S-wave meson scattering up to 2 GeV and its spectroscopy

[Albaladejo, Oller, arXiv:0801.4929]

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Introduction

Objective and tools:

- Problem: Scalar mesons dynamics and spectroscopy. How many? Where?
- Very broad resonances, strongly coupled channels open up in the nearby of resonances, which have very different natures: dynamically generated, $q\bar{q}$, glueballs...
- Our objective: study of strongly interacting channels with quantum numbers $I=0$, $I=1/2$ $J^{PC}=0^{++}$ for $\sqrt{s}\leq 2$ GeV.
- We'll use Chiral Lagrangians, implementing Unitarity in a standard way (UCHPT).

Lagrangians

Our lagrangian is:

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{VV} + \mathcal{L}_{S_8} + \mathcal{L}_{S_1}$$

- In $SU(3)$ UCHPT, we have eight Goldstone bosons: π, K, η.
- Large N_c limit implies η' becomes the ninth Goldstone boson: $SU(3) \to U(3)$.
  [Herrera-Siklody et al., NP, B**497**, 345 (1997)], [Herrera-Siklody et al., PL, B**419**, 326 (1998)]

$$\mathcal{L}_2 = \frac{f^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle + \frac{f^2}{4}\langle \chi^\dagger U + \chi U^\dagger\rangle - \underbrace{\frac{1}{2}M_1^2\eta_1^2}_{U_A(1)}$$

$$U(\phi) = \exp\left(i\sqrt{2}\Phi/f\right) \quad \Phi = \sum_{i=0}^{8}\frac{\lambda_i}{\sqrt{2}}\phi_i \quad \lambda_0 = \sqrt{\frac{2}{3}}\mathbf{I}_3$$

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu = \partial_\mu U - i g\left[v_\mu, U\right]$$

- In our case, $r_\mu = l_\mu = g v_\mu$: vectorial resonances nonet (Massive Yang-Mills fields)
- Mixing: $\eta_1, \eta_8 \to \eta, \eta'$. Mixing angle is $\theta \approx -20^\circ$.
- $\chi = 2B_0\mathcal{M}$, with $\mathcal{M}$ quark mass matrix

Lagrangians (II)

- Vector field v_μ is given by:

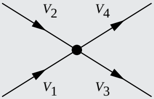
$$v_\mu = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 & K^{*0} \\ K^{*-} & \overline{K}^{*0} & -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \end{pmatrix}$$

- Assuming ideal mixing between ω_8, ω_1:

$$\frac{1}{\sqrt{2}}\omega = \frac{1}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1 \qquad \phi = -\frac{2}{\sqrt{6}}\omega_8 + \frac{1}{\sqrt{3}}\omega_1$$

- Derivative piece of $\mathcal{L}_2$ has interactions of: $\Phi\Phi$, $V\Phi\Phi$ y $VV\Phi\Phi$:
  - $\mathcal{L}_2^{\Phi\Phi} = \frac{f^2}{4}\langle \partial_\mu U \partial_\mu U^\dagger \rangle$
  - $\mathcal{L}_2^{VV\Phi\Phi} = g^2 \left\langle \Phi^2 v^\mu v_\mu - v_\mu \Phi v^\mu \Phi \right\rangle$
  - $\mathcal{L}_2^{V\Phi\Phi} = -\frac{igf^2}{4}\langle \partial_\mu U [v^\mu, U^\dagger] + [v^\mu, U]\partial_\mu U^\dagger \rangle$
  - g is determined through decay width $\rho \to \pi\pi$, from $\mathcal{L}_2^{V\Phi\Phi}$, being $g = 4.23$
- $\mathcal{L}_{VV}$ comes from the Yang–Mills kinetic term for the vector fields:

$$\mathcal{L}_{VV} = -\frac{1}{4}\langle F_{\mu\nu} F^{\mu\nu} \rangle \qquad F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - ig\left[v_\mu, v_\nu\right]$$



$$\Leftarrow \quad \frac{g^2}{2}\langle v_\mu v_\nu \left[v^\mu, v^\nu\right] \rangle$$

Lagrangians (III)

- We introduce explicit resonances from RChPT [Ecker et al., NP, B**321**, 311 (1999)].
- Scalar resonances $J^{PC} = 0^{++}$:

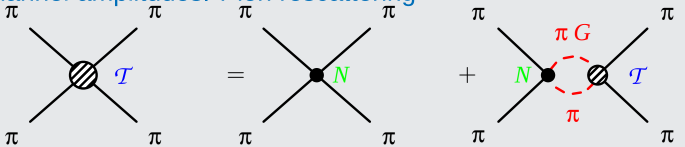
$$\begin{aligned} \mathcal{L}_{S_8} &= c_d \left\langle S_8 u_\mu u^\mu \right\rangle + c_m \left\langle S_8 \chi_+ \right\rangle \\ \mathcal{L}_{S_1} &= \tilde{c}_d S_1 \left\langle u_\mu u^\mu \right\rangle + \tilde{c}_m S_1 \left\langle \chi_+ \right\rangle \\ \chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u, \\ U(x) = u(x)^2 & \qquad u_\mu = i u^\dagger D_\mu U u^\dagger = u_\mu^\dagger \end{aligned}$$

- $S_1^{(i)}$ singlet and $S_8^{(i)}$ octet scalar resonance, with M, c_d and c_m fitted to data.

$$S_8 = \begin{pmatrix} \frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & a_0^+ & K_0^{*+} \\ a_0^- & -\frac{a_0}{\sqrt{2}} + \frac{f_8}{\sqrt{6}} & K_0^{*0} \\ K_0^{*-} & \overline{K}_0^{*0} & -\frac{2}{\sqrt{6}} f_8 \end{pmatrix}.$$

- Channels to be considered:
  - $I = 0$: $\pi\pi$, $K\overline{K}$, $\eta\eta$, $\sigma\sigma$, $\eta\eta'$, $\rho\rho$, $\omega\omega$, $\eta\eta'$, $\omega\phi$, $\phi\phi$, $K^*\overline{K}^*$, $a_1(1260)\pi$, $\pi^\star\pi$
  - $I = 1/2$, $I = 3/2$: $K\pi$, $K\eta$ and $K\eta'$ [Jamin, Oller, Pich, NP, B**622**, 279 (2002)], [Jamin, Oller, Pich, NP, B**587**, 331 (2000)]

$\sigma\sigma$ channel amplitudes. Pion rescattering



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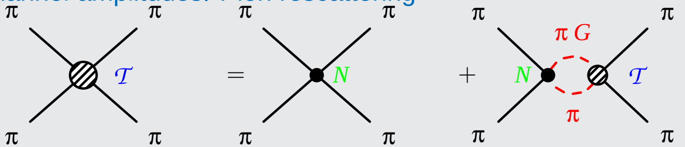
- Pion rescattering, given by factor $D^{-1}(s) = (1+t_2 G(s))^{-1}$, with:
  - $t_2 = \frac{s - m_\pi^2/2}{f_\pi^2}$ basic $\pi\pi \to \pi\pi$ amplitude.
  - $(4\pi)^2 G(s) = \alpha + \log\frac{m_\pi^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}$, two pion loop.
- To isolate transition amplitude $N_{i\to\sigma\sigma}$:

$$\lim_{s_i \to s_\sigma} \frac{T_{i\to(\pi\pi)_0(\pi\pi)_0}}{D_{II}(s_1)D_{II}(s_2)} = \frac{N_{i\to\sigma\sigma}\, g^2_{\sigma\pi\pi}}{(s_1 - s_\sigma)(s_2 - s_\sigma)}$$

σσ channel amplitudes. Pion rescattering



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- Now as (σ pole) $D_{II}(s)^{-1} = (1 + t_2 G(s))^{-1} \approx \frac{\alpha_0}{s - s_\sigma} + \cdots$, then:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0} \left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2$$

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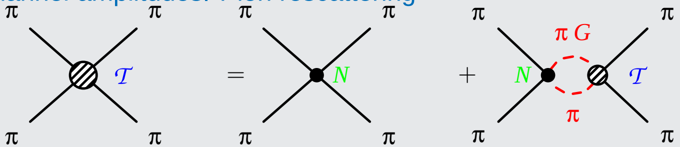

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- To calculate $(\alpha_0/g_{\sigma\pi\pi})^2$, consider $\pi\pi$ elastic scattering,

$$V = \frac{t_2(s)}{1+t_2(s)G(s)} \approx -\frac{g^2_{\sigma\pi\pi}}{s-s_\sigma} + \cdots$$

So we can write, using that (σ pole) $g_{II}(s_\sigma) = -1/t_2(s_\sigma)$:

$$\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 = \frac{f^2}{1-G'_{II}(s_\sigma)f^2 t_2(s_\sigma)^2} \approx 1.1 f^2$$

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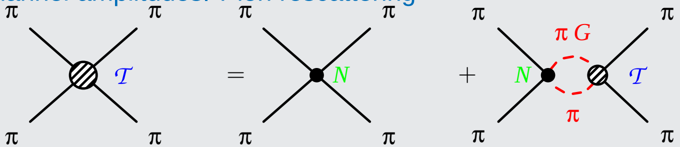

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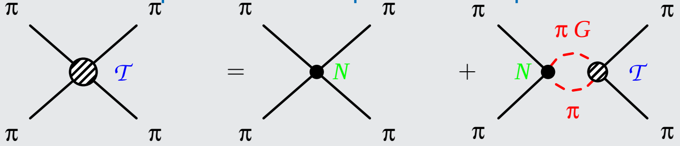
In conclusion, we follow a novel method to calculate amplitudes involving $\sigma\sigma$, without introducing any new free parameter and from Chiral Lagrangians:

$$N_{a\to(\sigma\sigma)_0} = T_{a\to(\pi\pi)_0(\pi\pi)_0}\left(\frac{\alpha_0}{g_{\sigma\pi\pi}}\right)^2 \simeq 1.1 f^2\ T_{a\to(\pi\pi)_0(\pi\pi)_0}$$

Unitarization of coupled channel amplitudes. Multiparticle states



Coupled channel partial wave

$$T = (I + N(s)g(s))^{-1} N(s)$$

- $N(s)$: N_{ij} amplitudes matrix, obtained from Lagrangian, $i,j = 1,\dots 13$
- $g(s)$: Unitarization loops (diagonal) matrix (including all intermediate states).

$$g_i(s) = g_i(s_0) - \frac{s - s_0}{8\pi^2} \int_{s_{\text{th},i}}^{\infty} ds' \frac{p_i(s')/\sqrt{s'}}{(s' - s_0)(s' - s + i\varepsilon)}$$

We fold this $g_i(s)$ for the case of $\sigma\sigma$ and $\rho\rho$, following a mass distribution, due to the large σ and ρ widths: $\Gamma_\sigma \simeq 500$ MeV, $\Gamma_\rho \simeq 150$ MeV, and analogously for $a_1(1260)\pi$, $\pi^\star\pi$.

Results and data

	M (MeV)	c_d (MeV)	c_m (MeV)
$S_8^{(1)}$	1290 ± 5	25.8 ± 0.5	25.8 ± 1.1
$S_8^{(2)}$	1905 ± 13	20.3 ± 1.4	-13.9 ± 2.0
$S_1^{(1)}$	894 ± 13	14.4 ± 0.3	46.6 ± 1.1

- First octet ($M_8^{(1)}$, $c_d^{(1)}$, $c_m^{(1)}$) is fixed to the work in [Jamin, Oller, Pich, NP, B**622**, 279 (2002)], [Jamin, Oller, Pich, NP, B**587**, 331 (2000)]
- Second octet has only fixed its mass, $M_8^{(2)}$, but not its couplings.
- Singlet mass $M_1^{(2)}$ is more difficult to fix. For lower values of the mass, we can obtain the same physics with higher couplings.
- Since $SU(3)$ is milder in the vector sector, we take just one substraction constant for the whole set of vector channels,

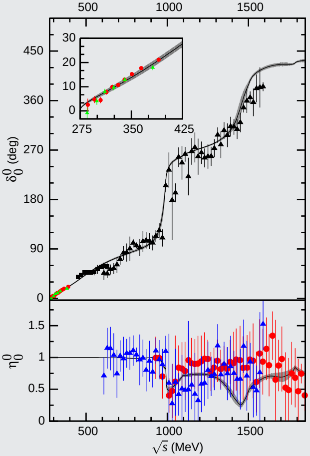
$$a_{\rho\rho} = a_{\omega\omega} = a_{K^\star \bar{K}^\star} = a_{\omega\phi} = a_{\phi\phi}$$

  If we leave them free we find very small changes.
- Together, we have $a_{\pi\pi}$, $a_{K\bar{K}}$, $a_{\eta\eta}$, $a_{\eta\eta'}$, $a_{\eta'\eta'}$, $a_{\sigma\sigma}$ and $a_{\rho\rho}$.

Total: just 12 free parameters, for about 370 data points from many different and independent experiments.

Results and data

- Phase shifts. K_{e4} decays: BNL-E865 [Pislak et al., PR, D**67**, 072004 (2003)], NA48/2 [Masetti, arXiv:hep-ex/0610071] and $\pi p \to \pi\pi n$: CERN-Cracow-Munich reanalysis [Kaminski et al., ZP, C**74**, 79 (1997)], and an average of may other data.
- $|S_{11}|^2$: CERN-Cracow-Munich [Hyams et al., NP, B**64**, 134 (1973)] and its reanalysis [Kaminski et al., ZP, C**74**, 79 (1997)].



- $\pi\pi \to \pi\pi$: Although reaching energies of 2 GeV, description of low energy data is still very good.

  Scattering length and effective range, in good agreement with recent determination of Leutwyler:

	Our value	Leutwyler
a_0^0	0.216	0.220 ± 0.005
b_0^0	0.277	0.276 ± 0.006

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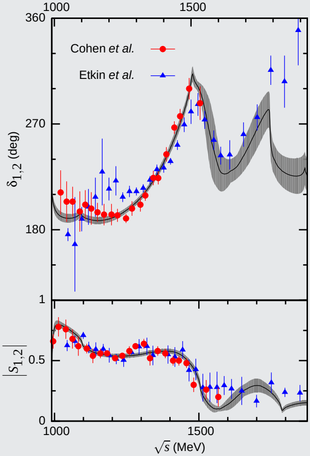
- $\pi\pi \to \eta\eta, \eta\eta'$: In good agreement for a low weight on χ^2. In addition, the data are unnormalized.
- $I = 1/2$ $K^-\pi^+ \to K^-\pi^+$ amplitude and phase from LASS.

Results and data

- BNL-E705: reactions $\pi^- p \to K^0_S K^0_S n$ [Etkin et al., PR, D**25**, 1786 (1982)]
- Argonne National Laboratory: $\pi^- p \to K^- K^+ n$ and $\pi^+ n \to K^- K^+ p$ [Cohen et al., PR, D**22**, 2595 (1980)]



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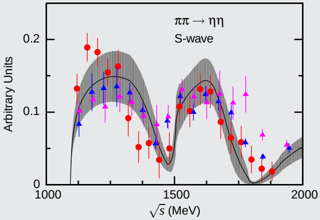
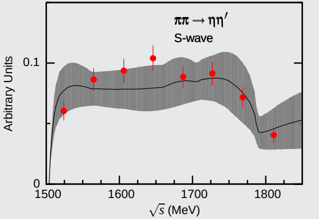
- $\pi\pi \to \eta\eta$: LAPP Collaboration, in the reaction $\pi^- p \to \eta\eta n$:
  [Alde et al., NP, B**269**, 485 (1986)], [Binon et al., NC, A**78**, 313 (1983)]
- $\pi\pi \to \eta\eta'$: LAPP Collaboration, in the reaction $\pi^- p \to \eta\eta' n$
  [Binon et al., NC, A**80**, 363 (1984)]





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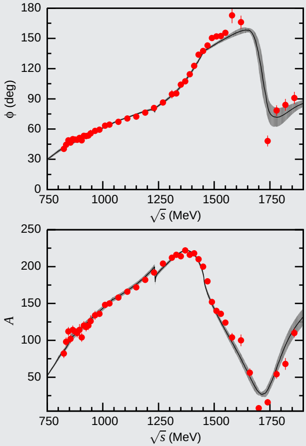
$\pi^+ K^-$ elastic scattering from LASS-SLAC in the reaction $K^- p \to K^- \pi^+ n$
[Aston et al., NP, B**296**, 493 (1988)]



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Results and data

No parametrization

We determine interaction kernels from Chiral Lagrangians, avoiding *ad hoc* parametrizations

Less free parameters

We have less free parameters, because of our chiral approach and our treatment of $\sigma\sigma$ amplitude, and the use of YM fields to describe vector-vector amplitudes

For instance, compare with: [Lindenbaum, Longacre, PL, B**274**, 492 (1992)], [Kloet, Loiseau, ZP A**353**, 227 (1995)], [Bugg, NP B**471**, 59 (1996)]

Higher energies

We have included enough channels to get at 2 GeV.

Spectroscopy. Pole content: Summary

PDG	M (MeV)	Γ (MeV)	$I=0$ Poles	M (MeV)	Γ (MeV)
$\sigma \equiv f_0(600)$			$456 \pm 6 - i\ 241 \pm 7$		
$f_0(980)$	980 ± 10	$40 - 100$	$983 \pm 4 - i\ 25 \pm 4$	983 ± 4	50 ± 8
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$f_0^L\ 1466 \pm 15 - i\ 158 \pm 12$	1370 ± 30	316 ± 24
$f_0(1500)$	1505 ± 6	109 ± 7	$f_0^R\ 1602 \pm 15 - i\ 44 \pm 15$	1502 ± 15	105 ± 30
$f_0(1710)$	1724 ± 7	137 ± 8	$1690 \pm 20 - i\ 110 \pm 20$	1700 ± 20	160 ± 40
$f_0(1790)$	1790^{+40}_{-30}	270^{+30}_{-60}	$1810 \pm 15 - i\ 190 \pm 20$	1810 ± 30	380 ± 40

$I=1/2$ Poles (MeV)	Effect	M (PDG)	Γ (PDG)
$708 \pm 6 - i\ 313 \pm 10$	$\kappa \equiv K_0^*(800)$	–	–
$1435 \pm 6 - i\ 142 \pm 8$	$K_0^*(1430)$	1414 ± 6	290 ± 21
$1750 \pm 20 - i\ 150 \pm 20$	$K_0^*(1950)$	–	–

Some points that deserve (and will receive) a detailed explanation are:

1. The whole structure of the $f_0(1500)$, which is related to f_0^R, f_0^L and $\eta\eta'$ threshold.
2. The mass of $f_0(1370)$, different from the one deduced from the pole.
3. The width of $f_0(1710)$, much smaller than $2\mathrm{Im}\sqrt{s_{f_0(1710)}}$.

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$f_0(1500)$	1505 ± 6	109 ± 7	f_0^R $1602 \pm 15 - i\ 44 \pm 15$	1502 ± 15	105 ± 30
$f_0(1710)$	1724 ± 7	137 ± 8	$1690 \pm 20 - i\ 110 \pm 20$	1700 ± 20	160 ± 40
$f_0(1790)$	1790^{+40}_{-30}	270^{+30}_{-60}	$1810 \pm 15 - i\ 190 \pm 20$	1810 ± 30	380 ± 40

$I=1/2$ Poles (MeV)	Effect	M (PDG)	Γ (PDG)
$708 \pm 6 - i\ 313 \pm 10$	$\kappa \equiv K_0^*(800)$	–	–
$1435 \pm 6 - i\ 142 \pm 8$	$K_0^*(1430)$	1414 ± 6	290 ± 21
$1750 \pm 20 - i\ 150 \pm 20$	$K_0^*(1950)$	–	–

Some points that deserve (and will receive) a detailed explanation are:

1. The whole structure of the $f_0(1500)$, which is related to f_0^R, f_0^L and $\eta\eta'$ threshold.
2. The mass of $f_0(1370)$, different from the one deduced from the pole.
3. The width of $f_0(1710)$, much smaller than $2\text{Im}\sqrt{s_{f_0(1710)}}$.

Spectroscopy. Pole content: Summary

PDG	M (MeV)	Γ (MeV)	$I=0$ Poles	M (MeV)	Γ (MeV)
$\sigma \equiv f_0(600)$			$456 \pm 6 - i\ 241 \pm 7$		
$f_0(980)$	980 ± 10	$40 - 100$	$983 \pm 4 - i\ 25 \pm 4$	983 ± 4	50 ± 8
$f_0(1370)$	$1200 - 1500$	$200 - 500$	f_0^L $1466 \pm 15 - i\ 158 \pm 12$	1370 ± 30	316 ± 24
$f_0(1500)$	1505 ± 6	109 ± 7	f_0^R $1602 \pm 15 - i\ 44 \pm 15$	1502 ± 15	105 ± 30
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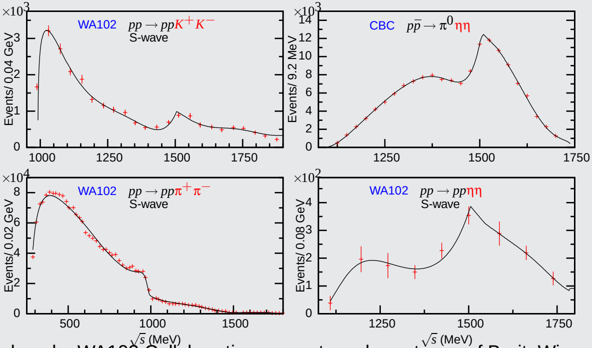
More data to support our spectroscopy

Here we see some data from WA102 and Crystal Barrel:



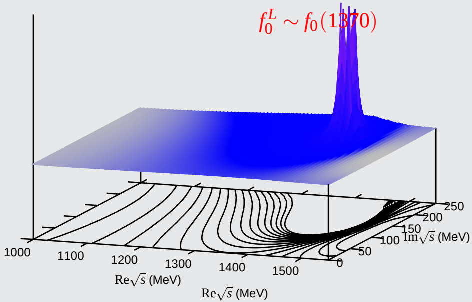
Similarly as done by WA102 Collaboration, we put a coherent sum of Breit–Wigners:

$$A_i(\sqrt{s}) = NR_i(\sqrt{s}) + \sum_{j \in \text{Res}} \frac{a_j e^{i\theta_j} g_{j;i}}{M_j^2 - s - iM_j\Gamma_j}$$

$$\text{Res} = \begin{cases} \sigma, f_0(980), f_0(1370), f_0^R & \sqrt{s} < m_\eta + m_{\eta'} \\ \sigma, f_0(980), f_0(1710), f_0(1790) & \sqrt{s} > m_\eta + m_{\eta'} \end{cases}$$

- Note that M_j, Γ_j and $g_{j;i}$ are fixed from the previous study on scattering (the poles on previous slide), without changing them.
- A constant is included to ensure continuity.

Spectroscopy. Pole content: $f_0(1370)$

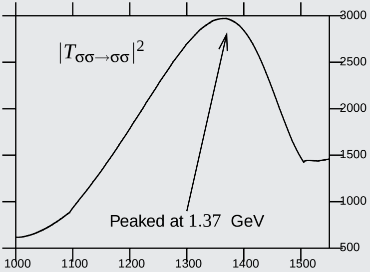


$f_0(1370)$

$1466 \pm 15 - i\ 158 \pm 12$ MeV

	M (MeV)	Γ (MeV)
PDG	$1200 - 1500$	$200 - 500$
Our	1370 ± 15	316 ± 24

- Shift in mass peak: $M_{f_0(1370)} \simeq 1370$ MeV.
- Strong coupling to $\sigma\sigma$, $\pi\pi$
- $\frac{\Gamma(f_0(1370)\to 4\pi)}{\Gamma(f_0(1370)\to \pi\pi)} = 0.30 \pm 0.12$, in good agreement with the interval $0.10 - 0.25$ of [Bugg,EPJ, C**52**, 55 (2007)]



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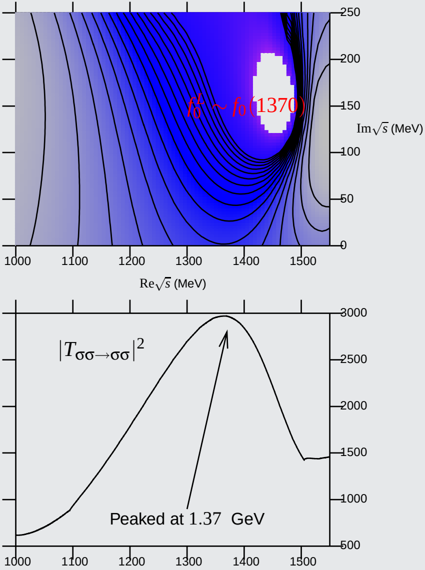


Spectroscopy. Couplings of $f_0(1370)$

$f_0(1370)$ $(I=0)$			$K_0^*(1430)$ $(I=1/2)$		
Coupling	bare	final	Coupling	bare	final
$g_{\pi^+\pi^-}$	3.9	3.59 ± 0.18	$g_{K\pi}$	5.0	4.8
$g_{K^0\bar{K}^0}$	2.3	2.23 ± 0.18	$g_{K\eta}$	0.7	0.9
$g_{\eta\eta}$	1.4	1.70 ± 0.30	$g_{K\eta'}$	3.4	3.8
$g_{\eta\eta'}$	3.7	4.00 ± 0.30			
$g_{\eta'\eta'}$	3.8	3.70 ± 0.40			

- Bare coupling are very similar to the physical ones.
- Bare: those of $S_8^{(1)}$, with $M_8^{(1)} = 1.29$ GeV, $c_d^{(1)} = c_m^{(1)} = 25.8$ MeV.
- The first scalar octet is a pure one, not mixed with the nearby $f_0(1500)$ nor $f_0(1710)$
- For instance we have the bare coupling for $\pi^+\pi^-$:

$$g_{\pi^+\pi^-} = \sqrt{\frac{2}{3}} \frac{c_d M_8^2 + 2(c_m - c_d) m_\pi^2}{f_\pi^2}$$

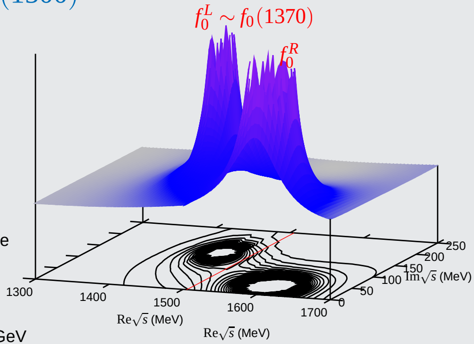
Spectroscopy. Pole content: $f_0(1500)$

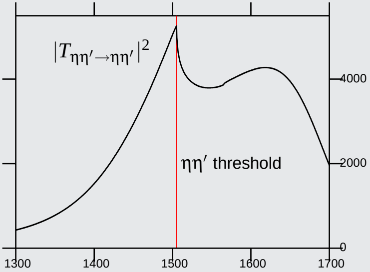
$f_0(1500)$

f_0^R $1602 \pm 15 - i\,44 \pm 15$ MeV

- The mass peak is at 1.5 GeV, due to $\eta\eta'$ threshold. Effect similar to $a_0(980)$ with the $K\bar{K}$ threshold
  [Oller, Oset, PR, D**60**, 074023 (1999)].
- The width is $\Gamma \approx 1.2 \times 88 \simeq 105$ MeV, because a Breit-Wigner at $(1.6 - i0.04)$ GeV is cut by $\eta\eta'$ thresold.
- Complicated energy region. Three interfering effects give raise to $f_0(1500)$:
  1. f_0^R: Pole at $(1.6 - i0.04)$ GeV
  2. $f_0(1370)$: Pole at $(1.47 - i0.16)$ GeV
  3. Nearby thresholds: $\eta\eta'$, $\omega\omega$

	M (MeV)	Γ (MeV)
PDG	1505 ± 6	109 ± 7
Our	1502	105 ± 36





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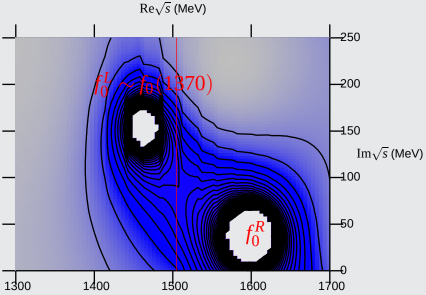
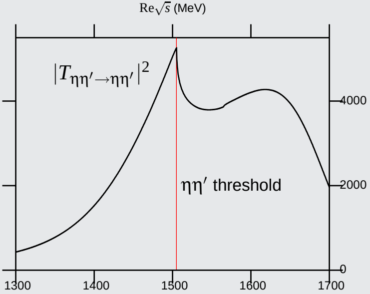
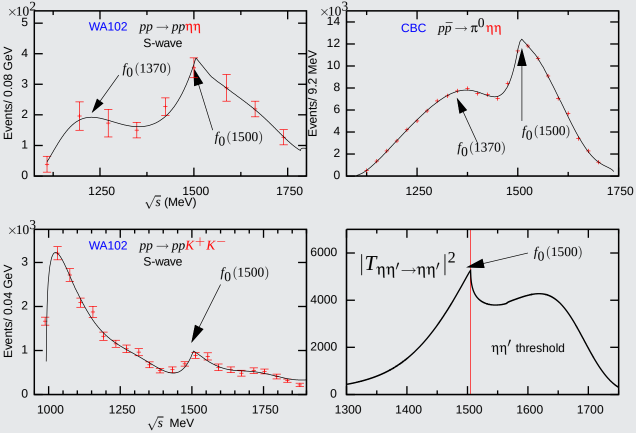




The shape of $f_0(1500)$

Recall:

- The data of WA102 and Crystall Barrell Colaboration which show the peak of $f_0(1500)$, and
- that we fit these data with the poles we find from our scattering study



Spectroscopy. Pole content: $f_0(1710)$

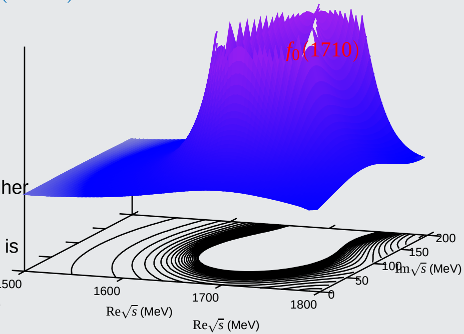
$f_0(1710)$

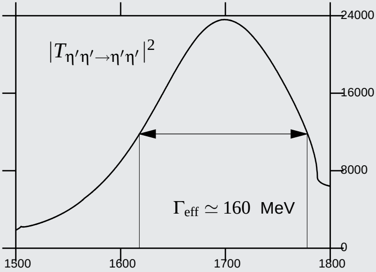
$1690 \pm 20 - i\ 110 \pm 20$ MeV

- The mass peak is slighty shifted to higher energy: $\sqrt{s} = M \simeq 1700$ MeV.
- The width "measured" on the real axis is $\Gamma_{\rm eff} \simeq 160$ MeV.
- The effective width can depend on the concrete process.

	M (MeV)	Γ (MeV)
PDG	1724 ± 7	137 ± 8
Our	1700 ± 20	160 ± 40

BR	Our value	PDG
$\frac{\Gamma(K\bar{K})}{\Gamma_{\rm total}}$	0.36 ± 0.12	$0.38^{+0.09}_{-0.19}$
$\frac{\Gamma(\eta\eta)}{\Gamma_{\rm total}}$	0.22 ± 0.12	$0.18^{+0.03}_{-0.13}$
$\frac{\Gamma(\pi\pi)}{\Gamma(K\bar{K})}$	0.32 ± 0.14	$0.41^{+0.11}_{-0.17}$
$\frac{\Gamma(\eta\eta)}{\Gamma(K\bar{K})}$	0.64 ± 0.38	0.48 ± 0.15





Spectroscopy. Pole content: $f_0(1710)$

$f_0(1710)$

$1690 \pm 20 - i\ 110 \pm 20$ MeV

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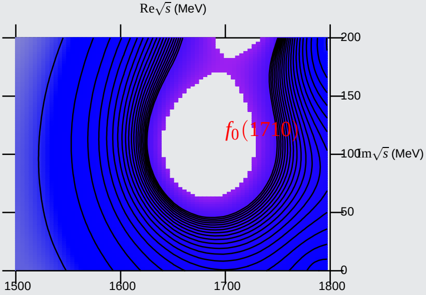
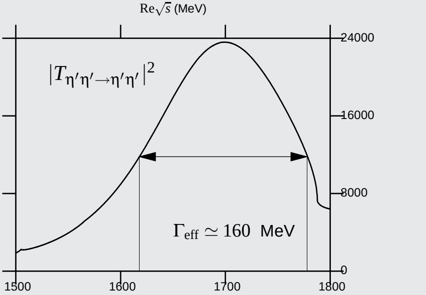




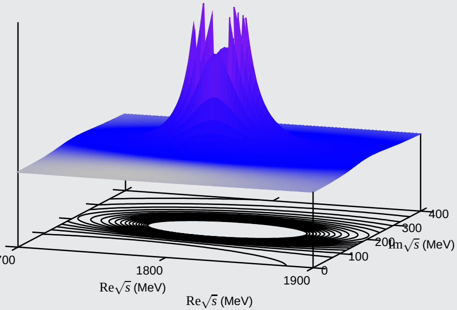
Spectroscopy. Pole content: $f_0(1790)$

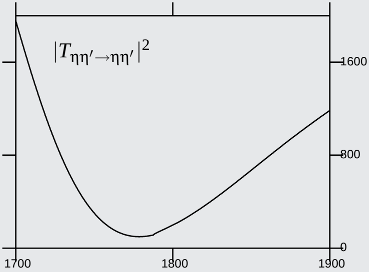
$f_0(1790)$

$1810 \pm 15 - i\ 190 \pm 20$ MeV

- Weak signal on the real axis
- It couples weakly to $K\bar{K}$, a major difference with respect to $f_0(1710)$, as also observed by BESII.
- It is the partner of the pole at $1.75 - i0.15$ GeV in $I = 1/2$.
- These poles originate from the higher bare octet, $S_8^{(2)}$, with $M_8^{(2)} = 1905$, $c_d^{(2)} = 20.3$, $c_m^{(2)} = -13.9$ MeV.

	M (MeV)	Γ (MeV)
BESII	1790^{+40}_{-30}	270^{+30}_{-60}
Our	1810 ± 15	380 ± 40





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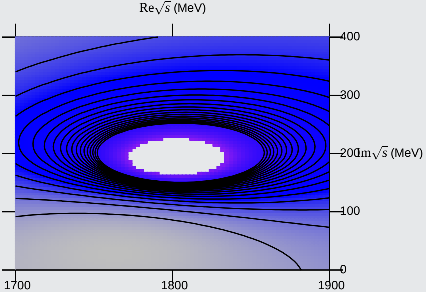

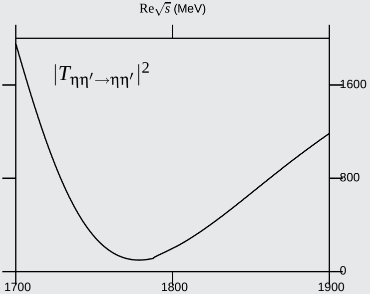

Spectroscopy. Couplings of f_0^R and $f_0(1710)$

Coupling (GeV)	f_0^R	$f_0(1710)$
$g_{\pi^+\pi^-}$	1.31 ± 0.22	1.24 ± 0.16
$g_{K^0\bar{K}^0}$	2.06 ± 0.17	2.00 ± 0.30
$g_{\eta\eta}$	3.78 ± 0.26	3.30 ± 0.80
$g_{\eta\eta'}$	4.99 ± 0.24	5.10 ± 0.80
$g_{\eta'\eta'}$	8.30 ± 0.60	11.7 ± 1.60

This pattern suggests an enhancement in $s\bar{s}$ production.
With a pseudoscalar mixing angle $\sin\beta = -1/3$ for η and η':

$$g_{\eta'\eta'} = \frac{2}{3}g_{ss} + \frac{1}{3}g_{nn} + \frac{2\sqrt{2}}{3}g_{ns}$$

$$g_{\eta\eta'} = -\frac{\sqrt{2}}{3}g_{ss} + \frac{\sqrt{2}}{3}g_{nn} + \frac{1}{3}g_{ns}$$

$$g_{\eta\eta} = \frac{1}{3}g_{ss} + \frac{2}{3}g_{nn} - \frac{2\sqrt{2}}{3}g_{ns}$$

$$\eta = -\frac{1}{\sqrt{3}}\eta_s + \sqrt{\frac{2}{3}}\eta_u$$

$$\eta' = -\sqrt{\frac{2}{3}}\eta_s + \frac{1}{\sqrt{3}}\eta_u$$

$$\eta_s = s\bar{s} \qquad \eta_u = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

With this equations, we can obtain g_{ss}, g_{ns} and g_{nn} from $g_{\eta\eta}$, $g_{\eta\eta'}$ and $g_{\eta'\eta'}$.

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Coupling (GeV)	f_0^R	$f_0(1710)$
g_{ss}	11.5 ± 0.5	13.0 ± 1.0
g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

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$$\begin{aligned} \eta &= -\frac{1}{\sqrt{3}}\eta_s + \sqrt{\frac{2}{3}}\eta_u \\ \eta' &= -\sqrt{\frac{2}{3}}\eta_s + \frac{1}{\sqrt{3}}\eta_u \end{aligned}$$

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\end{aligned}
$$

$$
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\eta' &= -\sqrt{\frac{2}{3}}\eta_s + \frac{1}{\sqrt{3}}\eta_u
\end{aligned}
$$

$$\eta_s = s\bar{s} \qquad \eta_u = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

The glueball $\to q\bar{q} \propto m_q$ chiral supression [Chanowitz, PRL**95**, 172001 (2005)] implies $|g_{ss}| \gg |g_{nn}|$. Together with the OZI rule requires $|g_{ss}| \gg |g_{ns}|$.
This is precisely what we obtain from the previous couplings and equations.

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Now let us consider $K\bar{K}$ coupling:

- Valence quarks: K^0 corresponds to $\sum_{i=1}^{3} \bar{s}_i u^j/\sqrt{3}$, and analogously $\bar{K}^0$
- Production of colour singlet $s\bar{s}$ requires the combination of the colour indices of K^0, $\bar{K}^0$
- Decompose $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^j$
- Only $s\bar{s}u\bar{u}$ contributes (factor $1/3$) and $s\bar{s}s\bar{s}$ has an extra factor two compared to $s\bar{s}u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$

Spectroscopy. Couplings of f_0^R and $f_0(1710)$

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g_{ns}	-0.2	2.1
g_{nn}	-1.4	1.2
$g_{ss}/6$	1.9 ± 0.1	2.1 ± 0.2

Now let us consider $K\bar{K}$ coupling:

- Valence quarks: K^0 corresponds to $\sum_{i=1}^{3} \bar{s}_i u^j/\sqrt{3}$, and analogously $\bar{K}^0$
- Production of colour singlet $s\bar{s}$ requires the combination of the colour indices of K^0, $\bar{K}^0$
- Decompose $\bar{s}_i s^j = \delta_i^j \bar{s}s/3 + (\bar{s}_i s^j - \delta_i^j \bar{s}s/3)$ and similarly $\bar{u}_i u^j$
- Only $s\bar{s}u\bar{u}$ contributes (factor $1/3$) and $s\bar{s}s\bar{s}$ has an extra factor two compared to $s\bar{s}u\bar{u}$, so one expects $g_{K^0\bar{K}^0} = g_{ss}/6$

Reasons to identify the glueball with $f_0(1710)$

Arguments favouring the first interepretation for the $f_0(1710)$:

1. $f_0(1370)$ does not mix, is pure $I=0$ octet state. No one can mix the with $f_0(1710)$. Note that the Chiral Suppression Mechanism for a glueball also implies that it should not mix.
2. Furthermore, if $f_0(1710)$ or f_0^R were an $\bar{s}s$ resonance, there should be an accompanying resonance in $I=1/2$, but there is no such pole.
3. Unquenched lattice calculations [Setxton, Vaccarino, Weingarten, PRL**75** 4563 (1995)] give the mass and the couplings of the lightest 0^{++} glueball:
   - Mass [Chen et al., PR, D**73**, 014516 (2006)]:

$$M_{0^{++}}^{\text{gb}} = 1.66 \pm 0.05 \ \text{GeV} \qquad M_{f_0(1710)} = 1.69 \pm 0.02 \ \text{GeV}$$

   - Couplings are calculated in the $SU(3)$ limit and are linear in the pseudoscalar mass squared in agreement with the chiral suppression mechanism of [Chanowitz, PRL**95**, 172001 (2005)].

S-Wave Meson scattering and spectroscopy

M. Albaladejo

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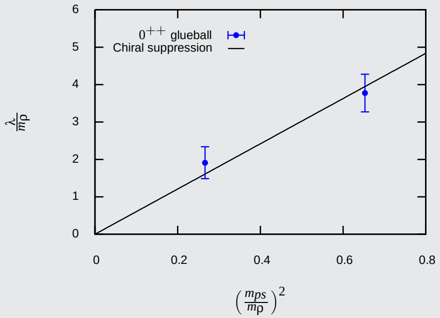
Coupling of glueball to a pair of pseudoscalars



We see here the coupling for a decay of the glueball to a pair of pseudoscalars:

- as calculated in unquenched lattice QCD [Setxton, Vaccarino, Weingarten, PRL**75** 4563 (1995)]
- a coupling linear in pseudoscalar mass squared as predicted by chiral suppression mechanism [Chanowitz, PRL**95**, 172001 (2005)].

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Note that our work is completely INDEPENDENT of Chanowitz's. When comparing, this coupling and mixing scheme naturally fits in our results.

Summary

- We have performed the first coupled channel study of the meson-meson S-waves for $I=0$ and $I=1/2$ up to 2 GeV with 13 coupled channels.
- Interaction kernels are determined from Chiral Lagrangians and implemented in N/D–type equations.
- We have fewer free parameters, compared with previous works in the literature, for a vast quantity of data.
- All scalar resonances below 2 GeV are generated:
  - $I=0$: σ, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$ and $f_0(1790)$
  - $I=1/2$: κ, $K_0^*(1430)$ and $K_0^*(1950)$
- The structure of the couplings to pairs of pseudoscalars implies that:
  - $f_0(1370)$, $K_0^*(1430)$ (and $a_0(1450)$) remain as pure octet.
  - f_0^R and $f_0(1710)$, which are the same pole reflected on different sheets, have a decay pattern showing an enhanced $\bar{s}s$ production, in agreement with the chiral suppression mechanism of glueball decay, so
  - It should be considered as the lightest unmixed 0^{++} glueball.

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σ width effects on $g(s)$

With s_σ complex, $N_{i\to\sigma\sigma}$ would be complex, thus violating unitarity. So we take s_i real, but varying according to a mass distribution.

- Lehman propagator representation (dispersion relation):

$$P(s) = -\frac{1}{\pi}\int_{\sqrt{s_{\text{th}}}}^{\infty} \mathrm{d}s' \frac{\mathrm{Im}P(s')}{s-s'+i\varepsilon},$$

- In a first approach:

$$\begin{aligned}
\mathrm{Im}P(s') &\propto \mathrm{Im}\left(\frac{1}{s'-m_\sigma^2+i\,m_\sigma\Gamma_\sigma(s')}\right)\\
\Gamma_\sigma(s') &= \Gamma_\sigma\sqrt{\frac{1-4m_\pi^2/s'}{1-4m_\pi^2/m_\sigma^2}}\\
\int_{\sqrt{s_{\text{th}}}}^{\infty}\mathrm{d}s'\,\mathrm{Im}P(s') &= 1
\end{aligned}$$

- $g(s)$ (unitarity loop) for σ channel can be written as:

$$\int_{\sqrt{s_{\text{th}}}}^{\infty}\mathrm{d}s_1\int_{\sqrt{s_{\text{th}}}}^{\infty}\mathrm{d}s_2\,\mathrm{Im}P(s_1)\,\mathrm{Im}P(s_2)\;g_4(s;s_1,s_2)\;,$$

Spectroscopy, poles and Riemann sheets

- When parameters are fitted to data, we extrapolate our amplitudes to the s-complex plane, finding poles, and we identify them with resonances through $\sqrt{s_0} \approx M - i\Gamma/2$.
- Analytical extrapolations are needed in the T_{ij} to the different Riemann sheets, which appear because of the cuts in $G(s)$ at opening thresholds:

$$p(s) = \frac{\sqrt{s-(m_a+m_b)^2}\sqrt{s-(m_a-m_b)^2}}{2\sqrt{s}}$$

- Using continuity, for $\sqrt{s}$ real, $> m_a + m_b$, we have:

$$G^{II}(s+i\varepsilon) = G(s-i\varepsilon) = G(s+i\varepsilon) - 2i\mathrm{Im}G(s+i\varepsilon) = G(s+i\varepsilon) + \frac{i}{4\pi}\frac{p(s)}{\sqrt{s}}$$

Some branching ratios...