Variable Compleja, 3º Matemáticas/4º PES Test 4, 27/10/2015

Nombre: ....

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1. V ó F (justificar):

   a) $\sqrt{i-1} = i\sqrt{1-i}$ (aquí $\sqrt{\ }$ es la raíz principal)

   b) $[z^{-w}] = 1/[z^{w}]$, para todo $w \in \mathbb{C}$ y $z \neq 0$.

   c) $\log(zw) = \operatorname{Log} z + \operatorname{Log} w + 2\pi i\mathbb{Z}$

c) Verdadero: $z = |z|\cdot e^{i\theta}$, $w = |w|e^{i\theta'}$, $z\cdot w = |z|\cdot|w|\, e^{i(\theta+\theta')}$

$$\operatorname{Log}(z) = \ln|z| + i\operatorname{Arg}(z) = \ln|z| + i\theta$$

$$\operatorname{Log}(w) = \ln|w| + i\operatorname{Arg}(w) = \ln|w| + i\theta'$$

$$\log(zw) = \ln|z\cdot w| + i\arg(z\cdot w) = \ln|zw| + i(\theta+\theta'+2\pi\mathbb{Z}) = \ln|zw| + i(\theta+\theta') + 2\pi i\mathbb{Z}$$

$$\|$$

$$\operatorname{Log}(z) + \operatorname{Log}(w) + 2\pi i\mathbb{Z} = \ln|z| + i\theta + \ln|w| + i\theta' + 2\pi i\mathbb{Z} = \ln|zw| + i(\theta+\theta') + 2\pi i\mathbb{Z}.$$

b) Verdadero: $[z^{-w}] = e^{-w(\ln|z| + i\arg(z))}$

$$[z^{w}] = e^{w(\ln|z| + i\arg(z))}$$

$$\frac{1}{[z^w]} = \frac{1}{e^{w(\ln|z| + i\arg(z))}} = \left[e^{w(\ln|z| + i\arg(z))}\right]^{-1} = e^{-w(\ln|z| + i\arg(z))}$$

a) Verdadero: $r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$; $s = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$

$${}_p\sqrt{i-1} = [i-1^{1/2}]_p = r^{1/2} e^{\frac{1}{2} i \operatorname{Arg}(i-1)} = \sqrt[4]{2}\, e^{\frac{1}{2} i \frac{3\pi}{4}} = \sqrt[4]{2}\, e^{\frac{3\pi i}{8}}$$

$${}_p\sqrt{1-i} = [1-i^{1/2}]_p = s^{1/2} e^{\frac{1}{2} i \operatorname{Arg}(1-i)} = \sqrt[4]{2}\, e^{\frac{1}{2} i \frac{-\pi}{4}} = \sqrt[4]{2}\, e^{\frac{-\pi i}{8}}$$

$$i({}_p\sqrt{1-i}) = i\sqrt[4]{2}\, e^{-\frac{\pi i}{8}} = e^{\frac{i\pi}{2}} \sqrt[4]{2}\, e^{-\frac{\pi i}{8}} = \sqrt[4]{2}\, e^{\frac{\pi i}{2} - \frac{\pi i}{8}} = \sqrt[4]{2}\, e^{\frac{4\pi i - \pi i}{8}} = \sqrt[4]{2}\, e^{\frac{3\pi i}{8}}$$