Comparing systems by distortion functions

J. Navarro[1], Universidad de Murcia (Spain),
F.J. Samaniego, University of California, Davis (USA).

[1]Supported by Ministerio de Economía y Competitividad under Grant MTM2012-34023-FEDER.

Outline

Distortion functions

- The distorted distributions were introduced in Yaari's dual theory of choice under risk (Econometrica 55 (1987):95–115).
- The **distorted distribution** (DD) associated to a distribution function (DF) F and to an increasing continuous **distortion function** $q : [0,1] \to [0,1]$ such that $q(0) = 0$ and $q(1) = 1$, is

$$F_q(t) = q(F(t)). \tag{1.1}$$

- For the reliability functions (RF) $\overline{F} = 1 - F$, $\overline{F}_q = 1 - F_q$, we have

$$\overline{F}_q(t) = \overline{q}(\overline{F}(t)), \tag{1.2}$$

where $\overline{q}(u) = 1 - q(1-u)$ is the **dual distortion function**; see Hürlimann (2004, N Am Actuarial J).

Distortion functions

- The distorted distributions were introduced in Yaari's dual theory of choice under risk (Econometrica 55 (1987):95–115).
- The **distorted distribution** (DD) associated to a distribution function (DF) F and to an increasing continuous **distortion function** $q : [0,1] \to [0,1]$ such that $q(0) = 0$ and $q(1) = 1$, is

$$F_q(t) = q(F(t)). \tag{1.1}$$

- For the reliability functions (RF) $\overline{F} = 1 - F$, $\overline{F}_q = 1 - F_q$, we have

$$\overline{F}_q(t) = \overline{q}(\overline{F}(t)), \tag{1.2}$$

where $\overline{q}(u) = 1 - q(1-u)$ is the **dual distortion function**; see Hürlimann (2004, N Am Actuarial J).

Distortion functions

- The distorted distributions were introduced in Yaari's dual theory of choice under risk (Econometrica 55 (1987):95–115).
- The **distorted distribution** (DD) associated to a distribution function (DF) F and to an increasing continuous **distortion function** $q : [0,1] \to [0,1]$ such that $q(0) = 0$ and $q(1) = 1$, is
$$F_q(t) = q(F(t)). \tag{1.1}$$
- For the reliability functions (RF) $\overline{F} = 1 - F$, $\overline{F}_q = 1 - F_q$, we have
$$\overline{F}_q(t) = \overline{q}(\overline{F}(t)), \tag{1.2}$$
where $\overline{q}(u) = 1 - q(1-u)$ is the **dual distortion function**; see Hürlimann (2004, N Am Actuarial J).

Multivariate distortion functions

- The **generalized distorted distribution** (GDD) associated to n DF $F_1, \ldots, F_n$ and to an increasing continuous **multivariate distortion function** $Q : [0,1]^n \to [0,1]$ such that $Q(0,\ldots,0) = 0$ and $Q(1,\ldots,1) = 1$, is

$$F_Q(t) = Q(F_1(t), \ldots, F_n(t)). \tag{1.3}$$

- For the RF we have

$$\overline{F}_Q(t) = \overline{Q}(\overline{F}_1(t), \ldots, \overline{F}_n(t)), \tag{1.4}$$

where $\overline{F} = 1 - F$, $\overline{F}_Q = 1 - F_Q$ and $\overline{Q}(u_1, \ldots, u_n) = 1 - Q(1 - u_1, \ldots, 1 - u_n)$ is the **multivariate dual distortion function**; see Navarro et al. (JAP, 2011).

Multivariate distortion functions

- The **generalized distorted distribution** (GDD) associated to n DF $F_1, \ldots, F_n$ and to an increasing continuous **multivariate distortion function** $Q : [0,1]^n \to [0,1]$ such that $Q(0, \ldots, 0) = 0$ and $Q(1, \ldots, 1) = 1$, is

$$F_Q(t) = Q(F_1(t), \ldots, F_n(t)). \tag{1.3}$$

- For the RF we have

$$\overline{F}_Q(t) = \overline{Q}(\overline{F}_1(t), \ldots, \overline{F}_n(t)), \tag{1.4}$$

where $\overline{F} = 1 - F$, $\overline{F}_Q = 1 - F_Q$ and $\overline{Q}(u_1, \ldots, u_n) = 1 - Q(1 - u_1, \ldots, 1 - u_n)$ is the **multivariate dual distortion function**; see Navarro et al. (JAP, 2011).

Proportional hazard rate (PHR) model

- The PHR (Cox) model associated to a RF $\overline{F}$ is

$$\overline{F}_\alpha(t) = \left(\overline{F}(t)\right)^\alpha = \overline{q}\left(\overline{F}(t)\right)$$

for $\alpha > 0$. F_α is a DD with $\overline{q}(u) = u^\alpha$ and $q(u) = 1 - (1-u)^\alpha$.

- The proportional reversed hazard rate (PRHR) model is

$$F_\alpha(t) = \left(F(t)\right)^\alpha = q\left(F(t)\right)$$

for $\alpha > 0$. F_α is a DD with $q(u) = u^\alpha$ and $\overline{q}(u) = 1 - (1-u)^\alpha$.

Proportional hazard rate (PHR) model

- The PHR (Cox) model associated to a RF $\overline{F}$ is

$$\overline{F}_\alpha(t) = \left(\overline{F}(t)\right)^\alpha = \overline{q}\left(\overline{F}(t)\right)$$

for $\alpha > 0$. F_α is a DD with $\overline{q}(u) = u^\alpha$ and $q(u) = 1 - (1-u)^\alpha$.

- The proportional reversed hazard rate (PRHR) model is

$$F_\alpha(t) = (F(t))^\alpha = q\left(F(t)\right)$$

for $\alpha > 0$. F_α is a DD with $q(u) = u^\alpha$ and $\overline{q}(u) = 1 - (1-u)^\alpha$.

Order statistics (OS)

- $X_1,\dots,X_n$ IID$\sim F$ random variables.
- Let $X_{1:n},\dots,X_{n:n}$ be the associated OS.
- Let $F_{i:n}(t)=\Pr(X_{i:n}\le t)$ be the DF, then

$$F_{i:n}(t)=\sum_{j=i}^{n}(-1)^{j-i}\binom{n}{j}\binom{j-1}{i-1}F_{j:j}(t)=q_{i:n}(F(t)),\quad (1.5)$$

(see David and Nagaraja 2003, p. 46) where

$$F_{j:j}(t)=\Pr(X_{j:j}\le t)=\Pr(\max(X_1,\dots,X_j)\le t)=F^j(t)$$

and

$$q_{i:n}(u)=\sum_{j=i}^{n}(-1)^{j-i}\binom{n}{j}\binom{j-1}{i-1}u^j$$

is a strictly increasing polynomial in $[0,1]$.

Order statistics (OS)

- $X_1, \ldots, X_n$ IID$\sim F$ random variables.
- Let $X_{1:n}, \ldots, X_{n:n}$ be the associated OS.
- Let $F_{i:n}(t) = \Pr(X_{i:n} \leq t)$ be the DF, then

$$F_{i:n}(t) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} F_{j:j}(t) = q_{i:n}(F(t)), \quad (1.5)$$

(see David and Nagaraja 2003, p. 46) where

$$F_{j:j}(t) = \Pr(X_{j:j} \leq t) = \Pr(\max(X_1, \ldots, X_j) \leq t) = F^j(t)$$

and

$$q_{i:n}(u) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} u^j$$

is a strictly increasing polynomial in $[0, 1]$.

Order statistics (OS)

- $X_1, \dots, X_n$ IID$\sim F$ random variables.
- Let $X_{1:n}, \dots, X_{n:n}$ be the associated OS.
- Let $F_{i:n}(t) = \Pr(X_{i:n} \le t)$ be the DF, then

$$F_{i:n}(t) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} F_{j:j}(t) = q_{i:n}(F(t)), \quad (1.5)$$

(see David and Nagaraja 2003, p. 46) where

$$F_{j:j}(t) = \Pr(X_{j:j} \le t) = \Pr(\max(X_1, \dots, X_j) \le t) = F^j(t)$$

and

$$q_{i:n}(u) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} u^j$$

is a strictly increasing polynomial in $[0, 1]$.

Coherent systems- IID case

- Samaniego (IEEE TR, 1985), IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} s_i \overline{F}_{i:n}(t), \tag{1.6}$$

where $s_i = \Pr(T = X_{i:n})$.

- $\mathbf{s} = (s_1, \dots, s_n)$ is the signature of the system.
- Then T has a DD from F with

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t) = \sum_{i=1}^{n} a_i \overline{F}^i(t) = \overline{q}(\overline{F}(t)), \tag{1.7}$$

where $\overline{q}(u) = \sum_{i=1}^{n} a_i u^i$ is the domination polynomial.

- $\mathbf{a} = (a_1, \dots, a_n)$ is the minimal signature of the system, Navarro et al. (CSTM, 2007).

Coherent systems- IID case

- Samaniego (IEEE TR, 1985), IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} s_i \overline{F}_{i:n}(t), \tag{1.6}$$

where $s_i = \Pr(T = X_{i:n})$.

- $\mathbf{s} = (s_1, \ldots, s_n)$ is the signature of the system.
- Then T has a DD from F with

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t) = \sum_{i=1}^{n} a_i \overline{F}^i(t) = \overline{q}(\overline{F}(t)), \tag{1.7}$$

where $\overline{q}(u) = \sum_{i=1}^{n} a_i u^i$ is the domination polynomial.

- $\mathbf{a} = (a_1, \ldots, a_n)$ is the minimal signature of the system, Navarro et al. (CSTM, 2007).

Coherent systems- IID case

- Samaniego (IEEE TR, 1985), IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} s_i \overline{F}_{i:n}(t), \tag{1.6}$$

  where $s_i = \Pr(T = X_{i:n})$.
- $\mathbf{s} = (s_1, \ldots, s_n)$ is the signature of the system.
- Then T has a DD from F with

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t) = \sum_{i=1}^{n} a_i \overline{F}^i(t) = \overline{q}(\overline{F}(t)), \tag{1.7}$$

  where $\overline{q}(u) = \sum_{i=1}^{n} a_i u^i$ is the domination polynomial.
- $\mathbf{a} = (a_1, \ldots, a_n)$ is the minimal signature of the system, Navarro et al. (CSTM, 2007).

Coherent systems- IID case

- Samaniego (IEEE TR, 1985), IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} s_i \overline{F}_{i:n}(t), \tag{1.6}$$

where $s_i = \Pr(T = X_{i:n})$.

- $\mathbf{s} = (s_1, \ldots, s_n)$ is the signature of the system.
- Then T has a DD from F with

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t) = \sum_{i=1}^{n} a_i \overline{F}^i(t) = \overline{q}(\overline{F}(t)), \tag{1.7}$$

where $\overline{q}(u) = \sum_{i=1}^{n} a_i u^i$ is the domination polynomial.

- $\mathbf{a} = (a_1, \ldots, a_n)$ is the minimal signature of the system, Navarro et al. (CSTM, 2007).

Coherent systems- INID case

- Coolen and Coolen-Maturi (2012), r types:

$$\overline{F}_T(t) = \sum_{i_1=0}^{m_1} \cdots \sum_{i_r=0}^{m_r} \phi(i_1, \ldots, i_r) \prod_{k=1}^{r} \binom{m_k}{i_k} F_k^{m_k - i_k}(t) \overline{F}_k^{i_k}(t). \tag{1.8}$$

- Then

$$\overline{F}_T(t) = \overline{Q}(\overline{F}_1(t), \ldots, \overline{F}_r(t)) \tag{1.9}$$

where $\overline{Q}$ is a multinomial.

- If $r = 1$, $\overline{Q}$ is the domination polynomial.
- If $r = n$, then $\overline{Q}$ is the **reliability function of the structure**; see Barlow and Proschan (1975,p. 21).

Coherent systems- INID case

- Coolen and Coolen-Maturi (2012), r types:

$$\overline{F}_T(t) = \sum_{i_1=0}^{m_1} \cdots \sum_{i_r=0}^{m_r} \phi(i_1,\dots,i_r) \prod_{k=1}^{r} \binom{m_k}{i_k} F_k^{m_k-i_k}(t)\overline{F}_k^{i_k}(t). \tag{1.8}$$

- Then

$$\overline{F}_T(t) = \overline{Q}(\overline{F}_1(t),\dots,\overline{F}_r(t)) \tag{1.9}$$

where $\overline{Q}$ is a multinomial.

- If $r = 1$, $\overline{Q}$ is the domination polynomial.
- If $r = n$, then $\overline{Q}$ is the **reliability function of the structure**; see Barlow and Proschan (1975,p. 21).

Coherent systems- INID case

- Coolen and Coolen-Maturi (2012), r types:

$$\overline{F}_T(t)=\sum_{i_1=0}^{m_1}\cdots\sum_{i_r=0}^{m_r}\phi(i_1,\dots,i_r)\prod_{k=1}^{r}\binom{m_k}{i_k}F_k^{m_k-i_k}(t)\overline{F}_k^{i_k}(t). \tag{1.8}$$

- Then

$$\overline{F}_T(t)=\overline{Q}(\overline{F}_1(t),\dots,\overline{F}_r(t)) \tag{1.9}$$

where $\overline{Q}$ is a multinomial.

- If $r=1$, $\overline{Q}$ is the domination polynomial.
- If $r=n$, then $\overline{Q}$ is the **reliability function of the structure**; see Barlow and Proschan (1975,p. 21).

Coherent systems- INID case

- Coolen and Coolen-Maturi (2012), r types:

$$\overline{F}_T(t) = \sum_{i_1=0}^{m_1} \cdots \sum_{i_r=0}^{m_r} \phi(i_1, \ldots, i_r) \prod_{k=1}^{r} \binom{m_k}{i_k} F_k^{m_k - i_k}(t) \overline{F}_k^{i_k}(t). \tag{1.8}$$

- Then

$$\overline{F}_T(t) = \overline{Q}(\overline{F}_1(t), \ldots, \overline{F}_r(t)) \tag{1.9}$$

  where $\overline{Q}$ is a multinomial.
- If $r = 1$, $\overline{Q}$ is the domination polynomial.
- If $r = n$, then $\overline{Q}$ is the **reliability function of the structure**; see Barlow and Proschan (1975,p. 21).

Coherent systems-GENERAL case

- A **path set** of T is a set $P \subseteq \{1, \ldots, n\}$ such that if all the components in P work, then the system works.
- A **minimal path set** of T is a path set which does not contains other path sets.
- If $P_1, \ldots, P_m$ are the minimal path sets of T, then $T = \max_{j=1,\ldots,m} X_{P_j}$, where $X_P = \min_{i \in P} X_i$ and

$$\overline{F}_T(t) = \Pr\left(\max_{j=1,\ldots,m} X_{P_j} > t\right) = \Pr\left(\cup_{j=1}^m \{X_{P_j} > t\}\right)$$
$$= \sum_{i=1}^m \overline{F}_{P_i}(t) - \sum_{i \neq j} \overline{F}_{P_i \cup P_j}(t) + \cdots \pm \overline{F}_{P_1 \cup \cdots \cup P_m}(t)$$

where $\overline{F}_P(t) = \Pr(X_P > t)$.

Coherent systems-GENERAL case

- A **path set** of T is a set $P \subseteq \{1, \ldots, n\}$ such that if all the components in P work, then the system works.
- A **minimal path set** of T is a path set which does not contains other path sets.
- If $P_1, \ldots, P_m$ are the minimal path sets of T, then $T = \max_{j=1,\ldots,m} X_{P_j}$, where $X_P = \min_{i \in P} X_i$ and

$$\overline{F}_T(t) = \Pr\left(\max_{j=1,\ldots,m} X_{P_j} > t\right) = \Pr\left(\cup_{j=1}^m \{X_{P_j} > t\}\right)$$

$$= \sum_{i=1}^m \overline{F}_{P_i}(t) - \sum_{i \neq j} \overline{F}_{P_i \cup P_j}(t) + \cdots \pm \overline{F}_{P_1 \cup \cdots \cup P_m}(t)$$

where $\overline{F}_P(t) = \Pr(X_P > t)$.

Coherent systems-GENERAL case

- A **path set** of T is a set $P \subseteq \{1,\dots,n\}$ such that if all the components in P work, then the system works.
- A **minimal path set** of T is a path set which does not contains other path sets.
- If $P_1,\dots,P_m$ are the minimal path sets of T, then $T = \max_{j=1,\dots,m} X_{P_j}$, where $X_P = \min_{i\in P} X_i$ and

$$\overline{F}_T(t) = \Pr\left(\max_{j=1,\dots,m} X_{P_j} > t\right) = \Pr\left(\cup_{j=1}^m \{X_{P_j} > t\}\right)$$
$$= \sum_{i=1}^m \overline{F}_{P_i}(t) - \sum_{i\neq j} \overline{F}_{P_i\cup P_j}(t) + \cdots \pm \overline{F}_{P_1\cup\cdots\cup P_m}(t)$$

where $\overline{F}_P(t) = \Pr(X_P > t)$.

Coherent systems-GENERAL case

- The copula representation for the RF of $(X_1,\dots,X_n)$ is
$$\overline{\mathbf{F}}(x_1,\dots,x_n)=K(\overline{F}_1(x_1),\dots,\overline{F}_n(x_n)),$$
where $\overline{F}_i(t)=\Pr(X_i>t)$ and K is the survival copula.
- Then
$$\overline{F}_P(t)=\overline{Q}_{P,K}(\overline{F}_1(t),\dots,\overline{F}_n(t)),$$
where $\overline{Q}_{P,K}(u_1,\dots,u_n)=K(u_1^P,\dots,u_n^P)$ and $u_i^P=u_i$ for $i\in P$ and $u_i^P=1$ for $i\notin P$.
- Therefore, from the minimal path set repres., we get
$$\overline{F}_T(t)=\overline{Q}_{\phi,K}(\overline{F}_1(t),\dots,\overline{F}_n(t)).$$
- In the ID case $\overline{F}_T(t)=\overline{q}_{\phi,K}(\overline{F}(t))$.
- If there are r different types of components, then
$$\overline{F}_T(t)=\overline{Q}_{\phi,K,r}(\overline{F}_1(t),\dots,\overline{F}_r(t)).$$

Coherent systems-GENERAL case

- The copula representation for the RF of $(X_1,\ldots,X_n)$ is
$$\overline{\mathbf{F}}(x_1,\ldots,x_n)=K(\overline{F}_1(x_1),\ldots,\overline{F}_n(x_n)),$$
where $\overline{F}_i(t)=\Pr(X_i>t)$ and K is the survival copula.
- Then
$$\overline{F}_P(t)=\overline{Q}_{P,K}(\overline{F}_1(t),\ldots,\overline{F}_n(t)),$$
where $\overline{Q}_{P,K}(u_1,\ldots,u_n)=K(u_1^P,\ldots,u_n^P)$ and $u_i^P=u_i$ for $i\in P$ and $u_i^P=1$ for $i\notin P$.
- Therefore, from the minimal path set repres., we get
$$\overline{F}_T(t)=\overline{Q}_{\phi,K}(\overline{F}_1(t),\ldots,\overline{F}_n(t)).$$
- In the ID case $\overline{F}_T(t)=\overline{q}_{\phi,K}(\overline{F}(t))$.
- If there are r different types of components, then
$$\overline{F}_T(t)=\overline{Q}_{\phi,K,r}(\overline{F}_1(t),\ldots,\overline{F}_r(t)).$$

Coherent systems-GENERAL case

- The copula representation for the RF of $(X_1, \dots, X_n)$ is
$$\overline{\mathbf{F}}(x_1, \dots, x_n) = K(\overline{F}_1(x_1), \dots, \overline{F}_n(x_n)),$$
where $\overline{F}_i(t) = \Pr(X_i > t)$ and K is the survival copula.
- Then
$$\overline{F}_P(t) = \overline{Q}_{P,K}(\overline{F}_1(t), \dots, \overline{F}_n(t)),$$
where $\overline{Q}_{P,K}(u_1, \dots, u_n) = K(u_1^P, \dots, u_n^P)$ and $u_i^P = u_i$ for $i \in P$ and $u_i^P = 1$ for $i \notin P$.
- Therefore, from the minimal path set repres., we get
$$\overline{F}_T(t) = \overline{Q}_{\phi,K}(\overline{F}_1(t), \dots, \overline{F}_n(t)).$$
- In the ID case $\overline{F}_T(t) = \overline{q}_{\phi,K}(\overline{F}(t))$.
- If there are r different types of components, then
$$\overline{F}_T(t) = \overline{Q}_{\phi,K,r}(\overline{F}_1(t), \dots, \overline{F}_r(t)).$$

Coherent systems-GENERAL case

- The copula representation for the RF of $(X_1, \dots, X_n)$ is
$$\overline{\mathbf{F}}(x_1, \dots, x_n) = K(\overline{F}_1(x_1), \dots, \overline{F}_n(x_n)),$$
where $\overline{F}_i(t) = \Pr(X_i > t)$ and K is the survival copula.
- Then
$$\overline{F}_P(t) = \overline{Q}_{P,K}(\overline{F}_1(t), \dots, \overline{F}_n(t)),$$
where $\overline{Q}_{P,K}(u_1, \dots, u_n) = K(u_1^P, \dots, u_n^P)$ and $u_i^P = u_i$ for $i \in P$ and $u_i^P = 1$ for $i \notin P$.
- Therefore, from the minimal path set repres., we get
$$\overline{F}_T(t) = \overline{Q}_{\phi,K}(\overline{F}_1(t), \dots, \overline{F}_n(t)).$$
- In the ID case $\overline{F}_T(t) = \overline{q}_{\phi,K}(\overline{F}(t))$.
- If there are r different types of components, then
$$\overline{F}_T(t) = \overline{Q}_{\phi,K,r}(\overline{F}_1(t), \dots, \overline{F}_r(t)).$$

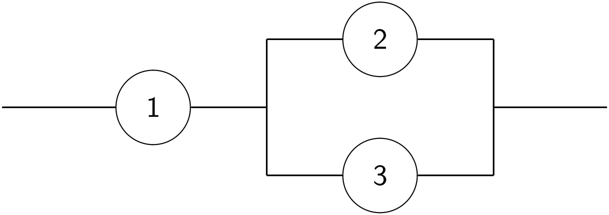
J. Navarro, E-mail: jorgenav@um.es

Coherent systems-GENERAL case

- The copula representation for the RF of $(X_1,\dots,X_n)$ is
$$\overline{\mathbf{F}}(x_1,\dots,x_n)=K(\overline{F}_1(x_1),\dots,\overline{F}_n(x_n)),$$
where $\overline{F}_i(t)=\Pr(X_i>t)$ and K is the survival copula.
- Then
$$\overline{F}_P(t)=\overline{Q}_{P,K}(\overline{F}_1(t),\dots,\overline{F}_n(t)),$$
where $\overline{Q}_{P,K}(u_1,\dots,u_n)=K(u_1^P,\dots,u_n^P)$ and $u_i^P=u_i$ for $i\in P$ and $u_i^P=1$ for $i\notin P$.
- Therefore, from the minimal path set repres., we get
$$\overline{F}_T(t)=\overline{Q}_{\phi,K}(\overline{F}_1(t),\dots,\overline{F}_n(t)).$$
- In the ID case $\overline{F}_T(t)=\overline{q}_{\phi,K}(\overline{F}(t))$.
- If there are r different types of components, then
$$\overline{F}_T(t)=\overline{Q}_{\phi,K,r}(\overline{F}_1(t),\dots,\overline{F}_r(t)).$$

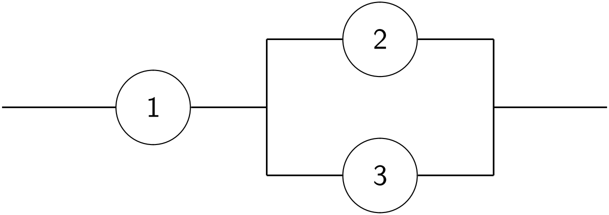
Example

Example



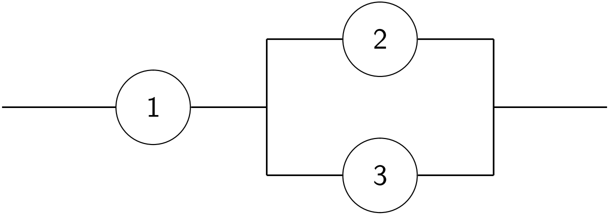
Coherent system lifetime $T = \min(X_1, \max(X_2, X_3))$.

Example



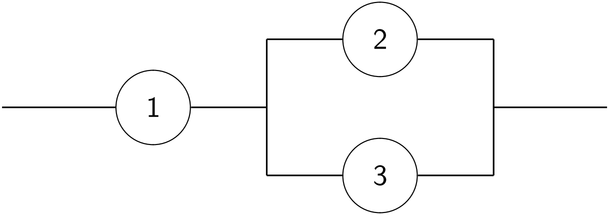
IID $\overline{F}$ cont.: $\mathbf{s} = (2/6, 4/6, 0) = (1/3, 2/3, 0)$.

Example



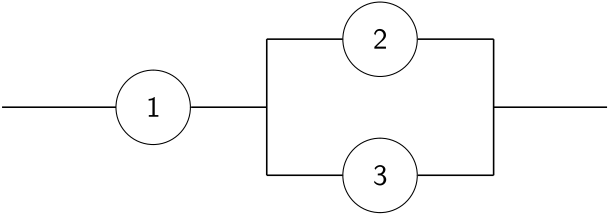
IID $\overline{F}$ cont.: $\overline{F}_T(t) = \frac{1}{3}\overline{F}_{1:3}(t) + \frac{2}{3}\overline{F}_{2:3}(t)$.

Example-general case



Coherent system lifetime $T = \max(\min(X_1, X_2), \min(X_1, X_3))$
Minimal path sets $P_1 = \{1, 2\}$ and $P_2 = \{1, 3\}$.

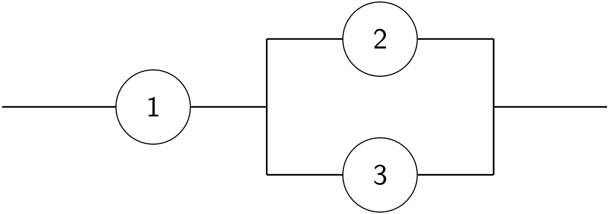
Example-general case



$$\begin{aligned}\overline{F}_T(t) &= \Pr(\{X_{\{1,2\}} > t\} \cup \{X_{\{1,3\}} > t\}) \\ &= \overline{F}_{\{1,2\}}(t) + \overline{F}_{\{1,3\}}(t) - \overline{F}_{\{1,2,3\}}(t).\end{aligned}$$

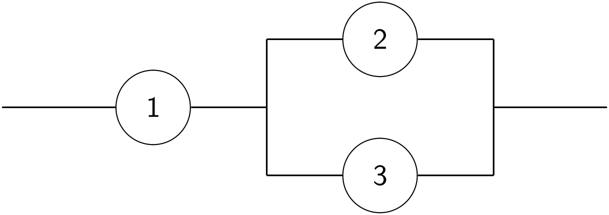
J. Navarro, E-mail: jorgenav@um.es

Example-general case



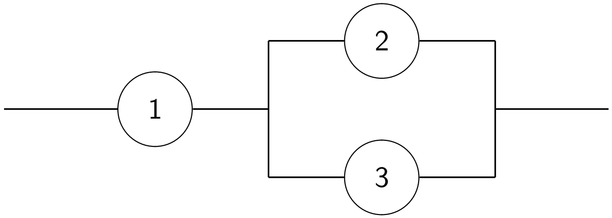
$\overline{F}_{\{1,2\}}(t) = \overline{\mathbf{F}}(t,t,0) = K(\overline{F}_1(t), \overline{F}_2(t), 1)$,...
$\overline{F}_T(t) = \overline{Q}_{\phi,K}(\overline{F}_1(t), \overline{F}_2(t), \overline{F}_3(t))$ where
$\overline{Q}_{\phi,K}(u_1,u_2,u_3) = K(u_1,u_2,1) + K(u_1,1,u_3) - K(u_1,u_2,u_3)$.

Example-general case



ID: $\overline{F}_T(t) = \overline{q}_{\phi,K}(\overline{F}(t))$,
where $\overline{q}_{\phi,K}(u) = K(u,u,1) + K(u,1,u) - K(u,u,u)$.

Example-general case



IID: $\overline{F}_T(t) = 2\overline{F}^2(t) - \overline{F}^3(t) = q_\phi(\overline{F}(t))$,
where $\overline{q}_\phi(u) = 2u^2 - u^3$ and $\mathbf{a} = (0, 2, -1)$.

Example IND components

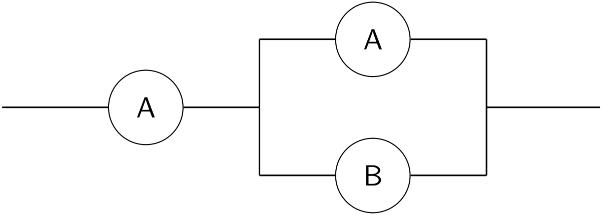


Figure: System 1.

$$\overline{F}_{T_1}(t) = \overline{Q}_{\phi,K}(\overline{F}_A(t), \overline{F}_A(t), \overline{F}_B(t)), \text{ where}$$

$$\overline{Q}_{\phi,K}(u_1, u_2, u_3) = K(u_1, u_2, 1) + K(u_1, 1, u_3) - K(u_1, u_2, u_3).$$

Example IND components

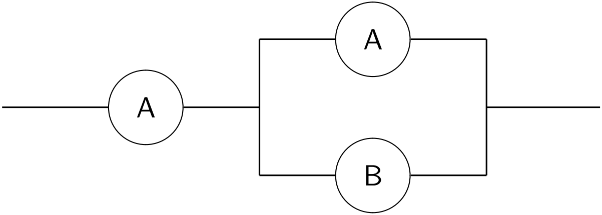


Figure: System 1.

INID: $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_A(t), \overline{F}_B(t))$, where

$$\overline{Q}_1(u_1, u_2) = u_1^2 + u_1 u_2 - u_1^2 u_2.$$

Example IND components

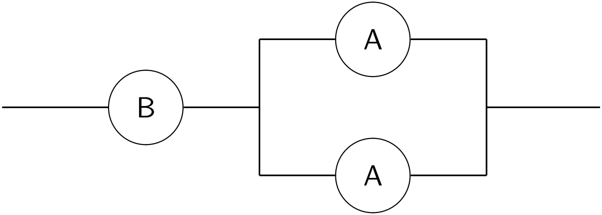


Figure: System 2.

$$\overline{F}_{T_2}(t) = \overline{Q}_{\phi,K}(\overline{F}_B(t), \overline{F}_A(t), \overline{F}_A(t)), \text{ where}$$

$$\overline{Q}_{\phi,K}(u_1, u_2, u_3) = K(u_1, u_2, 1) + K(u_1, 1, u_3) - K(u_1, u_2, u_3).$$

Example IND components

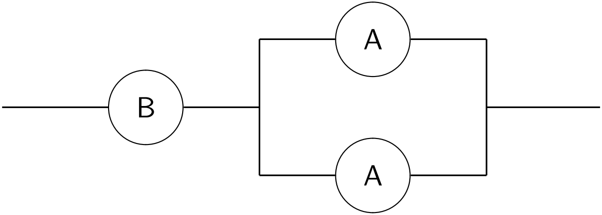


Figure: System 2.

INID: $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_A(t), \overline{F}_B(t))$, where

$$\overline{Q}_2(u_1, u_2) = 2u_1u_2 - u_1^2u_2.$$

Comparison results-DD

- If q_1 and q_2 are two DF,

$$q_1(F) \leq_{ord} q_2(F) \text{ for all } F?$$

- If q is a DF,

$$F \leq_{ord} G \Rightarrow q(F) \leq_{ord} q(G)?$$

- If Q_1 and Q_2 are two MDF,

$$Q_1(F_1, \ldots, F_n) \leq_{ord} Q_2(F_1, \ldots, F_n)?$$

- If Q is a MDF,

$$F_i \leq_{ord} G_i, i = 1, \ldots, n, \Rightarrow Q(F_1, \ldots, F_n) \leq_{ord} Q(G_1, \ldots, G_n)?$$

- Navarro, del Aguila, Sordo and Suárez-Llorens (2013, ASMBI) and (2015, MCAP) and Navarro and Gomis (2015, ASMBI).

Comparison results-DD

- If q_1 and q_2 are two DF,

$$q_1(F) \leq_{ord} q_2(F) \text{ for all } F?$$

- If q is a DF,

$$F \leq_{ord} G \Rightarrow q(F) \leq_{ord} q(G)?$$

- If Q_1 and Q_2 are two MDF,

$$Q_1(F_1, \dots, F_n) \leq_{ord} Q_2(F_1, \dots, F_n)?$$

- If Q is a MDF,

$$F_i \leq_{ord} G_i, i = 1, \dots, n, \Rightarrow Q(F_1, \dots, F_n) \leq_{ord} Q(G_1, \dots, G_n)?$$

- Navarro, del Aguila, Sordo and Suárez-Llorens (2013, ASMBI) and (2015, MCAP) and Navarro and Gomis (2015, ASMBI).

Comparison results-DD

- If q_1 and q_2 are two DF,

$$q_1(F) \leq_{ord} q_2(F) \text{ for all } F?$$

- If q is a DF,

$$F \leq_{ord} G \Rightarrow q(F) \leq_{ord} q(G)?$$

- If Q_1 and Q_2 are two MDF,

$$Q_1(F_1, \ldots, F_n) \leq_{ord} Q_2(F_1, \ldots, F_n)?$$

- If Q is a MDF,

$$F_i \leq_{ord} G_i, i = 1, \ldots, n, \Rightarrow Q(F_1, \ldots, F_n) \leq_{ord} Q(G_1, \ldots, G_n)?$$

- Navarro, del Aguila, Sordo and Suárez-Llorens (2013, ASMBI) and (2015, MCAP) and Navarro and Gomis (2015, ASMBI).

Comparison results-DD

- If q_1 and q_2 are two DF,

$$q_1(F) \leq_{ord} q_2(F) \text{ for all } F?$$

- If q is a DF,

$$F \leq_{ord} G \Rightarrow q(F) \leq_{ord} q(G)?$$

- If Q_1 and Q_2 are two MDF,

$$Q_1(F_1, \ldots, F_n) \leq_{ord} Q_2(F_1, \ldots, F_n)?$$

- If Q is a MDF,

$$F_i \leq_{ord} G_i, i = 1, \ldots, n, \Rightarrow Q(F_1, \ldots, F_n) \leq_{ord} Q(G_1, \ldots, G_n)?$$

- Navarro, del Aguila, Sordo and Suárez-Llorens (2013, ASMBI) and (2015, MCAP) and Navarro and Gomis (2015, ASMBI).

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc}
X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y)
\end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc}
X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y)
\end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc}
X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y)
\end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t | X > t) \leq_{ST} (Y - t | Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t | X > t) \leq E(Y - t | Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X | X < t) \geq_{ST} (t - Y | Y < t)$ for all t.
- Then

$$\begin{array}{ccccc} X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\ \Downarrow & & \Downarrow & & \Downarrow \\ X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y) \end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc} X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\ \Downarrow & & \Downarrow & & \Downarrow \\ X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y) \end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc} X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\ \Downarrow & & \Downarrow & & \Downarrow \\ X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y) \end{array}$$

Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{MRL} Y \Leftrightarrow E(X - t|X > t) \leq E(Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{RHR} Y \Leftrightarrow (t - X|X < t) \geq_{ST} (t - Y|Y < t)$ for all t.
- Then

$$\begin{array}{ccccc}
X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & E(X) \leq E(Y)
\end{array}$$

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}'_2/\overline{q}'_1$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}'_2/\overline{q}'_1$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}_2'/\overline{q}_1'$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}'_2/\overline{q}'_1$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}'_2/\overline{q}'_1$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-DD

- If T_i has the RF $\overline{q}_i(\overline{F}(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $\overline{q}_2 - \overline{q}_1 \geq 0$ in $(0, 1)$.
- $T_1 \leq_{HR} T_2$ for all F if and only if $\overline{q}_2/\overline{q}_1$ decreases in $(0, 1)$.
- $T_1 \leq_{RHR} T_2$ for all F if and only if q_2/q_1 increases in $(0, 1)$.
- $T_1 \leq_{LR} T_2$ for all F if and only if $\overline{q}_2'/\overline{q}_1'$ decreases in $(0, 1)$.
- $T_1 \leq_{MRL} T_2$ for all F such that $E(T_1) \leq E(T_2)$ if $\overline{q}_2/\overline{q}_1$ is bathtub in $(0, 1)$.

Preservation of stochastic orders-GDD

- If T_i has RF $\overline{Q}_i(\overline{F}_1,\dots,\overline{F}_r)$, $i=1,2$, then:
- $T_1 \leq_{ST} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if $\overline{Q}_2 - \overline{Q}_1 \geq 0$ in $(0,1)^r$.
- $T_1 \leq_{HR} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if $\overline{Q}_2/\overline{Q}_1$ is decreasing in $(0,1)^r$.
- $T_1 \leq_{RHR} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if Q_2/Q_1 is increasing in $(0,1)^r$.

Preservation of stochastic orders-GDD

- If T_i has RF $\overline{Q}_i(\overline{F}_1, \ldots, \overline{F}_r)$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all $\overline{F}_1, \ldots, \overline{F}_r$ if and only if $\overline{Q}_2 - \overline{Q}_1 \geq 0$ in $(0, 1)^r$.
- $T_1 \leq_{HR} T_2$ for all $\overline{F}_1, \ldots, \overline{F}_r$ if and only if $\overline{Q}_2 / \overline{Q}_1$ is decreasing in $(0, 1)^r$.
- $T_1 \leq_{RHR} T_2$ for all $\overline{F}_1, \ldots, \overline{F}_r$ if and only if Q_2 / Q_1 is increasing in $(0, 1)^r$.

Preservation of stochastic orders-GDD

- If T_i has RF $\overline{Q}_i(\overline{F}_1, \dots, \overline{F}_r)$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all $\overline{F}_1, \dots, \overline{F}_r$ if and only if $\overline{Q}_2 - \overline{Q}_1 \geq 0$ in $(0, 1)^r$.
- $T_1 \leq_{HR} T_2$ for all $\overline{F}_1, \dots, \overline{F}_r$ if and only if $\overline{Q}_2 / \overline{Q}_1$ is decreasing in $(0, 1)^r$.
- $T_1 \leq_{RHR} T_2$ for all $\overline{F}_1, \dots, \overline{F}_r$ if and only if Q_2 / Q_1 is increasing in $(0, 1)^r$.

Preservation of stochastic orders-GDD

- If T_i has RF $\overline{Q}_i(\overline{F}_1,\dots,\overline{F}_r)$, $i=1,2$, then:
- $T_1 \le_{ST} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if $\overline{Q}_2-\overline{Q}_1\ge 0$ in $(0,1)^r$.
- $T_1 \le_{HR} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if $\overline{Q}_2/\overline{Q}_1$ is decreasing in $(0,1)^r$.
- $T_1 \le_{RHR} T_2$ for all $\overline{F}_1,\dots,\overline{F}_r$ if and only if Q_2/Q_1 is increasing in $(0,1)^r$.

Example-System 1 and 2 INID components.

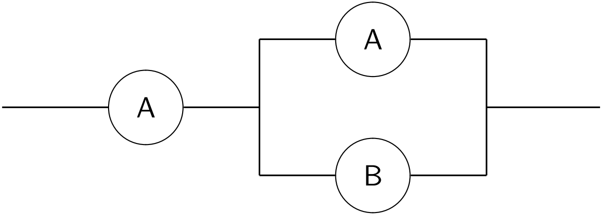


Figure: System 1.

Example-System 1 and 2 INID components.

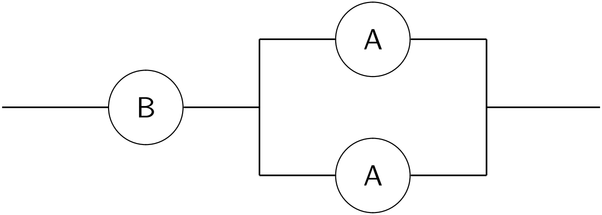


Figure: System 2.

Example-System 1 and 2 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- Then $T_1 \leq_{ST} T_2$ holds for all F_A, F_B if and only if

$$D(x,y) = \overline{Q}_2(x,y) - \overline{Q}_1(x,y) = x(y-x) \geq 0$$

  in $(0,1)^2$.
- $T_1 \leq_{ST} T_2$ holds if and only if $x \leq y$, that is, for all $\overline{F}_A \leq \overline{F}_B$.
- They are not HR ordered since

$$\frac{\overline{Q}_2(x,y)}{\overline{Q}_1(x,y)} = \frac{2-x}{x+y-xy}y$$

  is increasing in y and decreasing in x in the set $(0,1)^2$.

Example-System 1 and 2 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- Then $T_1 \leq_{ST} T_2$ holds for all F_A, F_B if and only if

$$D(x,y) = \overline{Q}_2(x,y) - \overline{Q}_1(x,y) = x(y-x) \geq 0$$

  in $(0,1)^2$.
- $T_1 \leq_{ST} T_2$ holds if and only if $x \leq y$, that is, for all $\overline{F}_A \leq \overline{F}_B$.
- They are not HR ordered since

$$\frac{\overline{Q}_2(x,y)}{\overline{Q}_1(x,y)} = \frac{2-x}{x+y-xy}y$$

  is increasing in y and decreasing in x in the set $(0,1)^2$.

Example-System 1 and 2 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- Then $T_1 \leq_{ST} T_2$ holds for all F_A, F_B if and only if

$$D(x,y) = \overline{Q}_2(x,y) - \overline{Q}_1(x,y) = x(y-x) \geq 0$$

  in $(0,1)^2$.
- $T_1 \leq_{ST} T_2$ holds if and only if $x \leq y$, that is, for all $\overline{F}_A \leq \overline{F}_B$.
- They are not HR ordered since

$$\frac{\overline{Q}_2(x,y)}{\overline{Q}_1(x,y)} = \frac{2-x}{x+y-xy}y$$

  is increasing in y and decreasing in x in the set $(0,1)^2$.

Example-System 1 and 2 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- Then $T_1 \le_{ST} T_2$ holds for all F_A, F_B if and only if

$$D(x,y) = \overline{Q}_2(x,y) - \overline{Q}_1(x,y) = x(y-x) \ge 0$$

  in $(0,1)^2$.
- $T_1 \le_{ST} T_2$ holds if and only if $x \le y$, that is, for all $\overline{F}_A \le \overline{F}_B$.
- They are not HR ordered since

$$\frac{\overline{Q}_2(x,y)}{\overline{Q}_1(x,y)} = \frac{2-x}{x+y-xy}y$$

  is increasing in y and decreasing in x in the set $(0,1)^2$.

Example-System 1 and 2 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- Then $T_1 \leq_{ST} T_2$ holds for all F_A, F_B if and only if

$$D(x,y) = \overline{Q}_2(x,y) - \overline{Q}_1(x,y) = x(y-x) \geq 0$$

  in $(0,1)^2$.
- $T_1 \leq_{ST} T_2$ holds if and only if $x \leq y$, that is, for all $\overline{F}_A \leq \overline{F}_B$.
- They are not HR ordered since

$$\frac{\overline{Q}_2(x,y)}{\overline{Q}_1(x,y)} = \frac{2-x}{x+y-xy}y$$

  is increasing in y and decreasing in x in the set $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x, y) = x^2 + xy - x^2y$.
- X_A has DF F_A, then $\overline{Q}_A(x, y) = x$.
- $T_1 \leq_{HR} X_A$ holds for all F_A, F_B since

$$\frac{\overline{Q}_A(x,y)}{\overline{Q}_1(x,y)} = \frac{x}{x^2 + xy - x^2y} = \frac{1}{x + y - xy}$$

is decreasing in x and y in the set $(0, 1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_A has DF F_A, then $\overline{Q}_A(x,y) = x$.
- $T_1 \leq_{HR} X_A$ holds for all F_A, F_B since

$$\frac{\overline{Q}_A(x,y)}{\overline{Q}_1(x,y)} = \frac{x}{x^2+xy-x^2y} = \frac{1}{x+y-xy}$$

is decreasing in x and y in the set $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_A has DF F_A, then $\overline{Q}_A(x,y) = x$.
- $T_1 \leq_{HR} X_A$ holds for all F_A, F_B since

$$\frac{\overline{Q}_A(x,y)}{\overline{Q}_1(x,y)} = \frac{x}{x^2 + xy - x^2y} = \frac{1}{x + y - xy}$$

is decreasing in x and y in the set $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_B has DF F_B, then $\overline{Q}_B(x,y) = y$.
- T_1 and X_B are not HR ordered (for all F_A, F_B) since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_1(x,y)} = \frac{y}{x^2 + xy - x^2y} = \frac{1}{x - x^2 + x^2/y}$$

is decreasing in x and increasing in y in the set $(0,1)^2$.

- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- $T_2 \leq_{HR} X_B$ for all F_A, F_B since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_2(x,y)} = \frac{y}{2xy - x^2y} = \frac{1}{2x - x^2}$$

is decreasing in x and y in $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_B has DF F_B, then $\overline{Q}_B(x,y) = y$.
- T_1 and X_B are not HR ordered (for all F_A, F_B) since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_1(x,y)} = \frac{y}{x^2+xy-x^2y} = \frac{1}{x-x^2+x^2/y}$$

is decreasing in x and increasing in y in the set $(0,1)^2$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- $T_2 \leq_{HR} X_B$ for all F_A, F_B since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_2(x,y)} = \frac{y}{2xy-x^2y} = \frac{1}{2x-x^2}$$

is decreasing in x and y in $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_B has DF F_B, then $\overline{Q}_B(x,y) = y$.
- T_1 and X_B are not HR ordered (for all F_A, F_B) since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_1(x,y)} = \frac{y}{x^2 + xy - x^2y} = \frac{1}{x - x^2 + x^2/y}$$

is decreasing in x and increasing in y in the set $(0,1)^2$.

- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- $T_2 \leq_{HR} X_B$ for all F_A, F_B since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_2(x,y)} = \frac{y}{2xy - x^2y} = \frac{1}{2x - x^2}$$

is decreasing in x and y in $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_B has DF F_B, then $\overline{Q}_B(x,y) = y$.
- T_1 and X_B are not HR ordered (for all F_A, F_B) since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_1(x,y)} = \frac{y}{x^2 + xy - x^2y} = \frac{1}{x - x^2 + x^2/y}$$

  is decreasing in x and increasing in y in the set $(0,1)^2$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- $T_2 \leq_{HR} X_B$ for all F_A, F_B since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_2(x,y)} = \frac{y}{2xy - x^2y} = \frac{1}{2x - x^2}$$

  is decreasing in x and y in $(0,1)^2$.

Example-System 1 and the IND components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- X_B has DF F_B, then $\overline{Q}_B(x,y) = y$.
- T_1 and X_B are not HR ordered (for all F_A, F_B) since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_1(x,y)} = \frac{y}{x^2 + xy - x^2y} = \frac{1}{x - x^2 + x^2/y}$$

is decreasing in x and increasing in y in the set $(0,1)^2$.
- T_2 has a GDD with $\overline{Q}_2(x,y) = 2xy - x^2y$.
- $T_2 \leq_{HR} X_B$ for all F_A, F_B since

$$\frac{\overline{Q}_B(x,y)}{\overline{Q}_2(x,y)} = \frac{y}{2xy - x^2y} = \frac{1}{2x - x^2}$$

is decreasing in x and y in $(0,1)^2$.

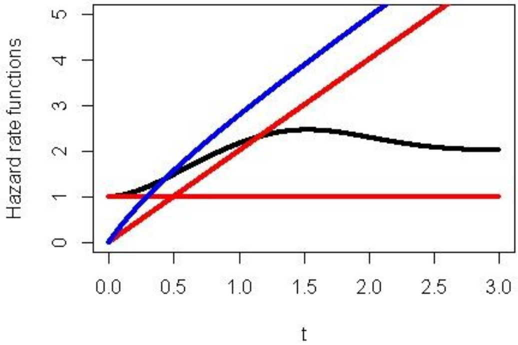


Figure: Hazard rate functions of the components (red) and the systems (T_1 black, T_2 blue) when $\overline{F}_A(t) = e^{-t}$ and $\overline{F}_B(t) = \exp(-t^2)$ for $t \geq 0$.

Example-System 1 and 3 INID components.

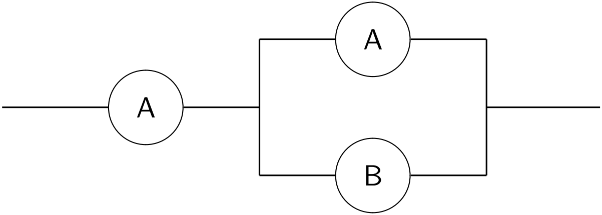


Figure: System 1.

Example-System 1 and 3 INID components.

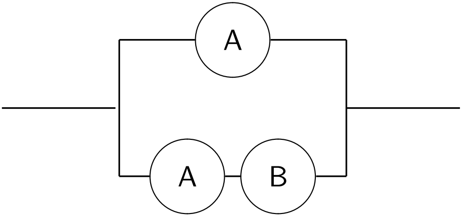


Figure: System 3.

Example-System 1 and 3 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_3 has a GDD with $\overline{Q}_3(x,y) = x + xy - x^2y$.
- $T_1 \leq_{HR} T_3$ holds for all F_A, F_B since

$$\frac{\overline{Q}_3(x,y)}{\overline{Q}_1(x,y)} = \frac{1+y-xy}{x+y-xy}$$

is decreasing in x and y in $(0,1)^2$.

Example-System 1 and 3 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_3 has a GDD with $\overline{Q}_3(x,y) = x + xy - x^2y$.
- $T_1 \leq_{HR} T_3$ holds for all F_A, F_B since

$$\frac{\overline{Q}_3(x,y)}{\overline{Q}_1(x,y)} = \frac{1+y-xy}{x+y-xy}$$

is decreasing in x and y in $(0,1)^2$.

Example-System 1 and 3 INID components.

- T_1 has a GDD with $\overline{Q}_1(x,y) = x^2 + xy - x^2y$.
- T_3 has a GDD with $\overline{Q}_3(x,y) = x + xy - x^2y$.
- $T_1 \leq_{HR} T_3$ holds for all F_A, F_B since

$$\frac{\overline{Q}_3(x,y)}{\overline{Q}_1(x,y)} = \frac{1+y-xy}{x+y-xy}$$

is decreasing in x and y in $(0,1)^2$.

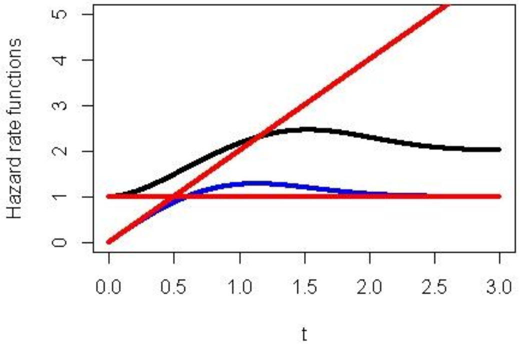


Figure: Hazard rate functions of the components (red) and the systems (T_1 black, T_3 blue) when $\overline{F}_A(t) = e^{-t}$ and $\overline{F}_B(t) = \exp(-t^2)$ for $t \geq 0$.

RR-plots INID case with $r = 2$.

- T_1 with RF $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_1(t), \overline{F}_2(t))$.
- T_2 with RF $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_1(t), \overline{F}_2(t))$.
- Domination region

$$C = \{(x, y) \in [0, 1]^2 : D(x, y) \geq 0\}$$

  where $D(x, y) = \overline{Q}_2(x, y) - \overline{Q}_1(x, y)$.
- $T_1 \leq_{ST} T_2$ holds if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- RR-plot: $(\overline{F}_1(t), \overline{F}_2(t))$ for $t \in (0, \infty)$.

RR-plots INID case with $r = 2$.

- T_1 with RF $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_1(t), \overline{F}_2(t))$.
- T_2 with RF $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_1(t), \overline{F}_2(t))$.
- Domination region

$$C = \{(x, y) \in [0, 1]^2 : D(x, y) \geq 0\}$$

where $D(x, y) = \overline{Q}_2(x, y) - \overline{Q}_1(x, y)$.

- $T_1 \leq_{ST} T_2$ holds if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- RR-plot: $(\overline{F}_1(t), \overline{F}_2(t))$ for $t \in (0, \infty)$.

RR-plots INID case with $r = 2$.

- T_1 with RF $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_1(t), \overline{F}_2(t))$.
- T_2 with RF $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_1(t), \overline{F}_2(t))$.
- Domination region

$$C = \{(x, y) \in [0, 1]^2 : D(x, y) \geq 0\}$$

where $D(x, y) = \overline{Q}_2(x, y) - \overline{Q}_1(x, y)$.

- $T_1 \leq_{ST} T_2$ holds if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- RR-plot: $(\overline{F}_1(t), \overline{F}_2(t))$ for $t \in (0, \infty)$.

RR-plots INID case with $r = 2$.

- T_1 with RF $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_1(t), \overline{F}_2(t))$.
- T_2 with RF $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_1(t), \overline{F}_2(t))$.
- Domination region

$$C = \{(x, y) \in [0, 1]^2 : D(x, y) \geq 0\}$$

  where $D(x, y) = \overline{Q}_2(x, y) - \overline{Q}_1(x, y)$.
- $T_1 \leq_{ST} T_2$ holds if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- RR-plot: $(\overline{F}_1(t), \overline{F}_2(t))$ for $t \in (0, \infty)$.

RR-plots INID case with $r = 2$.

- T_1 with RF $\overline{F}_{T_1}(t) = \overline{Q}_1(\overline{F}_1(t), \overline{F}_2(t))$.
- T_2 with RF $\overline{F}_{T_2}(t) = \overline{Q}_2(\overline{F}_1(t), \overline{F}_2(t))$.
- Domination region

$$C = \{(x, y) \in [0, 1]^2 : D(x, y) \geq 0\}$$

  where $D(x, y) = \overline{Q}_2(x, y) - \overline{Q}_1(x, y)$.
- $T_1 \leq_{ST} T_2$ holds if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- RR-plot: $(\overline{F}_1(t), \overline{F}_2(t))$ for $t \in (0, \infty)$.

Example

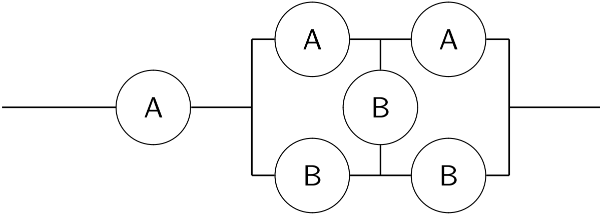


Figure: System 1 in Frank's talk.

Example

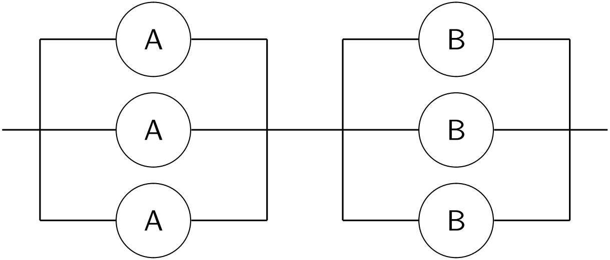


Figure: System 2 in Frank's talk.

Example

- System 1:

$$\overline{Q}_1(x,y) = x^3 + xy^2 + 2x^2y^2 - 3x^3y^2 - 2x^2y^3 + 2x^3y^3$$

- System 2:

$$\overline{Q}_2(x,y) = 9xy - 9xy^2 + 3xy^3 - 9x^2y + 9x^2y^2 - 3x^2y^3 + 3x^3y - 3x^3y^2 + x^3y^3.$$

- Difference

$$D(x,y) = 9xy - 10xy^2 + 3xy^3 - 9x^2y + 7x^2y^2 - x^2y^3 + 3x^3y - x^3y^3 - x^3.$$

- $T_1 \leq_{ST} T_2$ whenever $X_A \leq_{ST} X_B$.

Example

- System 1:

$$\overline{Q}_1(x,y) = x^3 + xy^2 + 2x^2y^2 - 3x^3y^2 - 2x^2y^3 + 2x^3y^3$$

- System 2:

$$\overline{Q}_2(x,y) = 9xy - 9xy^2 + 3xy^3 - 9x^2y + 9x^2y^2 - 3x^2y^3 + 3x^3y - 3x^3y^2 + x^3y^3.$$

- Difference

$$D(x,y) = 9xy - 10xy^2 + 3xy^3 - 9x^2y + 7x^2y^2 - x^2y^3 + 3x^3y - x^3y^3 - x^3.$$

- $T_1 \leq_{ST} T_2$ whenever $X_A \leq_{ST} X_B$.

Example

- System 1:

$$\overline{Q}_1(x,y) = x^3 + xy^2 + 2x^2y^2 - 3x^3y^2 - 2x^2y^3 + 2x^3y^3$$

- System 2:

$$\overline{Q}_2(x,y) = 9xy - 9xy^2 + 3xy^3 - 9x^2y + 9x^2y^2 - 3x^2y^3 + 3x^3y - 3x^3y^2 + x^3y^3.$$

- Difference

$$D(x,y) = 9xy - 10xy^2 + 3xy^3 - 9x^2y + 7x^2y^2 - x^2y^3 + 3x^3y - x^3y^3 - x^3.$$

- $T_1 \leq_{ST} T_2$ whenever $X_A \leq_{ST} X_B$.

Example

- System 1:

$$\overline{Q}_1(x,y) = x^3 + xy^2 + 2x^2y^2 - 3x^3y^2 - 2x^2y^3 + 2x^3y^3$$

- System 2:

$$\overline{Q}_2(x,y) = 9xy - 9xy^2 + 3xy^3 - 9x^2y + 9x^2y^2 - 3x^2y^3 + 3x^3y - 3x^3y^2 + x^3y^3.$$

- Difference

$$D(x,y) = 9xy - 10xy^2 + 3xy^3 - 9x^2y + 7x^2y^2 - x^2y^3 + 3x^3y - x^3y^3 - x^3.$$

- $T_1 \leq_{ST} T_2$ whenever $X_A \leq_{ST} X_B$.

Domination region and RR-plots.

- $T_1 \leq_{ST} T_2$ if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- We consider $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue),
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red) and
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green).

Domination region and RR-plots.

- $T_1 \leq_{ST} T_2$ if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- We consider $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue),
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red) and
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green).

Domination region and RR-plots.

- $T_1 \leq_{ST} T_2$ if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- We consider $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue),
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red) and
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green).

Domination region and RR-plots.

- $T_1 \leq_{ST} T_2$ if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- We consider $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue),
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red) and
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green).

J. Navarro, E-mail: jorgenav@um.es

Domination region and RR-plots.

- $T_1 \leq_{ST} T_2$ if and only if $(\overline{F}_1(t), \overline{F}_2(t)) \in C$ for all t.
- We consider $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue),
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red) and
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green).

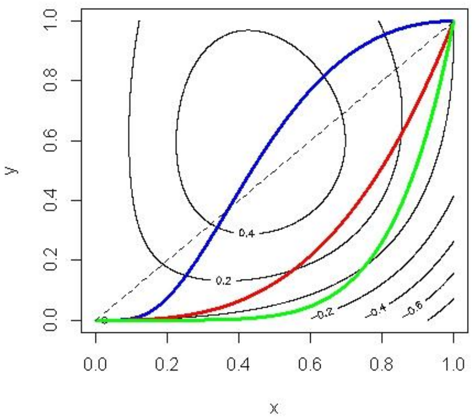


Figure: Domination region and RR-plots.

Ordering properties

- $T_1 \leq_{ST} T_2$ if $\overline{F}_1(t) \leq \overline{F}_2(t)$ for all t (above the diagonal).
- If $\overline{F}_1(t) = \exp(-t)$ and:
- Case 1: $\overline{F}_2(t) = \exp(-t^2)$ (blue), then $T_1 \leq_{ST} T_2$.
- Case 2: $\overline{F}_2(t) = \exp(-3t)$ (red), then $T_1 \leq_{ST} T_2$.
- Case 3: $\overline{F}_2(t) = \exp(-6t)$ (green), then T_1 and T_2 not ST ordered.
- There exist cases in which $T_1 \geq_{ST} T_2$.

Ordering properties IID case

- T_1 with minimal signature $(p_1, \ldots, p_n)$ IID comp.
- T_2 with minimal signature $(q_1, \ldots, q_n)$ IID comp.
- $T_1 \leq_{ST} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n} (q_i - p_i)x^i \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{HR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (j-i)(p_j q_i - p_i q_j)x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{LR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} ij(j-i)(p_j q_i - p_i q_j)x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

Ordering properties IID case

- T_1 with minimal signature $(p_1, \dots, p_n)$ IID comp.
- T_2 with minimal signature $(q_1, \dots, q_n)$ IID comp.
- $T_1 \leq_{ST} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n} (q_i - p_i) x^i \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{HR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (j-i)(p_j q_i - p_i q_j) x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{LR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} ij(j-i)(p_j q_i - p_i q_j) x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

Ordering properties IID case

- T_1 with minimal signature $(p_1,\dots,p_n)$ IID comp.
- T_2 with minimal signature $(q_1,\dots,q_n)$ IID comp.
- $T_1 \le_{ST} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n}(q_i-p_i)x^i \ge 0 \text{ for all } x\in(0,1).$$

- $T_1 \le_{HR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(j-i)(p_jq_i-p_iq_j)x^{i+j-2} \ge 0 \text{ for all } x\in(0,1).$$

- $T_1 \le_{LR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}ij(j-i)(p_jq_i-p_iq_j)x^{i+j-2} \ge 0 \text{ for all } x\in(0,1).$$

Ordering properties IID case

- T_1 with minimal signature $(p_1, \ldots, p_n)$ IID comp.
- T_2 with minimal signature $(q_1, \ldots, q_n)$ IID comp.
- $T_1 \leq_{ST} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n}(q_i - p_i)x^i \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{HR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(j-i)(p_j q_i - p_i q_j)x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{LR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} ij(j-i)(p_j q_i - p_i q_j)x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

Ordering properties IID case

- T_1 with minimal signature $(p_1, \dots, p_n)$ IID comp.
- T_2 with minimal signature $(q_1, \dots, q_n)$ IID comp.
- $T_1 \leq_{ST} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n} (q_i - p_i) x^i \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{HR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (j-i)(p_j q_i - p_i q_j) x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

- $T_1 \leq_{LR} T_2$ holds for all F if and only if

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} ij(j-i)(p_j q_i - p_i q_j) x^{i+j-2} \geq 0 \text{ for all } x \in (0,1).$$

Our Main References

- Samaniego and Navarro (2016). On comparing coherent systems with heterogeneous components. To appear in Advances in Applied Probability 48(1).
- Navarro (2015). Stochastic comparisons of generalized mixtures and coherent systems. To appear in TEST. Published online first (May 2015). DOI 10.1007/s11749-015-0443-5.
- Navarro, Samaniego and Balakrishnan (2011). Signature-based representations for the reliability of systems with heterogeneous components. Journal of Applied Probability 48, 856–867.
- Navarro, Samaniego, Balakrishnan and Bhattacharya (2008). Applications and extensions of system signatures in engineering reliability. Naval Research Logistics 55, 313–327.

Our Main References

- Samaniego and Navarro (2016). On comparing coherent systems with heterogeneous components. To appear in Advances in Applied Probability 48(1).
- Navarro (2015). Stochastic comparisons of generalized mixtures and coherent systems. To appear in TEST. Published online first (May 2015). DOI 10.1007/s11749-015-0443-5.
- Navarro, Samaniego and Balakrishnan (2011). Signature-based representations for the reliability of systems with heterogeneous components. Journal of Applied Probability 48, 856–867.
- Navarro, Samaniego, Balakrishnan and Bhattacharya (2008). Applications and extensions of system signatures in engineering reliability. Naval Research Logistics 55, 313–327.

Our Main References: Distorted distributions

- Navarro, del Aguila, Sordo and Suarez-Llorens (2013). Stochastic ordering properties for systems with dependent identically distributed components. Appl Stoch Mod Bus Ind 29, 264–278.
- Navarro, del Aguila, Sordo, Suarez-Llorens (2015). Preservation of stochastic orders under the formation of generalized distorted distributions. Applications to coherent systems. To appear in Methodology and Computing in Applied Probability. DOI: 10.1007/s11009-015-9441-z.
- Navarro and Spizzichino (2010). Comparisons of series and parallel systems with components sharing the same copula. Appl Stoch Mod Bus Ind 26, 775–791.

Our Main References: Distorted distributions

- Navarro, Pellerey and Di Crescenzo (2015). Orderings of coherent systems with randomized dependent components. European Journal of Operational Research 240, 127–139.
- Navarro and Gomis (2015). Comparisons in the mean residual life order of coherent systems with identically distributed components. To appear in Applied Stochastic Models in Business and Industry. DOI: 10.1002/asmb.2121.

References

- For the more references, please visit my personal web page:

$$https : //webs.um.es/jorgenav/$$

- Thank you for your attention!!

References

- For the more references, please visit my personal web page:

$$https://webs.um.es/jorgenav/$$

- Thank you for your attention!!