PARTIAL DIFFERENTIAL EQUATIONS and FOURIER SERIES
3rd year Math degree, 2020-21

Goals: *introduction to Partial Differential Equations (PDEs) and their classical resolution techniques, especially by Fourier series expansions. In particular, the student should learn how to derive, analyze and solve some of the most important PDEs, including the heat, Laplace and wave equations.*

1. Classical examples of PDEs
- The vibrating string equation. Physical derivation. The D'Alembert solution and the method of separation of variables. Boundary conditions. Basic properties: finite propagation speed, conservation of energy.
- The heat equation. Physical derivation. The Fourier method. Meaning of the boundary conditions. Basic properties: infinite propagation, conservation of energy.
- The Laplace equation: Physical meaning. Dirichlet and Neumann boundary conditions. Change to polar coordinates, and resolution by separation of variables. Main properties: maximum principles, uniqueness, mean value property.

2. The theory of Fourier series
The concept of Fourier series. First examples. The Dini criterion for pointwise convergence. Fourier series in L^2 and Parseval formula. Convolutions and approximation of the identity. The Dirichlet and Fejér kernels. Uniform convergence of Cesàro means. Some applications.

3. More on Partial Differential Equations
- Sturm-Liouville systems. Eigenvalues and bases of eigenfunctions. Bessel functions.
- The Laplace, heat and vibrating membrane equations in rectangular and circular domains. Meaning and analysis of solutions.
- The wave equation in R^2 and R^3. Explicit formulas. Properties: uniqueness, propagation domain, Huygens principle, concentration of singularities. The inhomogeneous equation and Duhamel's formula.
- Other topics: the Fourier transform, harmonic functions, examples of non-linear PDEs,...

Bibliography:

E. Stein, R. Shakarchi, *Fourier Analysis: An introduction*. Princeton Univ Press, 2003.
W. Strauss, *Partial Differential Equations, an introduction*. Wiley 2008.
R. Haberman, *EDPs, series de Fourier y problemas de contorno*, Prentice-Hall, 2003.
R. Churchill, J. Brown, *Fourier series and boundary value problems,* McGraw Hill, 2008.
L. Evans, *Partial Differential Equations*, 2nd Ed, Amer Math Soc, 2010.
I. Peral, *Primer curso de Ecuaciones en Derivadas Parciales,* Addison-Wesley, 1995
G. Folland, *Fourier Analysis and its Applications*, Amer Math Soc, 2009.

Teacher: Gustavo Garrigós **Web:** webs.um.es/gustavo.garrigos
Office: 1.10. **Tutor hours:** Tu 15:00-17:45 or by appointment
Exams: final May 28th, extra July 13th
Final evaluation: The final grade will follow from the formula

max{ 0´7 **EF** + 0´3 **EC**, **EF**} where

EF= score in final exam
EC= average score in class tests (continuous evaluation).

Besides, the students who actively participate solving exercises in the blackboard will have an additional score.