MATEMÁTICAS II, 1º grado Ingeniería Química Test 7, 6/5/2013

Nombre: [redacted]

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1.- Consideramos la porción de la bola unidad dada por

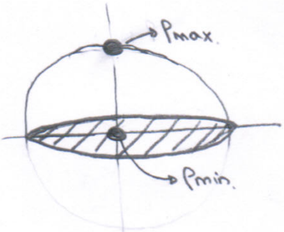
$$\Omega = \left\{ r \in [0,1],\ \varphi \in [0, \tfrac{\pi}{2}],\ \theta \in [0, 2\pi] \right\},$$

con densidad $\rho = r^2 \cos\varphi$.

~~(a)~~ Esboza Ω, indicando los puntos más densos y menos densos

~~(b)~~ Calcula la masa total de Ω

(c) Calcula la integral sobre la *superficie* de Ω de la función $f(x,y,z) = z^2$.



Semiesfera positiva

$\rho = r^2 \cos\varphi$ — $r=1,\ \varphi = \pi/2 \rightarrow \rho_{min} = 0$

— $r = 0, \rightarrow \rho_{min} = 0$

— $r = 1,\ \varphi = 0 \rightarrow \rho_{max} = 1$

b) Masa Ω = $\iiint \rho \, dx\,dy\,dz = \int_0^1\int_0^{\pi/2}\int_0^{2\pi} r^2 \cos\varphi \,\mathrm{sen}\,\varphi \, r^2 \, dr\,d\varphi\,d\theta = [\theta]_0^{2\pi} \int_0^1 r^4 dr \int_0^{\pi/2} \cos\varphi\,\mathrm{sen}\,\varphi\, d\varphi$

$[\theta]_0^{2\pi} \left[\frac{r^5}{5}\right]_0^1 \left[-\frac{\cos^2\varphi}{2}\right]_0^{\pi/2} = 2\pi \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{\pi}{5}$

$\int_0^{\pi/2} \cos\varphi\,\mathrm{sen}\,\varphi\, d\varphi \rightarrow \cos\varphi = u,\ \frac{du}{\mathrm{sen}\varphi} = d\varphi \rightarrow \int_0^{\pi/2} u \,\cancel{\mathrm{sen}\varphi}\, \frac{du}{-\cancel{\mathrm{sen}\varphi}} = \left[\frac{u^2}{2}\right]_0^{\pi/2} = \left[-\frac{\cos^2\varphi}{2}\right]_0^{\pi/2}$

c) superficie Ω donde $f(x,y,z) = z^2$ $z = \cos\varphi$

$\int_0^{2\pi}\int_0^{\pi/2} \cos^2\varphi \,\mathrm{sen}\,\varphi \, d\theta\, d\varphi = [\theta]_0^{2\pi} \int_0^{\pi/2} \cos^2\varphi\,\mathrm{sen}\,\varphi\, d\varphi \rightarrow u = \cos\varphi,\ \frac{du}{-\mathrm{sen}\varphi} = d\varphi \rightarrow 2\pi \int_0^{\pi/2} u^2 \,\cancel{\mathrm{sen}\varphi}\, \frac{du}{-\cancel{\mathrm{sen}\varphi}} =$

$= 2\pi \int_0^{\pi/2} u^2 du = 2\pi \left[-\frac{\cos^3\varphi}{3}\right]_0^{\pi/2} = \frac{2\pi}{3}$