S-wave meson scattering up to 2 GeV and its spectroscopy

[Albaladejo, Oller, arXiv:0801.4929]

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MESON2008 (Krakow): June 6-10, 2008
Introduction

Objective and tools:

- Problem: Scalar mesons dynamics and spectroscopy. How many? Where?
- Very broad resonances, strongly coupled channels open up in the nearby of resonances, which have very different natures: dynamically generated, $q\bar{q}$, glueballs…
- Our objective: study of strongly interacting channels with quantum numbers $I = 0$, $I = 1/2$ $J^{PC} = 0^{++}$ for $\sqrt{s} \leq 2$ GeV.
- We’ll use Chiral Lagrangians, implementing Unitarity in a standard way (UCHPT).
Lagrangians

Our lagrangian is:

\[ L = L_2 + L_{VV} + L_{S_8} + L_{S_1} \]

- In \( SU(3) \) UChPT, we have **eight** Goldstone bosons: \( \pi, K, \eta \).
- Large \( N_c \) limit implies \( \eta' \) becomes the **nineth** Goldstone boson: \( SU(3) \to U(3) \).

[Herrera-Siklody et al., NP, B497, 345 (1997)], [Herrera-Siklody et al., PL, B419, 326 (1998)]

\[
L_2 = \frac{f^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle + \frac{f^2}{4} \langle \chi^\dagger U + \chi U^\dagger \rangle - \frac{1}{2} M_1^2 \eta_1^2 \]

\[ U(\phi) = \exp \left( i \sqrt{2} \Phi / f \right) \quad \Phi = \sum_{i=0}^{8} \frac{\lambda_i}{\sqrt{2}} \phi_i \quad \lambda_0 = \sqrt{\frac{2}{3}} \mathbf{I}_3 \]

\[ D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu = \partial_\mu U - ig [v_\mu, U] \]

- In our case, \( r_\mu = l_\mu = g v_\mu \): **vectorial resonances** nonet (Massive Yang-Mills fields)
- **Mixing**: \( \eta_1, \eta_8 \to \eta, \eta' \). Mixing angle is \( \theta \approx -20^\circ \).
- \( \chi = 2B_0 \mathcal{M} \), with \( \mathcal{M} \) quark mass matrix
Lagrangians (II)

- Vector field $v_\mu$ is given by:

$$
v_\mu = \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_1 \\
\rho^- \\
K^{*-}
\end{pmatrix} \begin{pmatrix}
\frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_1 \\
\rho^+ \\
K^{*0} \\
-\frac{2}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_1
\end{pmatrix}
$$

- Assuming ideal mixing between $\omega_8, \omega_1$:

$$
\frac{1}{\sqrt{2}} \omega = \frac{1}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_1 \quad \phi = -\frac{2}{\sqrt{6}} \omega_8 + \frac{1}{\sqrt{3}} \omega_1
$$

- Derivative piece of $L_2$ has interactions of: $\Phi \Phi, V \Phi \Phi$ and $VV \Phi \Phi$:

- $L^{\Phi \Phi}_2 = \frac{f^2}{4} \langle \partial_\mu U \partial_\mu U^\dagger \rangle$

- $L^{VV \Phi \Phi}_2 = g^2 \langle \Phi^2 v_\mu v_\mu - v_\mu \Phi v_\mu \Phi \rangle$

- $L^{V \Phi \Phi}_2 = -igf^2 \langle \partial_\mu U [v_\mu, U^\dagger] + [v_\mu, U] \partial_\mu U^\dagger \rangle$

- $g$ is determined through decay width $\rho \to \pi \pi$, from $L^{V \Phi \Phi}_2$, being $g = 4.23$

- $L_{VV}$ comes from the Yang–Mills kinetic term for the vector fields:

$$
L_{VV} = -\frac{1}{4} \langle F_{\mu \nu} F^{\mu \nu} \rangle \quad \quad F_{\mu \nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - ig [v_\mu, v_\nu]
$$

$$
\leq \frac{g^2}{2} \langle v_\mu v_\nu [v_\mu, v_\nu] \rangle
$$
Lagrangians (III)

- We introduce explicit resonances from RChPT [Ecker et al., NP, B321, 311 (1999)].
- Scalar resonances $J^{PC} = 0^{++}$:

$$
\mathcal{L}_{S_8} = c_d \langle S_8 u_\mu u^\mu \rangle + c_m \langle S_8 \chi_+ \rangle \\
\mathcal{L}_{S_1} = \tilde{c}_d S_1 \langle u_\mu u^\mu \rangle + \tilde{c}_m S_1 \langle \chi_+ \rangle \\
\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u,
$$

$$
U(x) = u(x)^2, \\
u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu
$$

- $S_1^{(i)}$ singlet and $S_8^{(i)}$ octet scalar resonance, with $M$, $c_d$ and $c_m$ fitted to data.

$$
S_8 = \begin{pmatrix}
\frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & a_0^+ & K_{0^+}^*
\\
a_0^- & -\frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & K_{0^0}^*
\\
K_{0^-}^* & \overline{K}_{0^0}^* & -\frac{2}{\sqrt{6}} f_8
\end{pmatrix}.
$$

- Channels to be considered:
  - $I = 0$: $\pi\pi$, $K\overline{K}$, $\eta\eta$, $\sigma\sigma$, $\eta\eta'$, $\rho\rho$, $\omega\omega$, $\eta\eta'$, $\omega\phi$, $\phi\phi$, $K^+\overline{K}^*$, $a_1 (1260) \pi$, $\pi^*\pi$
σσ channel amplitudes. Pion rescattering

We want to obtain σσ amplitudes starting from our lagrangian. σ is S-wave ππ interaction, |σ⟩ = |ππ⟩₀, [Oller, Oset, NP, B620, 438 (1997)]

- Pion rescattering, given by factor \( D^{-1}(s) = (1 + t_2 G(s))^{-1} \), with:
  - \( t_2 = \frac{s - m_π^2/2}{f_π^2} \) basic ππ → ππ amplitude.
  - \( (4\pi)^2 G(s) = \alpha + \log \frac{m_π^2}{\mu^2} - \sigma(s) \log \frac{\sigma(s) - 1}{\sigma(s) + 1} \), two pion loop.

- To isolate transition amplitude \( N_{i→σσ} \):
  \[
  \lim_{s_i→σσ} \frac{T_{i→(ππ)₀(ππ)₀}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i→σσ} g^2_{σππ}}{(s_1 - s_σ)(s_2 - s_σ)}
  \]
**σσ channel amplitudes. Pion rescattering**

We want to obtain σσ amplitudes starting from our lagrangian. σ is S-wave ππ interaction, |σ⟩ = |ππ⟩₀, [Oller, Oset, NP, B620, 438 (1997)]

- To isolate transition amplitude \( N_{i \rightarrow \sigma \sigma} \):
  \[
  \lim_{s_i \rightarrow s_\sigma} \frac{T_{i \rightarrow (\pi \pi)_0(\pi \pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i \rightarrow \sigma \sigma} g^2_{\sigma \pi \pi}}{(s_1 - s_\sigma)(s_2 - s_\sigma)}
  \]

- Now as (σ pole) \( D_{II}(s)^{-1} = (1 + t_2 G(s))^{-1} \approx \frac{\alpha_0}{s - s_\sigma} + \cdots \), then:
  \[
  N_{a \rightarrow (\sigma \sigma)_0} = T_{a \rightarrow (\pi \pi)_0(\pi \pi)_0} \left( \frac{\alpha_0}{g_{\sigma \pi \pi}} \right)^2
  \]
\[ \sigma \sigma \text{ channel amplitudes. Pion rescattering} \]

\[ \pi \pi \pi \pi \]

\[ T \]

\[ = \]

\[ N \pi \pi \]

\[ + \]

\[ N \pi \pi \]

\[ P \ G \]

\[ T \]

We want to obtain \( \sigma \sigma \) amplitudes starting from our lagrangian. \( \sigma \) is S-wave \( \pi \pi \) interaction, \( |\sigma\rangle = |\pi\pi\rangle_0 \), [Oller, Oset, NP, B620, 438 (1997)]

Now as (\( \sigma \) pole) \( D_{II}(s)^{-1} = (1 + t_2 G(s))^{-1} \approx \frac{\alpha_0}{s - s_{\sigma}} + \cdots \), then:

\[ N_{a \to (\sigma\sigma)} = T_{a \to (\pi\pi)_0 (\pi\pi)_0} \left( \frac{\alpha_0}{g_{\sigma\pi\pi}} \right)^2 \]

To calculate \( \left( \frac{\alpha_0}{g_{\sigma\pi\pi}} \right)^2 \), consider \( \pi\pi \) elastic scattering,

\[ V = \frac{t_2(s)}{1 + t_2(s) G(s)} \approx -\frac{g_{\sigma\pi\pi}^2}{s - s_{\sigma}} + \cdots \]

So we can write, using that (\( \sigma \) pole) \( g_{II}(s_{\sigma}) = -1/t_2(s_{\sigma}) \):

\[ \left( \frac{\alpha_0}{g_{\sigma\pi\pi}} \right)^2 = \frac{f^2}{1 - G_{II}'(s_{\sigma}) f^2 t_2(s_{\sigma})^2} \approx 1.1 f^2 \]
\( \sigma\sigma \) channel amplitudes. Pion rescattering

\[ \begin{align*}
\pi & \quad T & \quad \pi \\
\pi & \quad N & \quad \pi \\
\pi & \quad T & \quad \pi
\end{align*} \]

We want to obtain \( \sigma\sigma \) amplitudes starting from our lagrangian. \( \sigma \) is S-wave \( \pi\pi \) interaction, \( |\sigma\rangle = |\pi\pi\rangle_0 \), [Oller, Oset, NP, B620, 438 (1997)]

- To calculate \( (\alpha_0/g_{\sigma\pi\pi})^2 \), consider \( \pi\pi \) elastic scattering,

\[ V = \frac{t_2(s)}{1 + t_2(s)G(s)} \approx -\frac{g_{\sigma\pi\pi}^2}{s - s_{\sigma}} + \cdots \]

So we can write, using that (\( \sigma \) pole) \( g_{II}(s_{\sigma}) = -1/t_2(s_{\sigma}) \):

\[ \left( \frac{\alpha_0}{g_{\sigma\pi\pi}} \right)^2 = \frac{f^2}{1 - G'_{II}(s_{\sigma})f^2 t_2(s_{\sigma})^2} \approx 1.1f^2 \]

In conclusion, we follow a novel method to calculate amplitudes involving \( \sigma\sigma \), without introducing any new free parameter and from Chiral Lagrangians:

\[ N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0} \left( \frac{\alpha_0}{g_{\sigma\pi\pi}} \right)^2 \approx 1.1f^2 T_{a\to(\pi\pi)_0(\pi\pi)_0} \]
Unitarization of coupled channel amplitudes. Multiparticle states

\[ T = (I + N(s)g(s))^{-1} N(s) \]

- **\( N(s) \):** \( N_{ij} \) amplitudes matrix, obtained from Lagrangian, \( i,j = 1, \ldots 13 \)
- **\( g(s) \):** Unitarization loops (diagonal) matrix (including all intermediate states).

\[ g_i(s) = g_i(s_0) - \frac{s - s_0}{8\pi^2} \int_{s_{th,i}}^{\infty} ds' \frac{p_i(s')/\sqrt{s'}}{(s' - s_0)(s' - s + i\epsilon)} \]

We fold this \( g_i(s) \) for the case of \( \sigma\sigma \) and \( \rho\rho \), following a mass distribution, due to the large \( \sigma \) and \( \rho \) widths: \( \Gamma_\sigma \simeq 500 \text{ MeV}, \Gamma_\rho \simeq 150 \text{ MeV} \), and analogously for \( a_1(1260)\pi, \pi^*\pi \).
Results and data

<table>
<thead>
<tr>
<th>State</th>
<th>$M$ (MeV)</th>
<th>$c_d$ (MeV)</th>
<th>$c_m$ (MeV)</th>
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<tr>
<td>$S_{8}^{(1)}$</td>
<td>1290 ± 5</td>
<td>25.8 ± 0.5</td>
<td>25.8 ± 1.1</td>
</tr>
<tr>
<td>$S_{8}^{(2)}$</td>
<td>1905 ± 13</td>
<td>20.3 ± 1.4</td>
<td>−13.9 ± 2.0</td>
</tr>
<tr>
<td>$S_{1}^{(1)}$</td>
<td>894 ± 13</td>
<td>14.4 ± 0.3</td>
<td>46.6 ± 1.1</td>
</tr>
</tbody>
</table>

- First octet ($M_{8}^{(1)}$, $c_{d}^{(1)}$, $c_{m}^{(1)}$) is fixed to the work in [Jamin, Oller, Pich, NP, B622, 279 (2002)], [Jamin, Oller, Pich, NP, B587, 331 (2000)].

- Second octet has only fixed its mass, $M_{8}^{(2)}$, but not its couplings.

- Singlet mass $M_{1}^{(2)}$ is more difficult to fix. For lower values of the mass, we can obtain the same physics with higher couplings.

- Since $SU(3)$ is milder in the vector sector, we take just one subtraction constant for the whole set of vector channels,

\[ a_{\rho \rho} = a_{\omega \omega} = a_{K^{*}\bar{K}^{*}} = a_{\omega \phi} = a_{\phi \phi} \]

If we leave them free we find very small changes.

- Together, we have $a_{\pi \pi}$, $a_{K \bar{K}}$, $a_{\eta \eta}$, $a_{\eta \eta'}$, $a_{\eta' \eta'}$, $a_{\sigma \sigma}$ and $a_{\rho \rho}$.

Total: just 12 free parameters, for about 370 data points from many different and independent experiments.
Results and data

- **Phase shifts.** $K_{e4}$ decays: BNL-E865 [Pislak et al., PR, D67, 072004 (2003)], NA48/2 [Masetti, arXiv:hep-ex/0610071] and $\pi p \rightarrow \pi\pi n$: CERN-Cracow-Munich reanalysis [Kaminski et al., ZP, C74, 79 (1997)], and an average of may other data.

- $|S_{11}|^2$: CERN-Cracow-Munich [Hyams et al., NP, B64, 134 (1973)] and its reanalysis [Kaminski et al., ZP, C74, 79 (1997)].

- **$\pi\pi \rightarrow \pi\pi$:** Although reaching energies of 2 GeV, description of low energy data is still very good.

  Scattering length and effective range, in good agreement with recent determination of Leutwyler:

  \[
  \begin{array}{cc}
  \text{Our value} & \text{Leutwyler} \\
  a_0^0 & 0.216 \\
  b_0^0 & 0.277 \\
  \end{array}
  \]

  \[
  \begin{array}{cc}
  & 0.220 \pm 0.005 \\
  & 0.276 \pm 0.006 \\
  \end{array}
  \]

- **$\pi\pi \rightarrow K\bar{K}$:** Near threshold, Cohen data are favored.

- **$\pi\pi \rightarrow \eta\eta'$:** In good agreement for a low weight on $\chi^2$. In addition, the data are unnormalized.

- $I = 1/2$ $K^-\pi^+ \rightarrow K^-\pi^+$ amplitude and phase from LASS.
Results and data

- **BNL-E705**: reactions \( \pi^- p \rightarrow K^0_S K^0_S n \) [Etkin et al., PR, D25, 1786 (1982)]
- **Argonne National Laboratory**: \( \pi^- p \rightarrow K^- K^+ n \) and \( \pi^+ n \rightarrow K^- K^+ p \) [Cohen et al., PR, D22, 2595 (1980)]

\[ \pi\pi \rightarrow \pi\pi: \text{Although reaching energies of 2 GeV, description of low energy data is still very good.} \]

\[ \text{Scattering length and effective range, in good agreement with recent determination of Leutwyler:} \]

\[ a_0 = 0.216 \quad \text{Leutwyler} \quad a_0 = 0.220 \pm 0.005 \]
\[ b_0 = 0.277 \quad \text{Leutwyler} \quad b_0 = 0.276 \pm 0.006 \]

\[ \pi\pi \rightarrow K\bar{K}: \text{Near threshold, Cohen data are favored.} \]

\[ \pi\pi \rightarrow \eta, \eta': \text{In good agreement for a low weight on } \chi^2. \text{ In addition, the data are unnormalized.} \]

\[ I = 1/2 \quad K^- \pi^+ \rightarrow K^- \pi^+ \text{ amplitude and phase from LASS.} \]
Results and data

- \( \pi \pi \rightarrow \eta \eta \): LAPP Collaboration, in the reaction \( \pi^- p \rightarrow \eta \eta n \):
  - [Alde et al., NP, B269, 485 (1986)], [Binon et al., NC, A78, 313 (1983)]

- \( \pi \pi \rightarrow \eta \eta' \): LAPP Collaboration, in the reaction \( \pi^- p \rightarrow \eta \eta'n \)
  - [Binon et al., NC, A80, 363 (1984)]

- \( \pi \pi \rightarrow \pi \pi \): Although reaching energies of 2 GeV, description of low energy data is still very good.
  - Scattering length and effective range, in good agreement with recent determination of Leutwyler:
    - Our value: \( \alpha_0 = 0.216 \pm 0.005 \)
    - Leutwyler: \( \alpha_0 = 0.220 \pm 0.005 \)

- \( \pi \pi \rightarrow K\bar{K} \): Near threshold, Cohen data are favored.

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- \( I = 1/2 \) \( K^- \pi^+ \rightarrow K^- \pi^+ \) amplitude and phase from LASS.
Results and data

\[ \pi^+ K^- \] elastic scattering from LASS-SLAC in the reaction \( K^- p \rightarrow K^- \pi^+ n \)

[Aston et al., NP, B296, 493 (1988)]

- \( \pi \pi \rightarrow \pi \pi \): Although reaching energies of 2 GeV, description of low energy data is still very good.
  Scattering length and effective range, in good agreement with recent determination of Leutwyler:
  
  \[
  \begin{align*}
  a_0^0 &= 0.216 \quad \text{Our value} \\
  b_0^0 &= 0.277 \quad \text{Leutwyler}
  \end{align*}
  \]

- \( \pi \pi \rightarrow K \bar{K} \): Near threshold, Cohen data are favored.

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- \( I = 1/2 \) \( K^- \pi^+ \rightarrow K^- \pi^+ \) amplitude and phase from LASS.
Results and data

No parametrization

We determine interaction kernels from Chiral Lagrangians, avoiding *ad hoc* parametrizations

Less free parameters

We have less free parameters, because of our chiral approach and our treatment of $\sigma\sigma$ amplitude, and the use of YM fields to describe vector-vector amplitudes


Higher energies

We have included enough channels to get at 2 GeV.
Spectroscopy. Pole content: Summary

<table>
<thead>
<tr>
<th>PDG</th>
<th>(M) (MeV)</th>
<th>(\Gamma) (MeV)</th>
<th>(I = 0) Poles</th>
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</thead>
<tbody>
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<td>(\sigma \equiv f_0(600))</td>
<td></td>
<td></td>
<td>456 ± 6 (-i) 241 ± 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f_0(980))</td>
<td>980 ± 10</td>
<td>40 (-100)</td>
<td>983 ± 4 (-i) 25 ± 4</td>
<td>983 ± 4</td>
<td>50 ± 8</td>
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<td>(f_0(1370))</td>
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Some points that deserve (and will receive) a detailed explanation are:

- The whole structure of the \(f_0(1500)\), which is related to \(f_0^L\), \(f_0^R\) and \(\eta\eta'\) threshold.
- The mass of \(f_0(1370)\), different from the one deduced from the pole.
- The width of \(f_0(1710)\), much smaller than \(2\text{Im}\sqrt{s_{f_0(1710)}}\).
S-Wave Meson scattering and spectroscopy

M. Albaladejo

Introduction.

UChPT Lagrangians

Two sigma states

Unitarization of amplitudes. Multiparticle states

Results confront experiments

Spectroscopy

Summary

Extra slides

Spectroscopy. Pole content: Summary

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<td>$f_L^0$ $1466 \pm 15 - i 158 \pm 12$</td>
<td>$1370 \pm 30$</td>
<td>$316 \pm 24$</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>$1505 \pm 6$</td>
<td>$109 \pm 7$</td>
<td>$f_K^0$ $1602 \pm 15 - i 44 \pm 15$</td>
<td>$1502 \pm 15$</td>
<td>$105 \pm 30$</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>$1724 \pm 7$</td>
<td>$137 \pm 8$</td>
<td>$1690 \pm 20 - i 110 \pm 20$</td>
<td>$1700 \pm 20$</td>
<td>$160 \pm 40$</td>
</tr>
<tr>
<td>$f_0(1790)$</td>
<td>$1790_{-30}^{+40}$</td>
<td>$270_{-60}^{+30}$</td>
<td>$1810 \pm 15 - i 190 \pm 20$</td>
<td>$1810 \pm 30$</td>
<td>$380 \pm 40$</td>
</tr>
</tbody>
</table>

$I = 1/2$ Poles (MeV)

| $708 \pm 6 - i 313 \pm 10$ | $\kappa \equiv K_0^+(800)$ | $-$ | $-$ |
| $1435 \pm 6 - i 142 \pm 8$ | $K_0^+(1430)$ | $1414 \pm 6$ | $290 \pm 21$ |
| $1750 \pm 20 - i 150 \pm 20$ | $K_0^+(1950)$ | $-$ | $-$ |

Some points that deserve (and will receive) a detailed explanation are:

- The whole structure of the $f_0(1500)$, which is related to $f_0^K$, $f_0^L$ and $\eta \eta'$ threshold.
- The mass of $f_0(1370)$, different from the one deduced from the pole.
- The width of $f_0(1710)$, much smaller than $2 \text{Im} \sqrt{s_{f_0(1710)}}$. 

PDG: Particle Data Group
Spectroscopy. Pole content: Summary

<table>
<thead>
<tr>
<th>PDG</th>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$I = 0$ Poles</th>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma \equiv f_0(600)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_0(980)$</td>
<td>980 ± 10</td>
<td>40 – 100</td>
<td>456 ± 6 – $i$ 241 ± 7</td>
<td>983 ± 4 – $i$ 25 ± 4</td>
<td>983 ± 4</td>
</tr>
<tr>
<td>$f_0(1370)$</td>
<td>1200 – 1500</td>
<td>200 – 500</td>
<td>$f_0^L$ 1466 ± 15 – $i$ 158 ± 12</td>
<td>1370 ± 30</td>
<td>316 ± 24</td>
</tr>
<tr>
<td>$f_0(1500)$</td>
<td>1505 ± 6</td>
<td>109 ± 7</td>
<td>$f_0^R$ 1602 ± 15 – $i$ 44 ± 15</td>
<td>1502 ± 15</td>
<td>105 ± 30</td>
</tr>
<tr>
<td>$f_0(1710)$</td>
<td>1724 ± 7</td>
<td>137 ± 8</td>
<td>1690 ± 20 – $i$ 110 ± 20</td>
<td>1700 ± 20</td>
<td>160 ± 40</td>
</tr>
<tr>
<td>$f_0(1790)$</td>
<td>$1790^{+40}_{-30}$</td>
<td>$270^{+30}_{-60}$</td>
<td></td>
<td>$1810 ± 15 – i 190 ± 20$</td>
<td>$1810 ± 30$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$I = 1/2$ Poles (MeV)</th>
<th>Effect</th>
<th>$M$ (PDG)</th>
<th>$\Gamma$ (PDG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>708 ± 6 – $i$ 313 ± 10</td>
<td>$\kappa \equiv K_0^*(800)$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1435 ± 6 – $i$ 142 ± 8</td>
<td>$K_0^*(1430)$</td>
<td>1414 ± 6</td>
<td>290 ± 21</td>
</tr>
<tr>
<td>1750 ± 20 – $i$ 150 ± 20</td>
<td>$K_0^*(1950)$</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Some points that deserve (and will receive) a detailed explanation are:

- The whole structure of the $f_0(1500)$, which is related to $f_0^L$, $f_0^R$ and $\eta\eta'$ threshold.
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Spectroscopy. Pole content: Summary

<table>
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<tr>
<th>PDG</th>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
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<th>$M$ (MeV)</th>
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<tr>
<td>$\sigma \equiv f_0(600)$</td>
<td></td>
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<td>270$^{+30}_{−60}$</td>
<td>1810 ± 15 − i 190 ± 20</td>
<td>1810 ± 30</td>
<td>380 ± 40</td>
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$I = 1/2$ Poles (MeV)

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<th>$\Gamma$ (PDG)</th>
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</thead>
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<td>$\kappa \equiv K_0^+(800)$</td>
<td>708 ± 6 − i 313 ± 10</td>
<td>−</td>
</tr>
<tr>
<td>$K_0^+(1430)$</td>
<td>1435 ± 6 − i 142 ± 8</td>
<td>1414 ± 6</td>
</tr>
<tr>
<td>$K_0^+(1950)$</td>
<td>1750 ± 20 − i 150 ± 20</td>
<td>−</td>
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</tbody>
</table>

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More data to support our spectroscopy

Here we see some data from WA102 and Crystal Barrel:

Similarly as done by WA102 Collaboration, we put a coherent sum of Breit–Wigners:

\[
A_i(\sqrt{s}) = NR_i(\sqrt{s}) + \sum_{j \in \text{Res}} \frac{a_j e^{i\theta_j} g_{j;i}}{M_j^2 - s - iM_j \Gamma_j}
\]

\[
\text{Res} = \begin{cases} 
\sigma, f_0(980), f_0(1370), f_0^R 
& \sqrt{s} < m_\eta + m_\eta' \\
\sigma, f_0(980), f_0(1710), f_0(1790) 
& \sqrt{s} > m_\eta + m_\eta'
\end{cases}
\]

- Note that \(M_j, \Gamma_j\) and \(g_{j;i}\) are fixed from the previous study on scattering (the poles on previous slide), without changing them.
- A constant is included to ensure continuity.
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Spectroscopy. Pole content: $f_0(1370)$

$$f_0(1370)$$

$1466 \pm 15 - i \ 158 \pm 12 \text{ MeV}$

<table>
<thead>
<tr>
<th></th>
<th>PDG</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>1200 – 1500</td>
<td>1370 ± 15</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>200 – 500</td>
<td>316 ± 24</td>
</tr>
</tbody>
</table>

- Shift in mass peak:
  $M_{f_0(1370)} \sim 1370 \text{ MeV}$.

- Strong coupling to $\sigma \sigma$, $\pi \pi$

  $\Gamma(\!\!f_0(1370) \rightarrow 4\pi) \quad \Gamma(\!\!f_0(1370) \rightarrow \pi \pi) = 0.30 \pm 0.12$, in good agreement with the interval $0.10 - 0.25$ of [Bugg, EPJ, C52, 55 (2007)].
Spectroscopy. Pole content: $f_0(1370)$

\[ f_0(1370) \]

1466 ± 15 \(- i\) 158 ± 12 MeV

\[
\begin{array}{cc}
M (\text{MeV}) & \Gamma (\text{MeV}) \\
\text{PDG} & 1200 - 1500 & 200 - 500 \\
\text{Our} & 1370 ± 15 & 316 ± 24 \\
\end{array}
\]

- Shift in **mass** peak:
  \[ M_{f_0(1370)} \approx 1370 \text{ MeV.} \]

- **Strong coupling to** $\sigma\sigma$, $\pi\pi$

\[
\frac{\Gamma(f_0(1370)\to 4\pi)}{\Gamma(f_0(1370)\to \pi\pi)} = 0.30 \pm 0.12, \text{ in good agreement with the interval } 0.10 - 0.25 \text{ of } [\text{Bugg,EPJ, C52, 55 (2007)}]\]
Spectroscopy. Couplings of $f_0(1370)$

<table>
<thead>
<tr>
<th>Coupling</th>
<th>bare</th>
<th>final</th>
<th>Coupling</th>
<th>bare</th>
<th>final</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{\pi^+\pi^-}$</td>
<td>3.9</td>
<td>$3.59 \pm 0.18$</td>
<td>$g_{K\pi}$</td>
<td>5.0</td>
<td>4.8</td>
</tr>
<tr>
<td>$g_{K^0\bar{K}^0}$</td>
<td>2.3</td>
<td>$2.23 \pm 0.18$</td>
<td>$g_{K\eta}$</td>
<td>0.7</td>
<td>0.9</td>
</tr>
<tr>
<td>$g_{\eta\eta}$</td>
<td>1.4</td>
<td>$1.70 \pm 0.30$</td>
<td>$g_{K\eta'}$</td>
<td>3.4</td>
<td>3.8</td>
</tr>
<tr>
<td>$g_{\eta'\eta'}$</td>
<td>3.7</td>
<td>$4.00 \pm 0.30$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_{\eta\eta'}$</td>
<td>3.8</td>
<td>$3.70 \pm 0.40$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Bare coupling are very similar to the physical ones.
- Bare: those of $S_8^{(1)}$, with $M_8^{(1)} = 1.29$ GeV, $c_d^{(1)} = c_m^{(1)} = 25.8$ MeV.
- The first scalar octet is a pure one, not mixed with the nearby $f_0(1500)$ nor $f_0(1710)$.
- For instance we have the bare coupling for $\pi^+\pi^-$:

$$g_{\pi^+\pi^-} = \sqrt{\frac{2}{3} \frac{c_d M_8^2}{f^2} + 2(c_m - c_d) m^2_{\pi}}$$
Spectroscopy. Pole content: $f_0(1500)$

- $f_0^R(1500)$: Pole at $(1.6 - i0.04)$ GeV
- $f_0(1370)$: Pole at $(1.47 - i0.16)$ GeV
- Nearby thresholds: $\eta\eta'$, $\omega\omega$

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDG</td>
<td>1505 ± 6</td>
</tr>
<tr>
<td>Our</td>
<td>1502</td>
</tr>
</tbody>
</table>

- The mass peak is at 1.5 GeV, due to $\eta\eta'$ threshold. Effect similar to $a_0(980)$ with the $K\bar{K}$ threshold [Oller, Oset, PR, D60, 074023 (1999)].
- The width is $\Gamma \approx 1.2 \times 88 \approx 105$ MeV, because a Breit-Wigner at $(1.6 - i0.04)$ GeV is cut by $\eta\eta'$ threshold.
- Complicated energy region. Three interfering effects give raise to $f_0(1500)$:
Spectroscopy. Pole content: $f_0(1500)$

$f_0(1500)$

$f_0^R$ 1602 $\pm$ 15 $- i$ 44 $\pm$ 15 MeV

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  [Oller, Oset, PR, D60, 074023 (1999)].
- **The width** is $\Gamma \approx 1.2 \times 88 \approx 105$ MeV, because a Breit-Wigner at $(1.6 - i0.04)$ GeV is cut by $\eta\eta'$ threshold.
- Complicated energy region. Three **interfering effects** give raise to $f_0(1500)$:
  - $f_0^R$: Pole at $(1.6 - i0.04)$ GeV
  - $f_0(1370)$: Pole at $(1.47 - i0.16)$ GeV
  - Nearby **thresholds**: $\eta\eta'$, $\omega\omega$

<table>
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<tr>
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<th>PDG</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ (MeV)</td>
<td>1505 $\pm$ 6</td>
<td>1502</td>
</tr>
<tr>
<td>$\Gamma$ (MeV)</td>
<td>109 $\pm$ 7</td>
<td>105 $\pm$ 36</td>
</tr>
</tbody>
</table>
The shape of $f_0(1500)$

Recall:
- The data of WA102 and Crystall Barrell Collaboration which show the peak of $f_0(1500)$, and
- that we fit these data with the poles we find from our scattering study
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**Spectroscopy. Pole content:** $f_0(1710)$

$f_0(1710)$

$1690 \pm 20 - i \ 110 \pm 20$ MeV

- The **mass** peak is slightly shifted to higher energy: $\sqrt{s} = M \approx 1700$ MeV.
- The **width** “measured” on the real axis is $\Gamma_{\text{eff}} \approx 160$ MeV.
- The effective width can depend on the concrete process.

<table>
<thead>
<tr>
<th>BR</th>
<th>Our value</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(K\bar{K})$</td>
<td>$0.36 \pm 0.12$</td>
<td>$0.38^{+0.09}_{-0.19}$</td>
</tr>
<tr>
<td>$\Gamma(\eta\eta)$</td>
<td>$0.22 \pm 0.12$</td>
<td>$0.18^{+0.03}_{-0.13}$</td>
</tr>
<tr>
<td>$\Gamma(\eta\eta)$</td>
<td>$0.32 \pm 0.14$</td>
<td>$0.41^{+0.11}_{-0.17}$</td>
</tr>
<tr>
<td>$\Gamma(\pi\pi)$</td>
<td>$0.64 \pm 0.38$</td>
<td>$0.48 \pm 0.15$</td>
</tr>
</tbody>
</table>

$\Gamma_{\text{eff}} \approx 160$ MeV
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Spectroscopy. Pole content: \( f_0(1710) \)

\[ f_0(1710) \]

\[ 1690 \pm 20 - i \ 110 \pm 20 \text{ MeV} \]

- The mass peak is slightly shifted to higher energy: \( \sqrt{s} = M \approx 1700 \text{ MeV} \).
- The width "measured" on the real axis is \( \Gamma_{\text{eff}} \approx 160 \text{ MeV} \).
- The effective width can depend on the concrete process.

<table>
<thead>
<tr>
<th>( M ) (MeV)</th>
<th>( \Gamma ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDG</td>
<td>1724 ± 7</td>
</tr>
<tr>
<td>Our</td>
<td>1700 ± 20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BR ( \Gamma(K\bar{K}) )</th>
<th>Our value</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Gamma_{\text{total}}(\eta\eta) )</td>
<td>0.36 ± 0.12</td>
<td>0.38( ^{+0.09}_{-0.19} )</td>
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</tbody>
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\[ \Re \sqrt{s} \] (MeV)

\[ \text{Im} \sqrt{s} \] (MeV)

\[ |T_{\eta'\eta' \rightarrow \eta'\eta'}|^2 \]

\( \Gamma_{\text{eff}} \approx 160 \text{ MeV} \)
Spectroscopy. Pole content: \( f_0(1790) \)

\[ 1810 \pm 15 - i \ 190 \pm 20 \text{ MeV} \]

- Weak signal on the real axis
- It couples weakly to \( K\bar{K} \), a major difference with respect to \( f_0(1710) \), as also observed by BESII.
- It is the partner of the pole at \( 1.75 - i 0.15 \) GeV in \( I = 1/2 \).
- These poles originate from the higher bare octet, \( S_8^{(2)} \), with \( M_8^{(2)} = 1905 \), \( c_d^{(2)} = 20.3 \), \( c_m^{(2)} = -13.9 \) MeV.

<table>
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<tr>
<th></th>
<th>( M ) (MeV)</th>
<th>( \Gamma ) (MeV)</th>
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</thead>
<tbody>
<tr>
<td>BESII</td>
<td>1790(^{+40}_{-30})</td>
<td>270(^{+30}_{-60})</td>
</tr>
<tr>
<td>Our</td>
<td>1810 \pm 15</td>
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Spectroscopy. Pole content: $f_0(1790)$

$1810 \pm 15 - i \ 190 \pm 20 \text{ MeV}$

- Weak signal on the real axis
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<td>Our</td>
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**Spectroscopy. Couplings of $f_0^R$ and $f_0(1710)$**

<table>
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<th>Coupling (GeV)</th>
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<tr>
<td>$g_{\pi^+\pi^-}$</td>
<td>1.31 ± 0.22</td>
<td>1.24 ± 0.16</td>
</tr>
<tr>
<td>$g_{K_0 K^0}$</td>
<td>2.06 ± 0.17</td>
<td>2.00 ± 0.30</td>
</tr>
<tr>
<td>$g_{\eta}$</td>
<td>3.78 ± 0.26</td>
<td>3.30 ± 0.80</td>
</tr>
<tr>
<td>$g_{\eta'}$</td>
<td>4.99 ± 0.24</td>
<td>5.10 ± 0.80</td>
</tr>
<tr>
<td>$g_{\eta''}$</td>
<td>8.30 ± 0.60</td>
<td>11.7 ± 1.60</td>
</tr>
</tbody>
</table>

This pattern suggests an enhancement in $s\bar{s}$ production.

With a pseudoscalar mixing angle $\sin \beta = -1/3$ for $\eta$ and $\eta'$:

\[
\begin{align*}
\eta & = -\frac{1}{\sqrt{3}} \eta_s + \sqrt{\frac{2}{3}} \eta_u \\
\eta' & = -\sqrt{\frac{2}{3}} \eta_s + \frac{1}{\sqrt{3}} \eta_u \\
\eta_s & = s\bar{s} \\
\eta_u & = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}
\end{align*}
\]

With this equations, we can obtain $g_{ss}$, $g_{ns}$ and $g_{nn}$ from $g_{\eta\eta}$, $g_{\eta\eta'}$ and $g_{\eta''}$.
Spectroscopy. Couplings of $f_0^R$ and $f_0(1710)$

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This pattern suggests an enhancement in $s\bar{s}$ production.

With a pseudoscalar mixing angle $\sin \beta = -1/3$ for $\eta$ and $\eta'$:

\[
\begin{align*}
  g_{\eta'\eta'} & = \frac{2}{3} g_{ss} + \frac{1}{3} g_{nn} + \frac{2\sqrt{2}}{3} g_{ns} \\
  g_{\eta\eta'} & = -\frac{\sqrt{2}}{3} g_{ss} + \frac{\sqrt{2}}{3} g_{nn} + \frac{1}{3} g_{ns} \\
  g_{\eta\eta} & = \frac{1}{3} g_{ss} + \frac{2}{3} g_{nn} - \frac{2\sqrt{2}}{3} g_{ns}
\end{align*}
\]

\[
\begin{align*}
  \eta & = -\frac{1}{\sqrt{3}} \eta_s + \sqrt{\frac{2}{3}} \eta_u \\
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  \eta_s & = s\bar{s} \\
  \eta_u & = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}
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\]

With this equations, we can obtain $g_{ss}$, $g_{ns}$ and $g_{nn}$ from $g_{\eta\eta}$, $g_{\eta\eta'}$ and $g_{\eta'\eta'}$. 
S-wave meson scattering and spectroscopy

M. Albaladejo

Introduction. UChPT Lagrangians

Two sigma states

Unitarization of amplitudes. Multiparticle states

Results confront experiments

Spectroscopy

Summary

Extra slides

Spectroscopy. Couplings of $f_0^R$ and $f_0(1710)$

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\eta_u &= \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}
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\]

The glueball $\rightarrow q\bar{q} \propto m_q$ chiral supression [Chanowitz, PRL95, 172001 (2005)] implies $|g_{ss}| \gg |g_{nn}|$. Together with the OZI rule requires $|g_{ss}| \gg |g_{ns}|$.

This is precisely what we obtain from the previous couplings and equations.
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Now let us consider $K\bar{K}$ coupling:

- **Valence quarks:** $K^0$ corresponds to $\sum_{i=1}^{3} \bar{s}_i u^i / \sqrt{3}$, and analogously $\bar{K}^0$
- **Production of colour singlet $s\bar{s}$** requires the combination of the colour indices of $K^0, \bar{K}^0$
- **Decompose** $\bar{s}_i s^j = \delta_i^j s\bar{s}/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^i$
- **Only $s\bar{s}u\bar{u}$** contributes (factor 1/3) and $s\bar{s}s\bar{s}$ has an extra factor two compared to $s\bar{s}u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$
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Arguments favouring the first interpretation for the $f_0(1710)$:

- $f_0(1370)$ does not mix, is pure $I = 0$ octet state. No one can mix the with $f_0(1710)$. Note that the Chiral Suppression Mechanism for a glueball also implies that it should not mix.

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- Unquenched lattice calculations [Setxton, Vaccaro, Weingarten, PRL75 4563 (1995)] give the mass and the couplings of the lightest $0^{++}$ glueball:
  - Mass [Chen et al., PR, D73, 014516 (2006)]:
    \[ M_{0^{++}}^{\text{gb}} = 1.66 \pm 0.05 \text{ GeV} \quad M_{f_0(1710)} = 1.69 \pm 0.02 \text{ GeV} \]
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Coupling of glueball to a pair of pseudoscalars

We see here the coupling for a decay of the glueball to a pair of pseudoscalars:

- as calculated in unquenched lattice QCD
  [Setxton, Vaccarino, Weingarten, PRL\textbf{75} 4563 (1995)]

- a coupling linear in pseudoscalar mass squared as predicted by chiral suppression mechanism [Chanowitz, PRL\textbf{95}, 172001 (2005)].
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Note that our work is completely INDEPENDENT of Chanowitz's. When comparing, this coupling and mixing scheme naturally fits in our results.
Summary

- We have performed the first coupled channel study of the meson-meson S-waves for $I = 0$ and $I = 1/2$ up to 2 GeV with 13 coupled channels.
- Interaction kernels are determined from Chiral Lagrangians and implemented in N/D–type equations.
- We have fewer free parameters, compared with previous works in the literature, for a vast quantity of data.
- All scalar resonances below 2 GeV are generated:
  - $I = 0$: $\sigma, f_0(980), f_0(1370), f_0(1500), f_0(1710)$ and $f_0(1790)$
  - $I = 1/2$: $\kappa, K_0^*(1430)$ and $K_0^*(1950)$
- The structure of the couplings to pairs of pseudoscalars implies that:
  - $f_0(1370), K_0^*(1430)$ (and $a_0(1450)$) remain as pure octet.
  - $f_0^R$ and $f_0(1710)$, which are the same pole reflected on different sheets, have a decay pattern showing an enhanced $\bar{s}s$ production, in agreement with the chiral suppression mechanism of glueball decay, so
  - It should be considered as the lightest unmixed $0^{++}$ glueball.
References


With $s_\sigma$ complex, $N_{i\to\sigma\sigma}$ would be complex, thus violating unitarity. So we take $s_i$ real, but varying according to a mass distribution.

Lehman propagator representation (dispersion relation):

$$P(s) = -\frac{1}{\pi} \int_{\sqrt{s_{th}}}^{\infty} ds' \frac{\text{Im}P(s')}{s - s' + i\epsilon},$$

In a first approach:

$$\text{Im}P(s') \propto \text{Im}\left(\frac{1}{s' - m_{\sigma}^2 + i m_{\sigma}\Gamma_{\sigma}(s')}\right)$$

$$\Gamma_{\sigma}(s') = \Gamma_{\sigma}\sqrt{\frac{1 - 4m_{\pi}^2/s'}{1 - 4m_{\pi}^2/m_{\sigma}^2}}$$

$$\int_{\sqrt{s_{th}}}^{\infty} ds' \text{Im}P(s') = 1$$

$g(s)$ (unitarity loop) for $\sigma$ channel can be written as:

$$\int_{\sqrt{s_{th}}}^{\infty} ds_1 \int_{\sqrt{s_{th}}}^{\infty} ds_2 \text{Im}P(s_1)\text{Im}P(s_2) g_4(s; s_1, s_2),$$
Spectroscopy, poles and Riemann sheets

- When parameters are fitted to data, we extrapolate our amplitudes to the $s$-complex plane, finding poles, and we identify them with resonances through $\sqrt{s_0} \approx M - i\Gamma/2$.

- Analytical extrapolations are needed in the $T_{ij}$ to the different Riemann sheets, which appear because of the cuts in $G(s)$ at opening thresholds:

$$p(s) = \frac{\sqrt{s - (m_a + m_b)^2}}{2\sqrt{s}} \frac{\sqrt{s - (m_a - m_b)^2}}{2\sqrt{s}}$$

- Using continuity, for $\sqrt{s}$ real, $> m_a + m_b$, we have:

$$G^{II}(s + i\epsilon) = G(s - i\epsilon) = G(s + i\epsilon) - 2i\text{Im}G(s + i\epsilon) = G(s + i\epsilon) + \frac{i}{4\pi} \frac{p(s)}{\sqrt{s}}$$
Some branching ratios...