Variable Compleja, 3º Matemáticas/4º PES Test 5, 10/11/2015

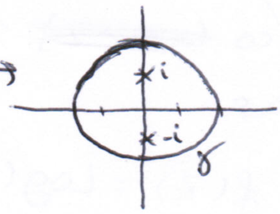
Nomb[redacted] 10

1. Calcular las siguientes integrales

$$(a) \int_{|z-1-i|=2} \frac{\operatorname{Log}(z+2)}{z^2}\,dz \qquad (b) \int_{|z|=2} \frac{2z}{z^2+1}\,dz \qquad (c) \int_{-\pi}^{\pi} e^{\cos t}\cos(\sin t)\,dt$$

(b)

$$I = \int_{|z|=2} \frac{2z}{z^2+1}\,dz = \rightarrow$$



$$\frac{A}{(z+i)} + \frac{B}{z-i} = \frac{2z}{z^2+1}$$

$z^2+1=0 \Rightarrow z = \pm i$

$$\left.\begin{array}{l} A+B=2 \\ -A+B=0 \end{array}\right\} \Rightarrow \begin{array}{l} A=1 \\ B=1 \end{array}$$

Tengo que separar la integral

$$\rightarrow = \int_\gamma \underbrace{\frac{1}{z+i}}_{I_1} + \underbrace{\frac{1}{z-i}}_{I_2}$$

$$I_1 = \int_\gamma \frac{1}{z+i}\,dz \underset{\downarrow}{=} 2\pi i\, f(-i) = 2\pi i$$

Cauchy

$f(z)=1$ (Holomorfa)

$$f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{z-z}$$

$$I_2 = \int_\gamma \frac{1}{z-i}\,dz = 2\pi i\, f(+i) = 2\pi i$$

$f(z)=1 \in H(D)$

$$\Rightarrow I = I_1 + I_2 = 2\pi i + 2\pi i = 4\pi i$$

(c)

$$\int_{-\pi}^{\pi} e^{\cos t}\cdot\cos(\operatorname{sen} t)\,dt = \longrightarrow$$

$$e^{i\operatorname{sen} t} = \cos(\operatorname{sen} t) + i\operatorname{sen}(\operatorname{sen} t) \qquad \operatorname{Re}(e^{i\operatorname{sen} t}) = \cos(\operatorname{sen} t)$$

$$\longrightarrow = \operatorname{Re}\int_{-\pi}^{\pi} e^{\cos t}\cdot e^{i\operatorname{sen} t}\,dt = \operatorname{Re}\int_{-\pi}^{\pi} e^{\overbrace{\cos t + i\operatorname{sen} t}^{e^{it}}}\,dt = \operatorname{Re}\int_{-\pi}^{\pi} e^{e^{it}}\,dt$$

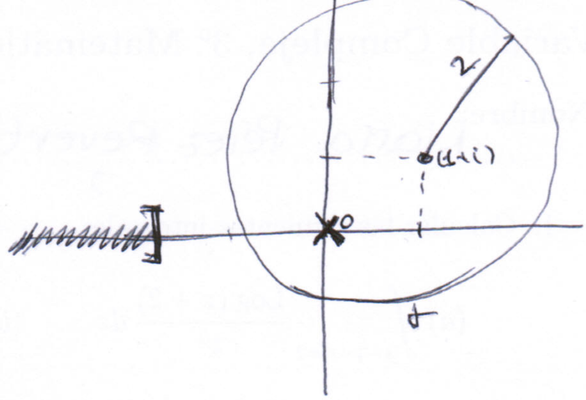
$$\left\{\begin{array}{ll} z = e^{it} & t\in[-\pi,\pi] \\ dz = ie^{it}dt = iz\,dt & \end{array}\right.$$

$$= \operatorname{Re}\int_{-\pi}^{\pi} e^z \frac{dz}{iz} = \operatorname{Re}\frac{1}{i}\int_{-\pi}^{\pi}\frac{e^z}{z}\,dz \quad (*)$$

(*) Cauchy $\longrightarrow$ $\operatorname{Re}\frac{1}{\cancel{i}}\, 2\pi\cancel{i}\, e^0 = \operatorname{Re} 2\pi = 2\pi$

$f(z)=e^z \in$ holomorfa

(a) $I = \int_{|z-1-i|=2} \frac{Log(z+2)}{z^2}\, dz$




$Log(z+2) \in H(\mathbb{C} \setminus (-\infty, -2]$

Como no toca al disco ~~$(-\infty,-2]$~~ puedo aplicar cauchy, aplicado a derivadas

$\int_\gamma$ con $n=1$, $\zeta = 0$ $\qquad f(z) = Log(z+2)$

$f'(0) = \frac{1}{2}$

$$f'(0) = \frac{1!}{2\pi i} \int_\gamma \frac{f(\zeta)}{(\zeta - z)^2}$$

$$\frac{1}{\cancel{2}} \cancel{2}\pi i = \int \frac{f(\zeta)}{(\zeta - z)^2} \qquad \Rightarrow \quad I = \pi i$$