examen enero

Ejercicio 1: para hacer a mano

Ejercicio 2: Michaelis-Menten

```
--> S:[0.10 , 0.25 , 0.50 , 1 , 2 , 4 , 8]; V:[ 14 , 24 , 30.8 , 42.7 , 51.7 , 51.8 , 53.1];
```

```
(%o56) [0.1,0.25,0.5,1,2,4,8]
(%o57) [14,24,30.8,42.7,51.7,51.8,53.1]
```

```
--> SV:transpose(matrix(S,V));
```

(%o5)
$$\begin{bmatrix} 0.1 & 14 \\ 0.25 & 24 \\ 0.5 & 30.8 \\ 1 & 42.7 \\ 2 & 51.7 \\ 4 & 51.8 \\ 8 & 53.1 \end{bmatrix}$$

a) hacer regresion (1/S,1/V)

```
--> x:1/S;y:1/V;xy:transpose(matrix(x,y));
```

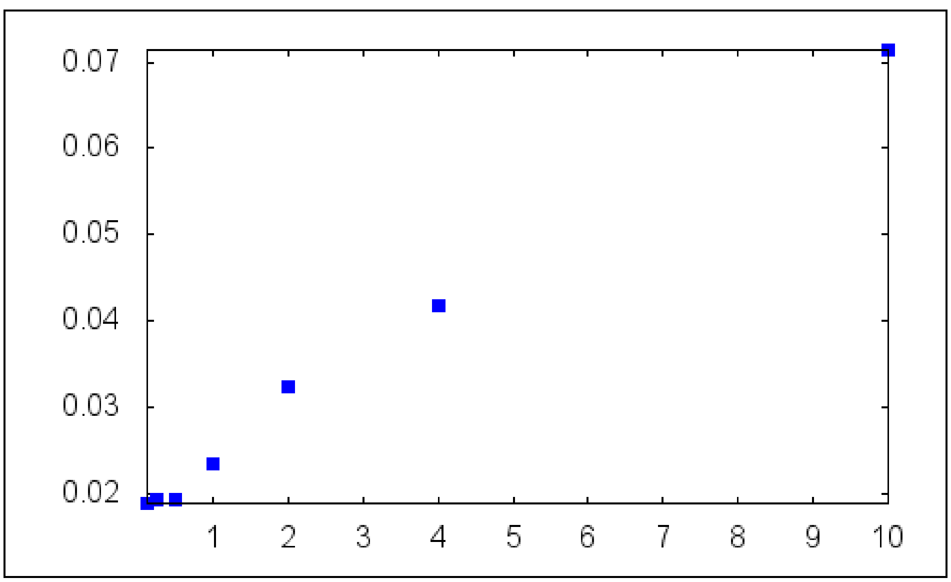
(%o6) $[10.0, 4.0, 2.0, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}]$

(%o7) $[\frac{1}{14}, \frac{1}{24}, 0.032467532467532, 0.023419203747073, 0.01934235976789$
$0.019305019305019, 0.018832391713748]$

(%o8)
$$\begin{bmatrix} 10.0 & \frac{1}{14} \\ 4.0 & \frac{1}{24} \\ 2.0 & 0.032467532467532 \\ 1 & 0.023419203747073 \\ \frac{1}{2} & 0.019342359767892 \\ \frac{1}{4} & 0.019305019305019 \\ \frac{1}{8} & 0.018832391713748 \end{bmatrix}$$

--> wxdraw2d(point_type=5,points(xy));

(%t13)



(%o13)

--> simple_linear_regression(xy);

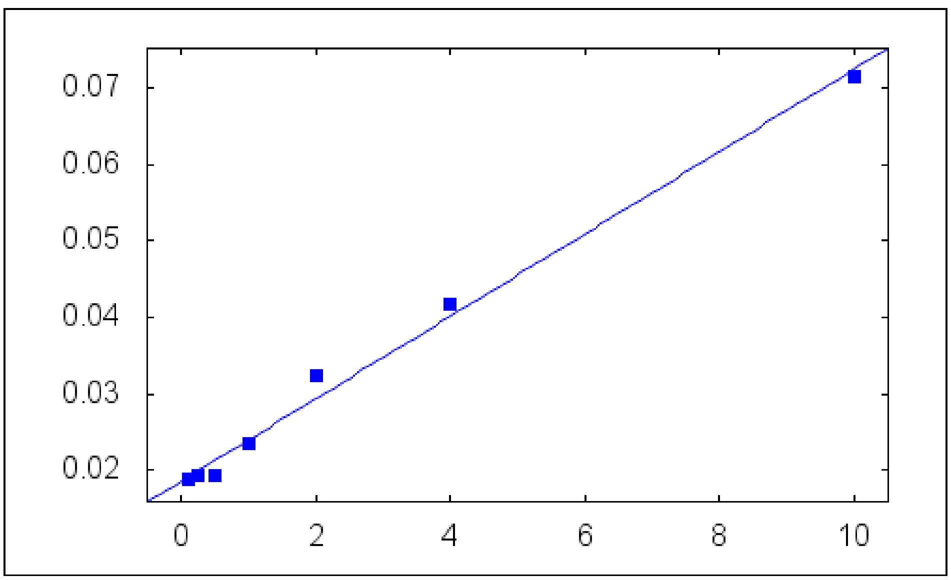
(%o14)

```
SIMPLE LINEAR REGRESSION

model=0.0053892115652689 x+0.018589941195332

correlation=0.99599886592681

v_estimation=3.5391967967917824 10^-6

b_conf_int=[0.0048333271240327,0.005945096006505]

hypotheses=H0: b = 0 ,H1: b # 0

statistic=24.92138352922223

distribution=[student_t,5]

p_value=1.9407968105955575 10^-6
```

la correlacion es buena, pero los puntos no estan visualmente muy alineados (debido a que hay varios ptos apiñados, y uno muy alejado)

```
--> wxdraw2d(point_type=5,points(xy),
explicit(0.0053892115652689*'x+0.018589941195332, 'x,-0.5,10.5));
```

(%t16)



(%o16)

b) Observamos que 1/vmax = 0.018589941195332 y k/vmax= 0.0053892115652689. Por tanto

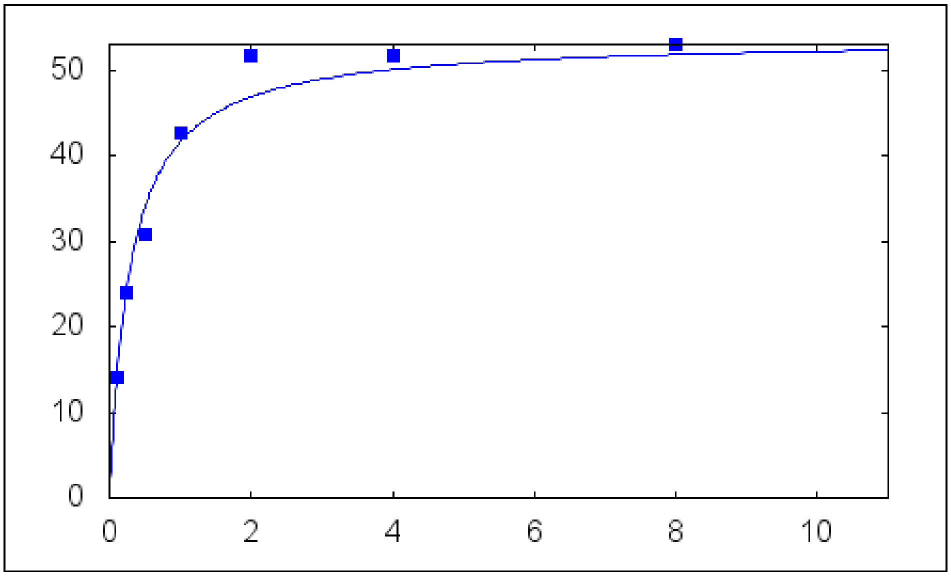
```
--> vmax :1/ 0.018589941195332 ;k:vmax * 0.0053892115652689;
```

(%o17) 53.7925316434623
(%o18) 0.28989933365804

es decir, la velocidad maxima de la reaccion es 53'8 y la concentracion de semi-saturacion es 0'29, lo que concuerda aprox con los datos.

```
--> wxdraw2d(point_type=5,points(SV), explicit(vmax*'S/(k+'S),'S, 0, 11));
```

(%t20)



(%o20)

c) en la gráfica se observa que la aproximacion se queda un poco corta cuando S es grande, por tanto es de esperar que la estimacion de vmax sea un poco pequeña.

d) Usamos regresion no-lineal para dar un ajuste mejor:

```
--> load(lsquares);
```

(%o23)
C:/maxima/maxima_installed/Maxima-5.28.0-2/share/maxima/5.28.0-2/sha

```
--> mse : lsquares_mse(SV, ['S,'V], 'V = (a*'S)/(b + 'S));
```

$$(\%o25)\quad \frac{\sum_{i=1}^{7}\left(SV_{i,2}-\frac{a\,SV_{i,1}}{SV_{i,1}+b}\right)^2}{7}$$

```
--> lsquares_estimates_approximate(mse, [a, b], initial =[53, 0.3], tol =0.001);
```

```
*************************************************
  N=     2    NUMBER OF CORRECTIONS=25
        INITIAL VALUES
 F=   7.358827113789662D+00    GNORM=   3.970647771367358D+01
*************************************************
   I  NFN      FUNC                     GNORM                    STEPLE
   1    4      6.940833639337722D+00    2.027851058440031D+00    5.1725
   2    7      6.904893846829252D+00    5.685261055884003D+00    2.1000
   3    8      6.750450902936597D+00    1.503153349053390D+01    1.0000
   4    9      6.183953870352441D+00    3.417677715653687D+01    1.0000
   5   10      5.246931134788660D+00    4.773036459717098D+01    1.0000
   6   11      4.148676181483496D+00    4.645804633356590D+01    1.0000
   7   12      3.405800898930655D+00    2.519611200055596D+01    1.0000
   8   13      3.205091087815736D+00    5.230607267867647D+00    1.0000
   9   14      3.189309339392971D+00    3.013989438506634D-02    1.0000
 THE MINIMIZATION TERMINATED WITHOUT DETECTING ERRORS.
 IFLAG = 0
 (%o26) [[a=56.92492140746688,b=0.34797931180919]]
```

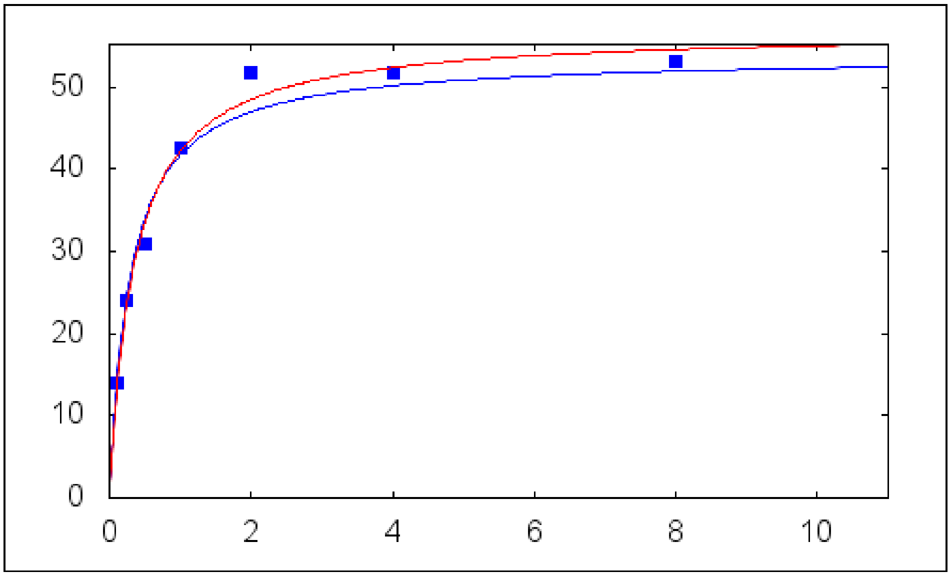
He utilizado los datos vmax, k anteriores como condicion inicial.
Por tanto parece mejor estimar

vmax= 56.92492140746688 y k=0.34797931180919

```
--> vmax2: 56.92492140746688; k2:0.34797931180919;
```

```
(%o27) 56.92492140746688
(%o28) 0.34797931180919
```

```
--> wxdraw2d(point_type=5,points(SV), explicit(vmax*'S/(k+'S),'S, 0, 11),
    color=red, explicit(vmax2*'S/(k2+'S),'S, 0, 11));
```

(%t30) 

(%o30)

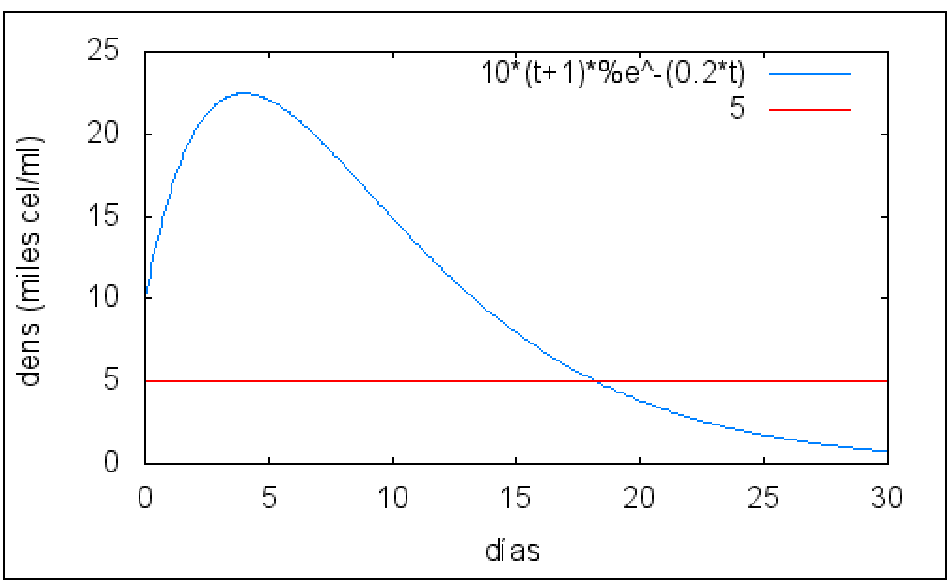
el ajuste se aprecia bastante mejor que el anterior.

EJERCICIO 3: GRAFICA CRECIMIENTO TUMORAL

--> D(t):=10*(t+1)*exp(-.2*t);

(%o33) D(t):=10(t+1)exp((-0.2)t)

--> wxplot2d([D(t),5], [t,0,30],[xlabel,"días"],[ylabel,"dens (miles cel/ml)"])$

(%t39) 

se observa que:

- inicialmente hay 10 mil cel/ml, y a largo plazo las celulas tumorales desaparecen
- alcanzan su maximo tras aprox 4 días, siendo este en torno a 22 mil cel/ml
- a partir del 5 día la densidad de celulas tumorales decrece,
y en el día 9 hay un pto de inflex y a partir de ahí la veloc de decrec se ralentiza.
- se alcanza una densidad de 5 mil cel/ml tras aprox 18 días

Veamos abajo los valores exactos

```
--> diff(D(t),t,1);
```

(%o41) $10\,\%e^{-0.2\,t} - 2.0\,(t+1)\,\%e^{-0.2\,t}$

```
--> solve([%=0], [t]);
```

```
rat: replaced -0.2 by -1/5 = -0.2
rat: replaced -2.0 by -2/1 = -2.0
rat: replaced -0.2 by -1/5 = -0.2
```

(%o42) $[t=4]$

```
--> D(4);
```

(%o43) 22.46644820586108

por tanto, tras 4 días se alcanza la densidad maxima
de celulas tumorales, siendo esta 22'466 mil cel/ml

```
--> diff(D(t),t,2);
```

(%o44) $0.4\,(t+1)\,\%e^{-0.2\,t} - 4.0\,\%e^{-0.2\,t}$

```
--> solve([%=0], [t]);
```

```
rat: replaced -4.0 by -4/1 = -4.0
rat: replaced -0.2 by -1/5 = -0.2
rat: replaced 0.4 by 2/5 = 0.4
rat: replaced -0.2 by -1/5 = -0.2
```

(%o45) $[t=9]$

por tanto pto inflex en el noveno día

b) Se alcanza D(t)=5

```
--> solve([D(t)=5], [t]);
```

rat: replaced -0.2 by -1/5 = -0.2

(%o46) $[t=\frac{\%e^{t/5}-2}{2}]$

```
--> find_root(D(t)=5, t, 15, 20);
```

(%o48) 18.25444583564739

... tras 18 días y 6 horas aprox

```
--> D1(t):= 10*%e^(-0.2*t)-2.0*(t+1)*%e^(-0.2*t);
```

(%o49) $D1(t):=10\,\%e^{(-0.2)\,t}-2.0\,(t+1)\,\%e^{(-0.2)\,t}$

```
--> D1(18.2544);
```

(%o50) -0.74032412016443

la veloc de decrecimiento de las celulas tumorales en ese instante es de unas 740 cel/ml y dia

c) sin radioterapia conocemos la velocidad de crecimiento
y nos piden D(t). Usamos que V(t)= D'(t) e integramos

```
--> V(t):=20*exp(-t/5);
```

(%o51) $V(t):=20\,\exp\left(\frac{-t}{5}\right)$

```
--> integrate(V(t), t);
```

(%o52) $-100\,\%e^{-\frac{t}{5}}$

--> Dc(t):= C -100*exp(-t/5);

(%o53) $Dc(t):=C-100\,\exp\left(\frac{-t}{5}\right)$

como D(0)=10

--> solve([Dc(0)=10], [C]);

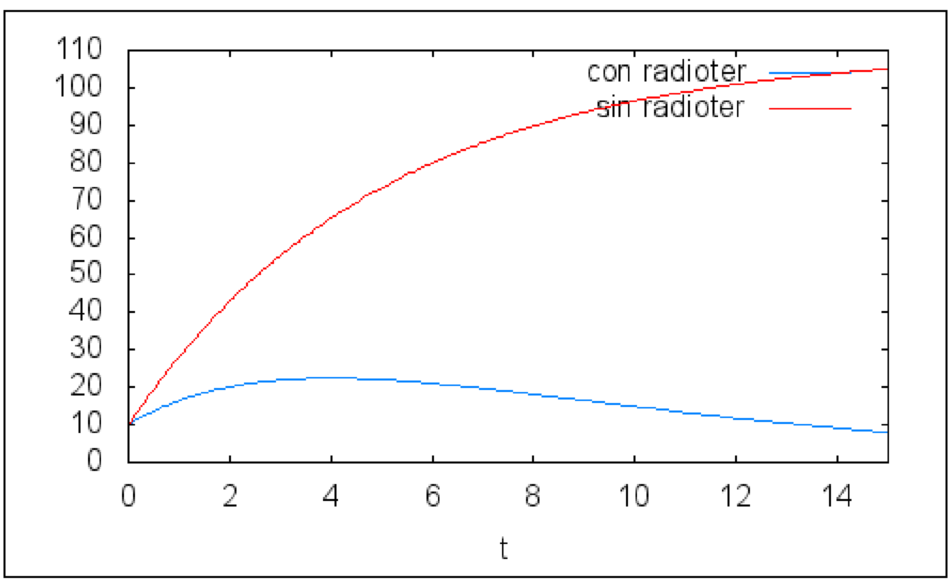
(%o55) [C=110]

--> C:110;

(%o56) 110

--> wxplot2d([D(t),Dc(t)], [t,0,15],[legend,"con radioter", "sin radioter"])$

(%t58)



--> Dc(7),numer;

(%o60) 85.34030360583935

al cabo de una semana hay 85 mil cel/ml
a largo plazo habrá unas 110 mil cel/ml.

4) Ecuación diferencial

```
--> kill(V);
```

(%o61) *done*

```
--> ed:'diff(V,P)=-r*V/P;
```

(%o62) $\frac{d}{d\,P}\,V=-\frac{r\,V}{P}$

```
--> ode2(ed, V, P);
```

(%o63) $V=\%c\,\%e^{-r\,\log(P)}$

```
--> radcan(%);
```

(%o64) $V=\frac{\%c}{P^{r}}$

ajustamos el dato inicial usando P=1, V=10

```
--> ic1(%, P=1, V=10);
```

(%o65) $V=\frac{10}{P^{r}}$

calculamos la r usando P=5, V=5

```
--> solve([5=10/5^r], [r]),numer;
```

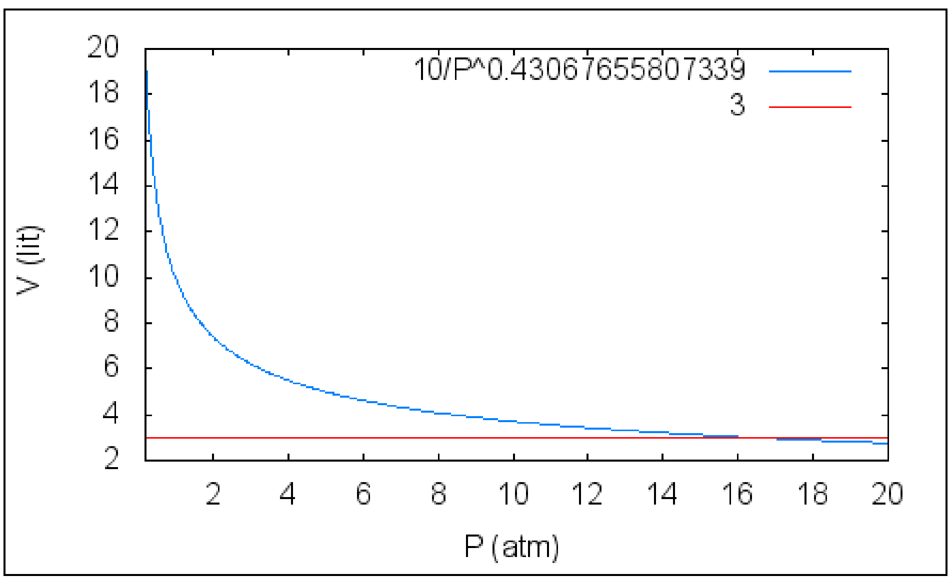
```
rat: replaced -2.0 by -2/1 = -2.0
rat: replaced -2.0 by -2/1 = -2.0
rat: replaced -0.4306765580734 by -13456039/31243955 = -0.430676558C
rat: replaced -0.4306765580734 by -13456039/31243955 = -0.430676558C
rat: replaced 3.200619127764074E-8 by 1/31243955 = 3.20061912776407
rat: replaced -0.4306765580734 by -13456039/31243955 = -0.430676558C
```

(%o67) [r=0.43067655807339]

--> r:0.43067655807339;

(%o68) 0.43067655807339

--> wxplot2d([10/P^r,3], [P,.2,20],[xlabel, "P (atm)"],[ylabel, "V (lit)"])$

(%t80) 

b) A presion 10 atm el vol es aprox 4 litros
el volumen es 3 lit cuando la presion es aprox 17 atm

--> V(P):=10/P^r;

(%o73) $V(P):=\frac{10}{P^{r}}$

--> V(10);

(%o74) 3.709568900243075

--> find_root(V(P)=3, P, 14, 18);

(%o82) 16.37143879734382

Ejercicio 3: Michaelis-Menten modelo 2

--> S2:1.5*S;V2:V*0.7;

(%o60) [0.15,0.375,0.75,1.5,3.0,6.0,12.0]
(%o61) [9.799999999999999,16.8,21.56,29.89,36.19,36.26,37.17]

--> S2:[0.15,0.37,0.75,1.5,3.0,6.0,12.0];
V2:[9.8,16.8,21.5,29.9,36.2,36.3,37.2];

(%o87) [0.15,0.37,0.75,1.5,3.0,6.0,12.0]
(%o88) [9.800000000000001,16.8,21.5,29.9,36.2,36.3,37.2]

--> SV2:transpose(matrix(S2,V2));

(%o89)

0.15	9.800000000000001
0.37	16.8
0.75	21.5
1.5	29.9
3.0	36.2
6.0	36.3
12.0	37.2

--> x2:1/S2;y2:1/V2;xy2:transpose(matrix(x2,y2));

```
(%o90) [6.666666666666667,2.702702702702703,1.333333333333333,0.66
,0.33333333333333,0.16666666666667,0.083333333333333]
(%o91) [0.10204081632653,0.05952380952381,0.046511627906977,0.0334
,0.027624309392265,0.027548209366391,0.026881720430108]
```

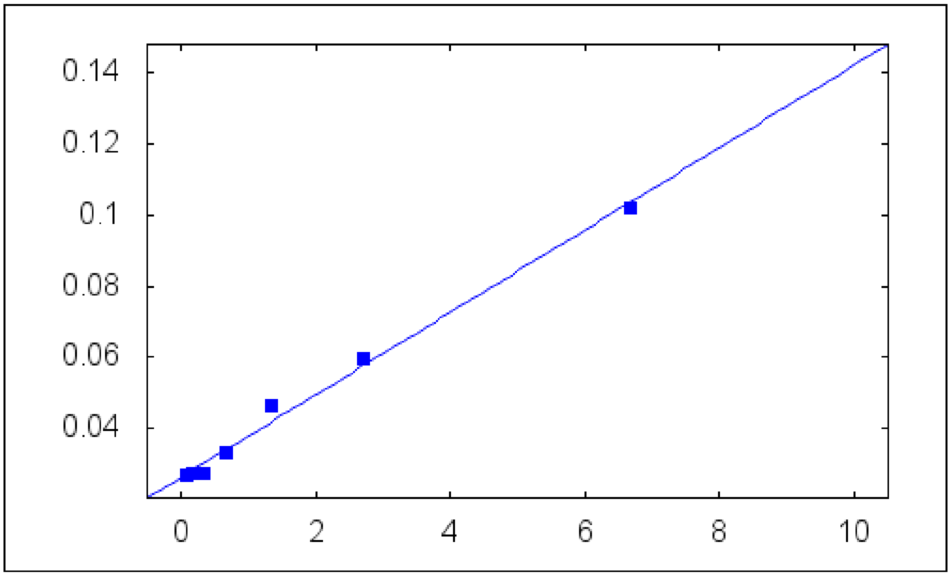
6.666666666666667	0.10204081632653
2.702702702702703	0.05952380952381
1.333333333333333	0.046511627906977
0.66666666666667	0.033444816053512
0.33333333333333	0.027624309392265
0.16666666666667	0.027548209366391
0.083333333333333	0.026881720430108

(%o92)

```
--> simple_linear_regression(xy2);
```

```
                SIMPLE LINEAR REGRESSION
   model=0.011539999663485 x+0.026520160547524
        correlation=0.99603980334081
      v_estimation=7.1532087463484767 10^-6
(%o93) b_conf_int=[0.010355817515276,0.012724181811694]
       hypotheses=H0: b = 0 ,H1: b # 0
        statistic=25.05063394413594
        distribution=[student_t,5]
       p_value=1.891575712509308 10^-6
```

```
-- wxdraw2d(point_type=5,points(xy2),
>  explicit(0.011538954227748*'x+0.026525045131291, 'x,-0.5,10.5));
```

(%t94) 

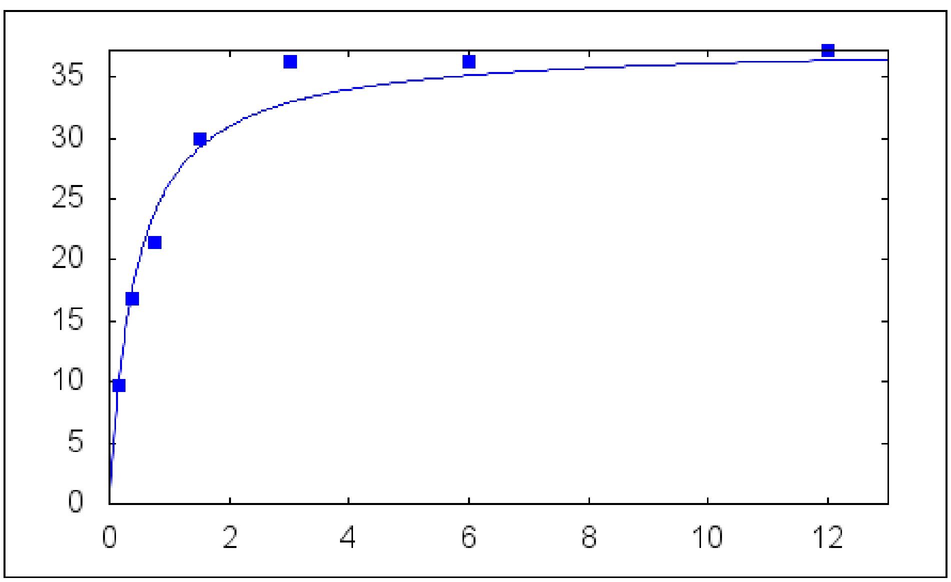
(%o94)

--> vmax3 :1/0.026525045131291 ;k3:vmax3 * 0.011538954227748;

(%o95) 37.70021860661502

(%o96) 0.43502109687782

--> wxdraw2d(point_type=5,points(SV2), explicit(vmax3*'S/(k3+'S),'S, 0, 13));

(%t97) 

(%o97)

--> load(lsquares);

(%o48)

```
C:/maxima/maxima_installed/Maxima-5.28.0-2/share/maxima/5.28.0-2/sha
```

--> mse2 : lsquares_mse(SV2, ['S,'V], 'V = (a*'S)/(b + 'S));

(%o98)
$$\frac{\sum_{i=1}^{7}\left(SV2_{i,2}-\frac{a\,SV2_{i,1}}{SV2_{i,1}+b}\right)^2}{7}$$

--> lsquares_estimates_approximate(mse2, [a, b], initial =[53, 0.3], tol =0.001);

```
*************************************************
 N=     2    NUMBER OF CORRECTIONS=25
       INITIAL VALUES
 F=  1.776337624291383D+02    GNORM=  6.125833771911408D+02
*************************************************
   I  NFN      FUNC                    GNORM                   STEPLE
   1    2      3.088386214370545D+01   3.681072218804968D+00   1.6324
   2    6      2.917702770618711D+01   5.965447436877772D+00   8.5000
   3    7      2.443051986020194D+01   2.482925342819425D+01   1.0000
   4    8      2.294098664242629D+01   2.912507662921533D+01   1.0000
   5    9      1.583962321460565D+01   4.300049094597123D+01   1.0000
   6   10      5.476835407038464D+00   3.919657465971996D+01   1.0000
   7   12      3.495380980903090D+00   1.097478424110607D+01   4.4832
   8   13      1.697027866138328D+00   3.765949026678986D-01   1.0000
   9   14      1.598371407135115D+00   1.740057532838554D-01   1.0000
  10   15      1.598071066605107D+00   2.871778344134490D-02   1.0000
 THE MINIMIZATION TERMINATED WITHOUT DETECTING ERRORS.
 IFLAG = 0
```

(%o99) [[a=39.82479536067655, b=0.51961968485457]]

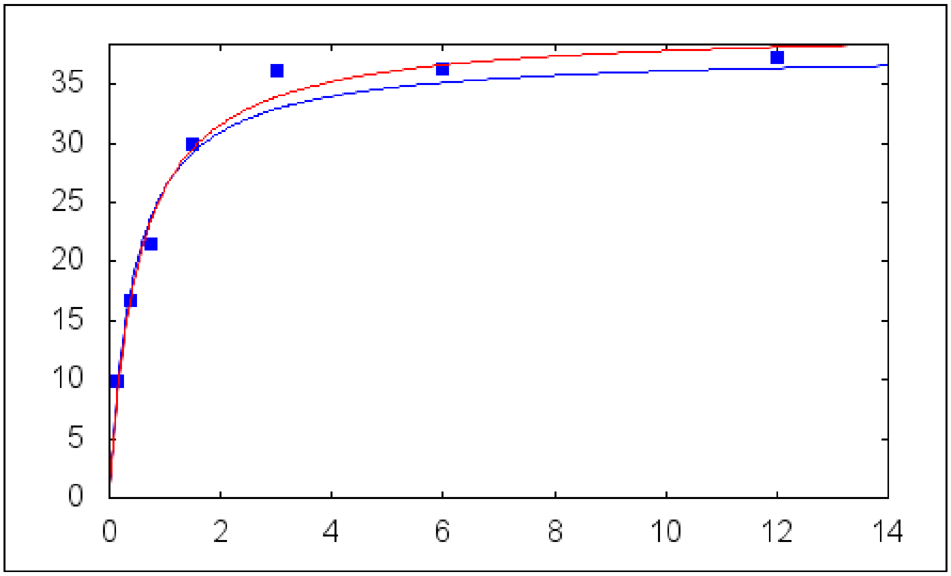
--> vmax4:39.80757925712786; k4:0.51887193354825;

(%o100) 39.80757925712786

(%o101) 0.51887193354825

--> wxdraw2d(point_type=5,points(SV2), explicit(vmax3*'S/(k3+'S),'S, 0, 14), color=red, explicit(vmax4*'S/(k4+'S),'S, 0, 14));

(%t102)



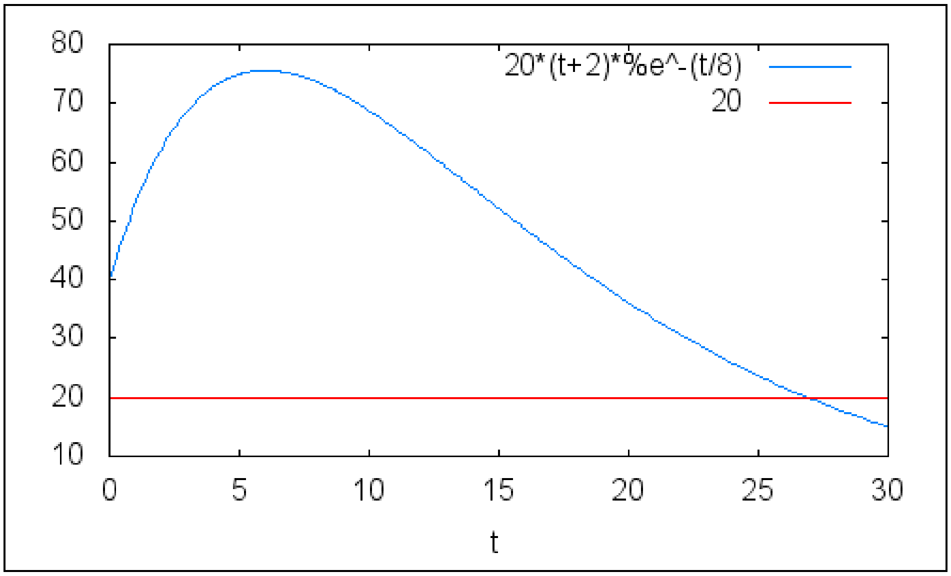
(%o102)

Ejercicio 3: modelo 2

--> D4(t):=20*(t+2)*exp(-t/8);

(%o1) $D4(t):=20\,(t+2)\exp\left(\frac{-t}{8}\right)$

--> wxplot2d([D4(t),20], [t,0,30])$

(%t114) 

--> diff(D4(t),t,1);

(%o110) $20\,\%e^{-\frac{t}{8}}-\frac{5\,(t+2)\,\%e^{-\frac{t}{8}}}{2}$

--> solve([%=0], [t]);

(%o111) $[t=6]$

--> diff(D4(t),t,2);

(%o112) $\frac{5\,(t+2)\,\%e^{-\frac{t}{8}}}{16}-5\,\%e^{-\frac{t}{8}}$

--> solve([%=0], [t]);

(%o113) $[t=14]$

--> find_root(D4(t)=20, t, 25, 30);

(%o2) 26.91483953696893

--> V4(t):=40*exp(-t/8);

(%o115) $V4(t):=40\ \exp\left(\frac{-t}{8}\right)$

--> integrate(V4(t), t);

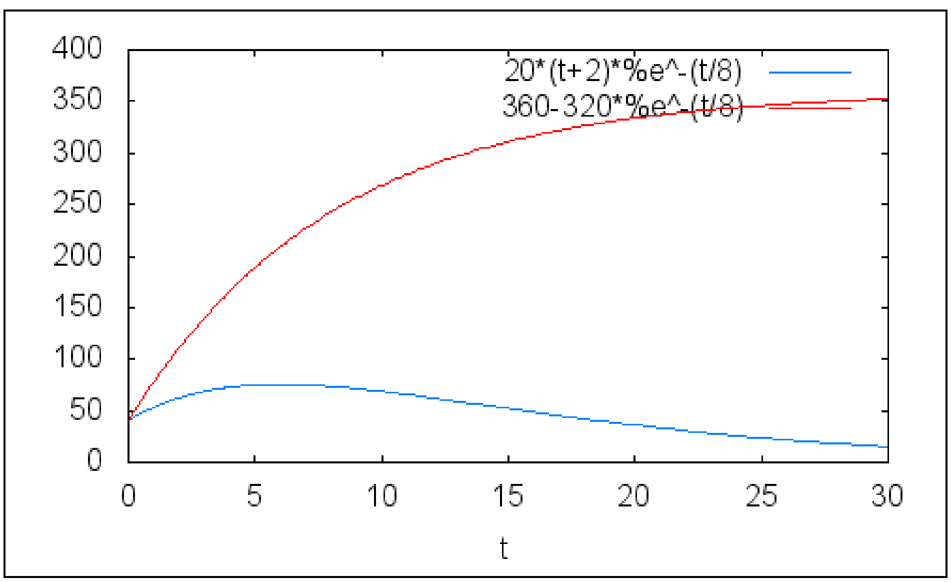
(%o116) $-320\ \%e^{-\frac{t}{8}}$

--> D5(t):=360-320*exp(-t/8);

(%o118) $D5(t):=360-320\ \exp\left(\frac{-t}{8}\right)$

--> wxplot2d([D4(t),D5(t)], [t,0,30])$

(%t122)



--> D5(15),numer;

(%o124) 310.9264106096229