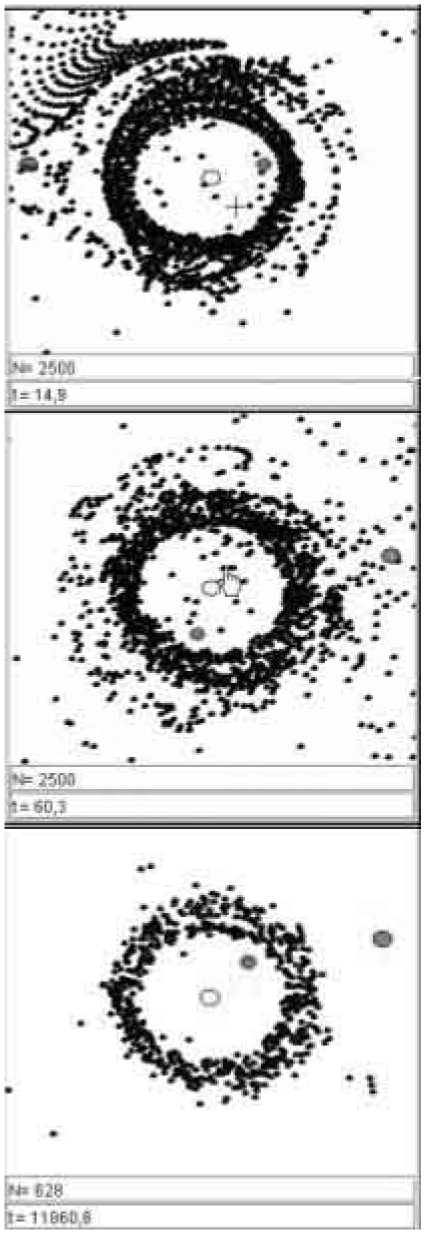

Figure3. Time evolution of a 2500 asteroids in the gravity field of Sun, Mars and Jupiter

and we obtain the asteroids trajectories by FD method.

The simulation includes the following elements:

-Dynamical evolution. We calculate the future positions of the different asteroids, taking into account the total gravitational field at its present positions.

-Asteroid's update. Asteroids that are too close to the Sun or planets, or too far away from the interest region, are eliminated.

-Periods calculation. Asteroids periods are determined through the angle swept by the asteroid position vector along a fixed temporal interval.

-Asteroid Distribution. An algorithm calculates and represents the asteroid distribution versus the T_J/T_a ratio.

2.3 Results and Discussion

First, we have studied the influence of Jupiter and Mars in the asteroids dynamical evolution. Later, we have included Saturn, and analyzed its influence. In both cases, we have run our model for an initial number of 2500 asteroids distributed in a 50x50 circular mesh (50 rings with radii in the range $3.5< R_a < 8.50$ asteroids each one).

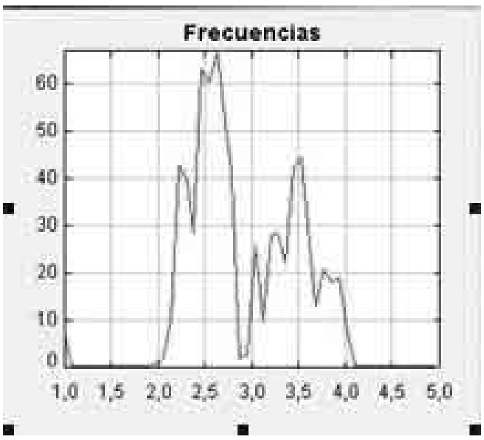

Figure 4. Distribution of asteroids

At the beginning of our study we did not consider the Mars planet. A considerable increase of the orbits eccentricity was observed, although we did not find a clear gaps distribution. Those results were expected, due to the role of Mars in the elimination of the asteroids crossing its orbit.

In the Fig. 3, different instants of the dynamic simulation are shown. A visual inspection of the asteroids spatial distribution, for large t, shows the mentioned gaps; those gaps are also shown in Fig. 4. Those gaps are present in the regions also found in the real distribution, Fig. 5, [4], corresponding to the resonances: 7/3, 3/1, 10/3 and 11/3. Also as in the real distribution very dense areas appear.

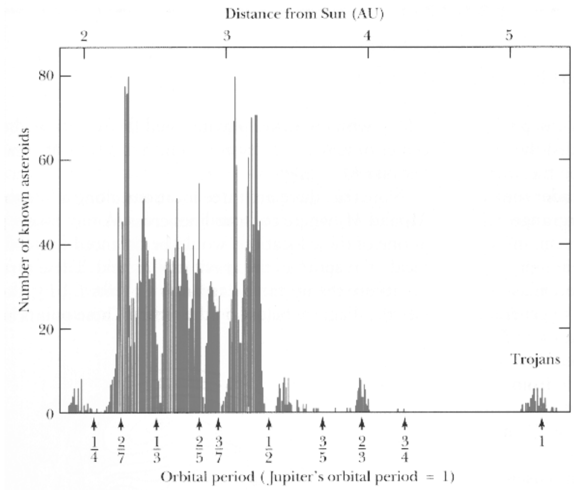

Figure 5. Asteroid Main-Belt Distribution. The T_J/T_a ratio increase to the left.

Including Saturn results in a new resonance at 8/3, Fig. 6 and Fig. 7.

Regarding the obtained results we would like to emphasize the following:

-In all studied situations, once the system is stabilized, we found radial zones with low number of asteroids. We verified that these zones correspond indeed with the distances to which the periods that correspond to the asteroids are simple fractions of the period of Jupiter. This result is related to a chaotic resonant phenomenon, whose effect is to destabilize some asteroids, sending them outside the belt, producing very dense population in other regions.

-We have not considered the eccentricity of the orbits, and we think this is not very important to visualize the phenomenon. Its effect has been partially compensated with the increase of the Jupiter mass relative to the Sun. This increases the amplitude of the disturbance on the asteroids. This can also have negative effects on the stability, for instance, of the Trojan asteroids.

-The simplicity of the model and the inaccuracies of the numerical calculations could justify the non detection of other gaps.

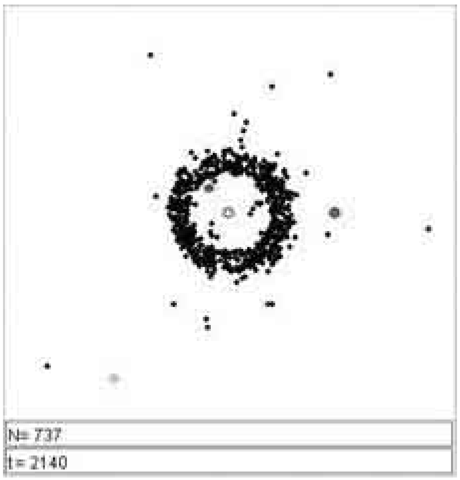

Figure 6. Spatial distribution considering in addition Saturn

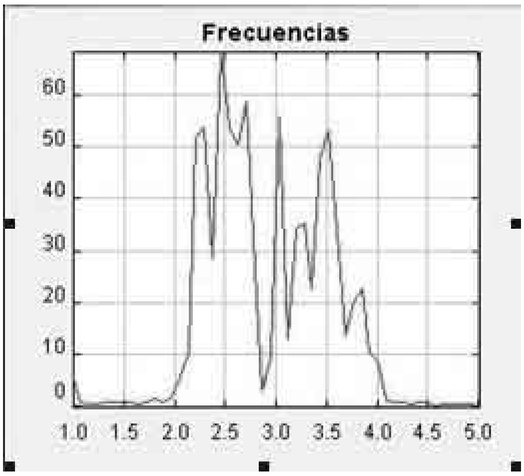

Figure 7. Radial distribution of asteroids considering Saturn

3. Conclusion

The radial distribution of asteroids between the orbits of Mars and Jupiter is an example of gravitational chaos. An asteroid is influenced mainly by the Sun attraction, but the minor action of Jupiter produces the resonance effect we have studied.

The scope of our study is very limited. A proper study should consider many more variables in order to be realistic, but this that was not the main interest of this work. From an educational point of view we were satisfied with obtaining qualitative results that allowed us to observe the fundamental phenomenon, the existence of gaps, learning, in addition, to implement some simple programming tools.

5. References

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