Contract cost in central banking: treating the symptom vs. treating the disease?

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Abstract

Chortareas and Miller (2003) extended the basic principal-agent framework for central banking to consider that contracts generate costs for the government. In such setting, they claim that a contract which penalizes the central bank when output exceeds its natural level yields higher social welfare than an inflation contract, since only the former eliminates the inflation bias.

We challenge these conclusions and prove that both incentive schemes (and any other generic linear output or inflation contract) eliminate this bias, achieving the same social welfare, provided the government behaves in accordance with optimizing behavior. Finally, we show that contract costs may render monetary policy delegation counterproductive.

Keywords: central bank, inflation bias, contract cost

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1 Introduction

Since the appearance of the pathbreaking articles of Kydland and Prescott (1977) and Barro and Gordon (1983) a widespread consensus has emerged among academics and policymakers that, when the monetary authority faces an incentive to expand output above its natural level, discretionary monetary policy gives rise to an inefficiently high level of inflation without any gain in terms of output. To tackle this inflation bias many countries have delegated monetary policy to autonomous central banks. Walsh (1995) modelled this process of delegation as a contract within a principal-agent framework. He showed that the inflation bias can be eliminated without incurring any output stabilization costs if the government (principal) offers the central bank (agent) an incentive scheme (an inflation contract) which penalizes the latter for creating inflation.[1] He established this result in a setting where such incentives schemes do not generate a cost for the government, i.e., it only cares about deviations of output and inflation from socially desirable levels.[2] However, Walsh (1995, p. 156, footnote 10) also stated that "An alternative approach would assume that the government's objective is to minimize the expected loss plus the transfer to the central banker. However, in the present context, the government will always be able to offer a minimum-cost contract. Therefore, assuming that the government minimized $E(V)$ *(expected loss)* instead of $E(V + t)$ *(expected loss plus the transfer)* involves no loss of generality" *(words in italics added).*

In a recent paper, Chortareas and Miller (2003), in what follows C-M, have challenged this statement, stating: "As we demonstrate, this conjecture is inaccurate". More precisely, they claim to have proved that, when the incentive transfer generates a cost for the principal, an

[1] Other solutions as the conservative central banker (Rogoff [1985]) or inflation targeting (Svensson [1997]) can also be interpreted as contracts. The interpretation of Rogoff's arrangement as a "quadratic contract" appears in Walsh (1995), Beetsma and Jensen (1998) and Jensen (2000). The interpretation of Svensson's institution as a contract appears in Persson and Tabellini (2000, chapter 17). On the other hand, Bernanke (2004) has stated that, over the past 25 years, the papers by Kydland and Prescott (1977), Rogoff (1985) and Walsh (1995) have been the most influential ones in monetary policy.

[2] Walsh (1995) considers explicitly the possibility that the incentive scheme represents a cost for the principal in another setting (section III, pp.160-166) where imperfect information on the central bank's type allows it to earn rents.

inflation contract *à la* Walsh cannot eliminate the inflation bias and, as a consequence, they conclude that the scenario analyzed by Walsh (1995) involves a loss of generality. As a way out of the inflation bias, C-M put forward an alternative contract that penalizes the central banker when output exceeds the natural level. According to C-M, this incentive scheme achieves a superior result because it attacks directly the root of the problem, i.e., the expansionary bias in output, while the inflation contract only attacks the consequence of this expansionary bias, i.e., inflation. In this respect, C-M make use of the following metaphor: "the output contract treats the disease; the inflation contract treats the symptom". As a result, they conclude that when incentive schemes represent a cost for the government, its welfare is higher if it offers the central bank an output contract instead of an inflation contract *à la* Walsh.

However, we show that C-M are wrong in stating that the conjecture by Walsh is inaccurate, since both types of incentive schemes, if properly designed, yield the same social welfare, i.e., the principal is indifferent between both of them. In fact, the complete elimination of the inflation bias is not exclusive of their proposed contract which penalizes the central bank when output exceeds the natural level. We prove that this bias can be removed by a generic linear output (or inflation) contract as long as it penalizes properly deviations of output (or inflation) from any arbitrary chosen value.

The reason why our results are at odds with those of C-M lies in that their proposed contracts are not derived from optimizing behavior from the part of the principal (the government). As a consequence, the incentive schemes obtained by making use of their approach cannot be guaranteed to be, using Walsh's (1995) words, "minimum-cost contracts". In fact, the analysis by C-M yields incorrect conclusions regarding the inflation bias.

To be more precise, the two contracts considered by C-M are determined by two choice variables of the government, namely, a fixed part and a "penalization rate" (on inflation or on increases of output above its natural level). But, in order to derive these two incentive schemes C-M fail to bear in mind that the government faces a constraint optimization problem when it chooses both strategic variables. Instead, they assume that the government solves a two-step problem of sorts. To wit, in a first step, the principal chooses only the penalization rate. However, it does so as if it were solving a non-constraint or free optimization problem (i.e., ignoring the participation constraint of the agent); and, in a second step, taking into account the penalization rate found in the first-step, the government is assumed to select the fixed part of the contract, so that the participation constraint holds. This way of proceeding implies that the incentive schemes obtained by C-M are not derived from optimizing behavior from the part of the government, which requires that its two choice variables be solved for simultaneously.

We also prove that, apart from the output contract proposed by C-M and the Walsh inflation contract, any generic contract which linearly links, explicitly or implicitly, the central bank's incentives to inflation eliminates the inflation bias and achieves the same level of social welfare, provided that it is designed in accordance with optimizing behavior. Finally, we show that when incentive schemes represent a cost to the government, it is not always in its own interest to offer the central bank an output or an inflation contract.

The rest of the paper is organized as follows. Section 2 presents briefly the model used by C-M. Section 3 studies how the inflation bias and social welfare are affected when the incentives schemes just mentioned are in place and compares our conclusions with the ones by C-M. Finally, Section 4 concludes.

2 The model

As in C-M (and following their notation for ease of comparison), the working of the economy is summarized by the following equations[3]:

$$\begin{aligned} y &= y^n + \alpha(\pi - \pi^e) + \varepsilon, \qquad (1) \\ \pi &= m + \nu - \gamma\varepsilon, \qquad (2) \\ U^G &= -\left[(y-y^*)^2 + \beta\pi^2\right] - \phi\left[tr(.)\right], \qquad (3) \\ U^{CB} &= -\left[(y-y^*)^2 + \beta\pi^2\right] + \xi\left[tr(.)\right], \qquad (4) \end{aligned}$$

where y^n, α, β, ϕ, $\xi > 0$; and superscripts, "G" and "CB", respectively, stand for "Government" and "Central Bank". Equation (1) shows that the economy possesses a Lucas supply function,

[3] The framework is the one considered by Walsh (1995), except for that C-M extend it to consider that incentive schemes represents a cost for the government since, for instance, it may be financed through distorsionary taxes (see C-M, p. 103).

so that the difference between output (y) and the natural level (y^n) depends on the deviations of inflation (π) from its expected value (π^e) and on a supply shock (ε) with zero mean and finite variance $\left(\sigma_\varepsilon^2\right)$. Expectations are rational, that is, $\pi^e = E\{\pi\}$, where $E\ \{.\}$ is the expectations operator. Expression (2) states that inflation is a function of: a) the growth of a monetary aggregate determined by the central bank (m); b) a velocity shock or a control error (ν), with $E\{\nu\} = 0$ and $E\{\nu^2\} = \sigma_\nu^2$, which is uncorrelated with ε; and c) the supply shock (which also appears in (1)), where γ picks up the effect of this shock on inflation.

Equations (3) and (4) represent the utility functions of, respectively, the government (principal) and central bank (agent). Each of these two expressions consists of two terms. The corresponding first terms mean that the government and the central bank care about deviations of inflation and output from some desired levels. This term is identical for both of these players and their common target value for inflation is normalized to zero. Besides, as it is standard in the literature on credibility in monetary policy, it is assumed that their common output objective (y^*) is higher than the natural level[4], i.e., $y^* > y^n$, and define $k \equiv y^* - y^n > 0$. This discrepancy gives rise to the classical time consistency problem to discretionary monetary policy which causes an "inflation bias".

The second terms in (3) and (4) stand for the valuation that the principal and the agent have, respectively, of the incentive scheme ($tr(.)$) designed by the former. More specifically, Subsection 3.1 considers the case of Walsh contract; and Subsection 3.2 analyzes the scenario with an output contract. Parameters ϕ and ξ represent, respectively, the weights that the government and the central banker put on the incentive scheme relative to the aggregate effect on their welfare of the deviations of both output and inflation from desirable levels. Since both these parameters are positive, the contract implies a cost for the principal a welfare gain for the agent.

The government is assumed to be benevolent, i.e., it has the social preferences. Therefore, social welfare will be evaluated in terms of the government's expected utility function appearing in (3).

The interactions between the government, the central bank and the private sector are mod-

[4] The suboptimality of the natural level of output stems from the existence of market failures, distortions, imperfections or rigidities in the economy.

elled through a multi-stage game. The sequence of events is as follows:

1) The government offers the central bank a contract ($tr(.)$).

2) The private sector observes the incentive scheme and then forms its expectations on inflation (π^e).

3) The realization of the output shock (ε) becomes common knowledge.

4) The central bank selects the level of the policy instrument (m).

5) The stochastic control error or velocity shock takes place (ν).

3 The results

In order to facilitate comparison with the conclusions of C-M, we derive our results in two different subsections depending on whether the incentive scheme is an inflation or an output contract.

3.1 Inflation contract

We begin by analyzing the case where the government offers the central bank a linear inflation contract. Formally, the incentive scheme designed by the principal is:

$$tr\,(.) = t_0 - t\pi. \tag{5}$$

Therefore, plugging (5) into (3) and (4), the utility functions of the government and the central bank become, respectively:

$$U^G = -\left[(y-y^*)^2 + \beta\pi^2\right] - \phi\left[t_0 - t\pi\right], \tag{6}$$

$$U^{CB} = -\left[(y-y^*)^2 + \beta\pi^2\right] + \xi\left[(t_0 - t\pi)\right]. \tag{7}$$

We look for a subgame perfect equilibrium. Therefore, we apply backward induction to the game outlined in Section 2 (for the case of an inflation contract). In the last stage of the game, once the private sector has set up its expectations on inflation, the central banker observes the realization of the shock (ε) and then responds to the incentives embedded in the contract. It does so by selecting the value for m that solves the following program (bearing in mind (1), (2) and (7)):

$$\underset{\{m\}}{Max} \quad -\left[(y-y^{*})^{2}+\beta\pi^{2}\right]+\xi\left[t_{0}-t\pi\right]$$
$$s.t. \quad \begin{cases} y=y^{n}+\alpha(\pi-\pi^{e})+\varepsilon, \\ \pi=m+\nu-\gamma\varepsilon. \end{cases}$$

The solution yields the following reaction function of the monetary authorities:

$$m=\left(\gamma-\left(\frac{\alpha}{\alpha^{2}+\beta}\right)\right)\varepsilon+\frac{\alpha^{2}}{\alpha^{2}+\beta}m^{e}+\frac{\alpha}{\alpha^{2}+\beta}k-\frac{\xi}{2\left(\alpha^{2}+\beta\right)}t. \tag{8}$$

Notice that a comparison of our expression for m in (8) with the corresponding equation in the paper by C-M ((12) in page 106) shows that the latter is incorrect. The reason is that the control error, v, appears in the latter implying that the central bank observes and takes into account the realization of the control error, v, when setting its policy instrument, m. This contradicts their assumption (and ours) that monetary authorities select the value of their policy instrument before (and not after) the realization of the control error occurs.

Anticipating the central bank's behavior, the private sector forms its rational expectations on inflation (since $m^e = \pi^e$):

$$\pi^{e}=\frac{\alpha}{\beta}k-\frac{\xi}{2\beta}t. \tag{9}$$

Now, plugging this value for the expected inflation into equation (8) and substituting the resulting expression into (2) one obtains:

$$\pi=\frac{\alpha}{\beta}k-\frac{\xi}{2\beta}t+\nu-\left(\frac{\alpha}{\alpha^{2}+\beta}\right)\varepsilon. \tag{10}$$

Our analysis so far has been equivalent to the one by C-M (apart from the inaccuracy just mentioned). However, the way in which we solve the first stage of the game is completely different from theirs.

In order to find the solution of the first stage, we need to express the expected utility functions of the government and the central bank in terms of the variables which define the contract, namely, t_0 and t. With this aim, first we substitute (1) into (6) and (7). Then, we plug the values for π^e and π (appearing in equations (9) and (10)) into the resulting two expressions for U^G, and U^{CB}. After doing so, taking expectations yields:

$$E\left(U^{G}\right) = -\phi t_0 - \frac{(2\phi+\xi)\,\xi}{4\beta}t^2 + \frac{\alpha k\,(\xi+\phi)}{\beta}t - \left(\alpha^2+\beta\right)\left(\frac{k^2}{\beta}+\sigma_\nu^2\right) - \frac{\beta}{\alpha^2+\beta}\sigma_\varepsilon^2, \quad (11)$$

$$E\left(U^{CB}\right) = \xi t_0 + \frac{\xi^2}{4\beta}t^2 - \left(\alpha^2+\beta\right)\left(\frac{k^2}{\beta}+\sigma_\nu^2\right) - \frac{\beta}{\alpha^2+\beta}\sigma_\varepsilon^2. \quad (12)$$

In the first stage, the principal chooses the value of its strategic variables, namely, the ones that shape the contract. It does so bearing in mind that the monetary authorities must accept the incentive scheme being offered. This "participation constraint" states that the expected utility obtained by the central bank when signing the contract must be higher or equal to a given reservation level, normalized to zero. Therefore, the government solves:

$$\underset{\{t_0,t\}}{Max} \quad E\left(U^{G}\right)$$
$$s.t. \quad E(U^{CB}) \geq 0,$$

which results in the following Lagrangian function:

$$\pounds = E\left(L^{G}\right) + \mu E\left(U^{CB}\right).$$

The Kuhn-Tucker first order conditions of this problem are:

$$\frac{\partial \pounds}{\partial t_0} = \frac{\partial \pounds}{\partial t} = 0, \quad (13)$$

$$\frac{\partial \pounds}{\partial \mu} \geq 0 \quad \text{and} \quad \left(\frac{\partial \pounds}{\partial \mu} = 0 \quad if \quad \mu > 0; \quad \frac{\partial \pounds}{\partial \mu} > 0 \quad if \quad \mu = 0\right). \quad (14)$$

Solving the two first order conditions (appearing in (13)) for the Lagrangian multiplier, μ, and equating yields:

$$\mu = -\frac{\frac{\partial E\left(U^{G}\right)}{\partial t_0}}{\frac{\partial E\left(U^{CB}\right)}{\partial t_0}} = -\frac{\frac{\partial E\left(U^{G}\right)}{\partial t}}{\frac{\partial E\left(U^{CB}\right)}{\partial t}}. \quad (15)$$

Now, rearranging we obtain the equality of the marginal rates of substitution (between t_0 and t) of the government and the central bank:

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^{G})=\overline{E(U^{G})}} = \frac{\frac{\partial E\left(U^{G}\right)}{\partial t}}{\frac{\partial E\left(U^{G}\right)}{\partial t_0}} = \frac{\frac{\partial E\left(U^{CB}\right)}{\partial t}}{\frac{\partial E\left(U^{CB}\right)}{\partial t_0}} = \left.\frac{\partial t_0}{\partial t}\right|_{E(U^{CB})=\overline{E(U^{CB})}}. \quad (16)$$

Calculating both marginal rates of substitution (from (11), (12) and (16)) we have:

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^{G})=\overline{E(U^{G})}} = \frac{\alpha k\,(\xi+\phi)}{\phi\beta} - \frac{\xi\,(2\phi+\xi)}{2\phi\beta}t, \quad (17)$$

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^{CB})=\overline{E(U^{CB})}} = -\frac{\xi}{2\beta}t. \quad (18)$$

Equating (17) and (18) and rearranging yields the optimal penalization on inflation:

$$t^{\pi} = \frac{2\alpha k}{\xi} \tag{19}$$

Now, from (11), (12) and (15) one finds the value of the Lagrangian multiplier:

$$\mu = \frac{\phi}{\xi} > 0. \tag{20}$$

Therefore, this multiplier is strictly positive which implies that the participation constraint holds with equality (from the first order condition appearing in (14)). Therefore, equating the expected utility of the central banker (given in (12)) to its reservation level, and solving for t_0, one obtains the expression for the equilibrium value of the fixed part of the contract:

$$t_0^{\pi} = \frac{1}{\xi}\left(k^2 + \left(\alpha^2 + \beta\right)\sigma_{\nu}^2 + \frac{\beta}{\left(\alpha^2 + \beta\right)}\sigma_{\varepsilon}^2\right). \tag{21}$$

Plugging (19) and (21) into (11) one finds the principal's expected utility under an optimum inflation contract:

$$E\left(U_{\pi}^{G}\right) = -\frac{\left(\xi + \phi\right)k^2}{\xi} - \frac{\left(\alpha^2 + \beta\right)\left(\xi + \phi\right)\sigma_{\nu}^2}{\xi} - \frac{\beta\left(\xi + \phi\right)\sigma_{\varepsilon}^2}{\left(\alpha^2 + \beta\right)\xi} \tag{22}$$

Finally, the resulting inflation bias is determined by substituting (19) into (9), which yields an expected inflation equal to zero. In other words, the inflation contract completely eliminates the inflation bias, irrespective of whether or not the incentive scheme generate a cost for the government.

Remark 1: This conclusion is in sharp contrast with the one by C-M who, in their Proposition 1, claimed quite the opposite, i.e.: "Unless the government places a zero weight on the cost associated with the central banker's incentive scheme, a linear inflation contract cannot completely eliminate the inflation bias" .

Remark 2: The Corollary 2 in C-M is incorrect. That is, their claim that "the more 'cost-conscious' the government is, the lower the required marginal penalization rate is in the government's optimal contract" is not in accordance with (19)*, i.e., the marginal penalization rate* (t^{π}) *does not dependent on the government's degree of cost-consciousness* (ϕ)*.*

The conclusions reached by C-M referred to in *Remark 1* and *Remark 2* are wrong since their analysis fails to consider that the government faces a constraint optimization problem when solving for all the variables that shape its contract. Instead, C-M implicitly assume that the principal solves some sort of two-step problem. To wit, in a first step, it chooses just the penalization rate, t, overlooking the participation constraint of the central bank; and, in a second step, it selects the fixed part of the contract, t_0, bearing in mind that the agent must accept the incentive scheme. That is, this first step is solved as if the government faced a non-constraint or free optimization problem (i.e., ignoring the restriction implied by the participation constraint of the agent). Formally, in this initial step, C-M obtain the equilibrium values of t by solving the following equation:

$$\frac{\partial E(U^G)}{\partial t} = 0, \tag{23}$$

which yields their (incorrect) expression (15):

$$t^{C-M} = \frac{2\alpha k}{\xi}\left(\frac{\xi+\phi}{\xi+2\phi}\right). \tag{24}$$

That is, C-M implicitly assume that the participation constraint is not binding in this first step, i.e., that the Lagrangian multiplier is zero. However, this analysis is not appropriate since, as shown in (20), the value of this multiplier is strictly positive.

In the second step, C-M assume that the other variable that shape the incentive scheme (i.e., the fixed part of the contract) is chosen so that the participation constraint (now) holds, taking account of the penalization rate determined in the first-step. Proceeding this way would (incorrectly) yield[5]:

$$t_0^{C-M} = \frac{1}{\xi}\left(\frac{\left(\beta\left(\xi+2\phi\right)^2+\phi\alpha^2\left(3\phi+2\xi\right)\right)k^2}{\beta\left(\xi+2\phi\right)^2}+\left(\alpha^2+\beta\right)\sigma_\nu^2+\frac{\beta}{\left(\alpha^2+\beta\right)}\sigma_\varepsilon^2\right). \tag{25}$$

Therefore, if the government were to offer the contract proposed by C-M, it would obtain the

[5] The expression of the fixed part of the inflation contract proposed by C-M does not appear in their paper. However, we indicate how such expression would look like (equation (25)) since we need to make use of it (below) in order: i) to obtain the value of the expected utility of the government with such a (suboptimal) contract; and ii) to compare this value with the one achieved under the optimal inflation contract.

following level of expected utility (plugging (24) and (25) into (11)):

$$E\left(U^G_{C-M}\right) = -\frac{(\phi+\xi)\left(\beta(\xi+2\phi)^2+\phi^2\alpha^2\right)k^2}{\xi\beta(\xi+2\phi)^2} - \frac{\left(\alpha^2+\beta\right)(\phi+\xi)\sigma^2_\nu}{\xi} - \frac{\beta(\phi+\xi)\sigma^2_\varepsilon}{\xi(\alpha^2+\beta)}. \quad (26)$$

Remark 3: The contract put forward by C-M is not derived from optimizing behavior i.e., it is not a minimum-cost contract.

Proof:

We have shown that the optimal inflation contract (characterized by (19) and (21)) yields the government an expected utility $E\left(U^G_\pi\right)$, which appears in (22). The difference between this value and U^G_{C-M}, i.e., the one obtained if the principal designed the C-M contract (in (26)), is equal to:

$$E\left(U^G_\pi\right) - E\left(U^G_{C-M}\right) = \frac{\phi^2(\xi+\phi)k^2\alpha^2}{\xi(\xi+2\phi)^2\beta} > 0. \quad (27)$$

Since this difference in (27) is strictly positive, the approach by C-M is not appropriate. As we have pointed out, the reason is that it overlooks the participation constraint when finding one choice variable of the principal, namely, the penalization rate. As a result, the incentive scheme put forward by these authors is not derived from optimizing behavior from the part of the principal, i.e., it is not a minimum-cost contract[6]. ∎

Remark 4: The way of proceeding by C-M would have been appropriate if they have considered the "basic" principal-agent framework for central banking, i.e., the one where contracts do not represent a cost for the government.

In fact, Walsh (1995) did use this two-step procedure in the basic principal agent framework and, therefore, was right to conclude that the inflation bias could be removed through a linear

[6] As mentioned above, this result is in accordance with Walsh (1995). In addition, in a related model, Candel-Sánchez and Campoy-Miñarro (2004, final note n. 4) support our conclusion when they point out (using a different notation): "if the government is concerned with $t(\pi)$ and it sets up the value for a to make the acceptance constraint hold with strict equality, then the government's optimal choice for b is $2\alpha k$, the Walsh's optimal penalization". Besides, Chortareas and Miller (2006) point to the same direction as well.

inflation contact. However, we have shown that this way of proceeding cannot be followed in the more complex setting considered in C-M (and in our paper) where contracts do represent a cost for the principal.

3.2 Output contract

In this subsection we take up the scenario where the contract penalizes the central banker when output is above its the natural level, i.e., the incentive scheme is:

$$tr = t_0 - t(y - y^n). \tag{28}$$

The game is solved in an analogous way as the one described in the previous subsection. Therefore, in the last stage, central bank solves (from (1), (2), (4) and (28)):

$$\begin{aligned} &\underset{\{m\}}{Max} \quad -\left[(y-y^*)^2 + \beta\pi^2\right] + \xi\left[t_0 - t(y-y^n)\right] \\ &s.t. \quad \begin{cases} y = y^n + \alpha(\pi - \pi^e) + \varepsilon, \\ \pi = m + \nu - \gamma\varepsilon, \end{cases} \end{aligned}$$

which yields:

$$m = \left(\gamma - \left(\frac{\alpha}{\alpha^2+\beta}\right)\right)\varepsilon + \frac{\alpha^2}{\alpha^2+\beta}m^e + \frac{\alpha}{\alpha^2+\beta}k - \frac{\xi\alpha}{2\left(\alpha^2+\beta\right)}t. \tag{29}$$

Notice that (29) is similar to the expression numbered (28) in the paper by C-M, except for that the control error (ν) mistakenly appears in the latter. As explained in the previous subsection, this control error cannot be present in the expression for the reaction function of the central bank, since it would wrongly imply that the monetary authorities observe and take into account the realization of the control error, v, when setting its policy instrument, m.[7]

Expected and actual inflation are obtained by making use of the expectation operator and expressions (29) and (2):

$$\pi^e = m^e = \frac{\alpha k}{\beta} - \left(\frac{\xi\alpha}{2\beta}\right)t, \tag{30}$$

$$\pi = \frac{\alpha k}{\beta} - \frac{\xi\alpha}{2\beta}t - \left(\frac{\alpha}{\alpha^2+\beta}\right)\varepsilon + \nu. \tag{31}$$

[7]A similar mistake appears in their expresion (33). In addition, page 109 contains an erratum. To wit, it should read "$k > 0$" instead of "$z > 0$".

Proposition 1: Social welfare is the same irrespective of whether the central bank is offered an inflation contract or an incentive scheme which penalizes it when output exceeds its natural level.

Proof:

Moving up to the first stage, we need to express the expected utility functions of the government and the central bank in terms of the choice variables, t_0 and t that determined the output contract. Therefore, making use of the expectation operator together with (1), (3), (4), (30) and (31) yields:

$$E\left(U^G\right) = -\phi t_0 - \frac{\alpha^2\xi^2}{4\beta}t^2 + \frac{\alpha^2 k\xi}{\beta}t - \left(\alpha^2+\beta\right)\left(\frac{k^2}{\beta}+\sigma_\nu^2\right) - \frac{\beta}{\alpha^2+\beta}\sigma_\varepsilon^2, \tag{32}$$

$$E\left(U^{CB}\right) = \xi t_0 - \frac{\alpha^2\xi^2}{4\beta}t^2 + \frac{\alpha^2 k\xi}{\beta}t - \left(\alpha^2+\beta\right)\left(\frac{k^2}{\beta}+\sigma_\nu^2\right) - \frac{\beta}{\alpha^2+\beta}\sigma_\varepsilon^2. \tag{33}$$

Again, making use of the Kuhn-Tucker conditions in the same way as in the previous subsection, one obtains the marginal rates of substitution between t_0 and t for, respectively, the government and the central bank:

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^G)=\overline{E(U^G)}} = \frac{\xi\left(2k-\xi t\right)\alpha^2}{2\phi\beta}, \tag{34}$$

$$\left.\frac{\partial t_0}{\partial t}\right|_{E(U^{CB})=\overline{E(U^{CB})}} = -\frac{\left(2k-\xi t\right)\alpha^2}{2\beta}. \tag{35}$$

Equating both marginal rate of substitution one obtains the optimal penalization on inflation:

$$t^y = \frac{2k}{\xi}. \tag{36}$$

The inflation bias is also eliminated with this output contract since substituting (36) into (30) yields that expected inflation is equal to zero as well.

Upon substituting (36) into (33) and equating to zero (since, again, the participation constraint must hold with equality) the equilibrium value of the fixed part of the contract is found to be:

$$t_0^y = \frac{1}{\xi}\left(k^2 + \left(\alpha^2+\beta\right)\sigma_\nu^2 + \frac{\beta\sigma_\varepsilon^2}{\left(\alpha^2+\beta\right)}\right). \tag{37}$$

The expected utility for society with this optimum contract is (plugging (36) and (37) into (32)):

$$E\left(U_y^G\right) = -\frac{(\xi+\phi)\,k^2}{\xi} - \frac{\left(\alpha^2+\beta\right)(\xi+\phi)\,\sigma_\nu^2}{\xi} - \frac{\beta\,(\xi+\phi)\,\sigma_\varepsilon^2}{\left(\alpha^2+\beta\right)\xi} \tag{38}$$

That is, this value coincides with the expected social utility in the case where the government designs an optimal inflation contract (expression (22)). ∎

Remark 5: This result is in sharp contrast with a main conclusion by C-M, namely, that when incentive schemes generate a cost for the government, it would (strictly) prefer to offer the central bank, instead of an inflation contract à la Walsh, an incentive scheme which penalizes increases of output above its natural level.

In fact, the conclusion by C-M that only their proposed output contract, and not an inflation contract, completely eliminates the inflation bias is incorrect: we have shown that both types of incentive schemes can succeed in removing this bias, provided they are optimally designed. In this respect it is worth noting that the intuition provided by C-M to support their conclusion is thoroughly flawed. To wit, these authors claim that the reason why only their proposed output contract can remove the inflation bias is because it attacks directly the root of the expansionary bias for output while the inflation contract only attacks the consequence of this expansionary bias, i.e., the inflation. Using the C-M's metaphor: "the output contract treats the disease; the inflation contract treats the symptom"[8]. We have shown that this explanation is not valid.

[8] Note that, (only) in the case of an output contract, C-M are fortunate enough to end up proposing an incentive scheme which happen to coincide with the optimal one, even though their way of proceeding is, again, inappropiate. More precisely, C-M obtain the penalization rate by solving for t the following first order condition of a sort of "unconstrained" optimization problem from the part of the principal (i.e., overlooking the participation constraint of the agent):

$$\frac{\partial E\left(U^G\right)}{\partial t} = 0.$$

They do so instead of purposedly taking into account the tangency condition between the isoexpected utilities curves of the government and the central bank in the (t_0, t) space, i.e.(see equation (16)):

$$\left.\frac{\partial t_0}{\partial t}\right|_{E\left(U^G\right)=\overline{E\left(U^G\right)}} = \left.\frac{\partial t_0}{\partial t}\right|_{E\left(U^{CB}\right)=\overline{E\left(U^{CB}\right)}}.$$

However, the value of t that fulfills this condition turns out to be, by coincidence, the same as the one obtained

Moreover, the following proposition generalizes the set of contracts that can achieve the social optimum:

Proposition 2: Consider a "generic" linear contract of the form $tr = t_0 + t(x + r)$, *where* x *is either output or inflation and* r *take any real number (positive, negative or zero). This generic incentive scheme yields the same level of social welfare provided the values of* t_0, and t *are chosen by the principal in accordance with optimizing behavior.*

Proof:

Denoting by t_0^g and t^g, respectively, the equilibrium values of t_0 and t for this generic contract, it is straightforward to check that:

a) For the case of an output contract (i.e., $x = y$) we have that:

$t^g = -t^y$ and $t_0^g = t_0^y + t^y(y^n + r)$, where $t^y$and t_0^y appear, respectively, in (36) and (37).

b) When there is an inflation contract in place (i.e., $x = \pi$) the following applies:

$t^g = -t^\pi$ and $t_0^g = t_0^\pi + t^\pi r$, where t^π appears in (19) and t_0^π in (21). ∎

Notice that when this generic incentive scheme is an output contract (i.e., $x = y$), the constant r need not be equal to $-y^n$, as in C-M, namely, it may take any arbitrarily chosen real number (positive, negative or zero). In other words, penalizations need not be linked, to deviations of output from its natural level. As a consequence the conclusion by C-M that their proposed contract is the solution to the inflation bias since it "treats the disease" (i.e., it penalizes increases of output beyond the natural level) is misleading.

by C-M (i.e. $t^y = 2\frac{k}{\xi}$). The reason is that, for this value of t, we have that:

$$\left.\frac{\partial E\left(U^G\right)}{\partial t}\right|_{t=t^y} = \left.\frac{\partial E\left(U^{BC}\right)}{\partial t}\right|_{t=t^y} = 0.$$

To sum up, when trying to obtain the optimal penalization rate, C-M do make the very same analytical error incurred by them when they look for the optimal penalization on inflation (shown in the previous subsection). Namely, they overlook the participation constraint of the agent. In this respect it is well-known that solving a constraint optimization problem ignoring any posible constraint for all the choice variables involved implies that obtaining the correct solution of the problem is not warranted, namely, it can only happen by chance.

The reason why, carrying on with the same metaphoric medical jargon, the "generic pharmaceutical" described in Proposition 2 maximizes social welfare is as follows. Given the quadratic form of the first term of the social utility function (first term appearing in brackets in (3)), any incentive scheme which linearly links (explicitly or implicitly) incentives to inflation does succeed in eliminating the inflation bias. In the case of an inflation contract incentives are explicitly linear in inflation. However, when the contract is linear in output incentives are also (but implicitly) linearly linked to inflation. Why? because output, in turn, is linear in inflation (from (1)).

Finally, extending the standard framework, as in C-M, to consider the possibility that incentive schemes represent a cost to the government raises a natural question which has not been addressed by these authors: is it always in the government's interest to offer the central bank an output or a inflation contract? The answer is provided by this proposition:

Proposition 3: When central bank contracts generate a cost for the government, delegating monetary policy through an output or inflation contract may be counterproductive. This will be the case when the weight that the government puts on these incentive schemes is sufficiently high relative to the weight put by the central bank on them, i.e., when $\frac{\phi}{\xi} > \frac{\alpha^2 k^2}{\beta(k^2+(\alpha^2+\beta)\sigma_\nu^2+\beta\sigma_\varepsilon^2)}$.

Proof:

In the case of delegation (through an inflation or an output contract) we have obtained that the expected utility of the government is (from (22) or (38)):

$$E\left(U_\pi^G\right) = E\left(U_y^G\right) = -\frac{(\xi+\phi)\,k^2}{\xi} - \frac{\left(\alpha^2+\beta\right)(\xi+\phi)\,\sigma_\nu^2}{\xi} - \frac{\beta\,(\xi+\phi)\,\sigma_\varepsilon^2}{\left(\alpha^2+\beta\right)\xi}. \tag{39}$$

On the other hand, in the absence of delegation (i.e., under discretion), the expected utility of principal is (setting $t_0 = t = 0$ in (11)):

$$E\left(U_D^G\right) = -\left(\alpha^2+\beta\right)\left(\frac{k^2}{\beta}+\sigma_\nu^2\right) - \frac{\beta}{\alpha^2+\beta}\sigma_\varepsilon^2. \tag{40}$$

Therefore, the difference between $E\left(U_D^G\right)$ and $E\left(U_\pi^G\right)$ (or between $E\left(U_D^G\right)$ and $E\left(U_y^G\right)$) is positive if the following condition applies (subtracting (39) from (40)):

$$\frac{\phi}{\xi} > \frac{\alpha^2 k^2}{\beta\left(k^2+\left(\alpha^2+\beta\right)\sigma_\nu^2+\beta\sigma_\varepsilon^2\right)}, \tag{41}$$

in which case, delegating monetary policy is counterproductive. ∎

We obtain this result because when central bank contracts represent a cost for the government, delegating monetary policy to an independent central bank has (by definition) a drawback for the government, i.e., the cost of the incentive scheme. And, this disadvantage may outweigh the benefit of delegation, i.e., the elimination of the inflation bias, in which case, the explanation why monetary policy delegation takes places must be looked for outside the model.

4 Conclusions

The literature on monetary policy has stressed the importance of institutional arrangements as a way out of the inflation bias generated by the classic time-inconsistency problem to discretionary monetary policy. Walsh (1995) showed that the government (principal) can offer the central bank an incentive scheme contingent upon realized inflation -a linear inflation contract- in such a way that the inflation bias is completely eliminated and output is stabilized optimally. He proved this result in a setting where such incentives schemes generate no cost for the government. Chortareas and Miller (2003), C-M, have extended the principal-agent setting to consider the possibility that incentive schemes represent a cost for the principal. In such a scenario a main conclusion of C-M is that the government should offer the central bank, instead of an inflation contract *à la* Walsh, an incentive scheme which penalizes the monetary authorities when output exceeds the natural level. They claim that the superiority of their proposed contract lies in that it can eliminate the inflation bias, as opposed to an inflation contract.

The aim of our paper has been to explore and compare, in the setup considered by C-M, the effects on social welfare of both an inflation and an output contract. We have shown that social welfare is the same with for both types of incentive schemes (and both of them eliminate the inflation bias), provided they are designed in accordance with optimizing behavior. This is in sharp contrast with the conclusions by C-M who claimed that social welfare is higher with an output contract and that, out of the two, only this type of incentive scheme removes the inflation bias. They obtain this incorrect results since they fail to take proper account of the participation constraint of the central bank when working out all the values of the strategic variables that shape the contracts. This inadequate way of proceeding leads C-M to the flawed intuition that the superiority of their proposed a contract, which penalizes deviations of output from the natural level, lies in that, using the very C-M's metaphor, it treats the disease (the expansionary bias for output), as opposed to the inflation contract which treats the symptom (inflation). We have proved that this explanation is incorrect since both types of contracts are equivalent. Moreover, carrying on with the C-M medical metaphor, any "generic pharmaceutical" (i.e., any generic contract) which linearly links, explicitly or implicitly, the central bank's incentives to inflation can succeed, if optimally designed, in eliminating the inflation bias achieving the same level of social welfare. Finally, we have pointed out that when incentive schemes represent a cost for the government delegating monetary policy through an output or inflation contract may be counterproductive. As the old (medical) proverb goes: "Sometimes the remedy is worse than the disease".

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