Applications of Distorted Distributions

Jorge Navarro[1],
Universidad de Murcia, Spain
E-mail: jorgenav@um.es

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Definition Distorted Distributions (DD)

- The distorted distributions were introduced in Yaari's dual theory of choice under risk (Yaari 1987, Econometrica 55:95–115).
- The *distorted distribution* (DD) associated to a distribution function (DF) F and to an increasing right-continuous *distortion function* $q : [0,1] \to [0,1]$ such that $q(0) = 0$ and $q(1) = 1$, is

$$F_q(t) = q(F(t)). \quad (1.1)$$

- If q is continuous and strictly increasing, then F and F_q have the same support.
- For the reliability functions (RF) $\overline{F} = 1 - F$, $\overline{F}_q = 1 - F_q$, we have

$$\overline{F}_q(t) = \overline{q}(\overline{F}(t)), \quad (1.2)$$

where $\overline{q}(u) = 1 - q(1-u)$ is the *dual distortion function*.

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Generalized Distorted Distributions (GDD)

- The *generalized distorted distribution* (GDD) associated to n DF $F_1, \ldots, F_n$ and to an increasing right-continuous *multivariate distortion function* $Q : [0,1]^n \to [0,1]$ such that $Q(0, \ldots, 0) = 0$ and $Q(1, \ldots, 1) = 1$, is

$$F_Q(t) = Q(F_1(t), \ldots, F_n(t)). \quad (1.3)$$

- If Q is continuous and strictly increasing and $F_1, \ldots, F_n$ have the same support, then F_Q also has the same support.
- For the RF we have

$$\overline{F}_Q(t) = \overline{Q}(\overline{F}_1(t), \ldots, \overline{F}_n(t)), \quad (1.4)$$

where $\overline{F} = 1 - F$, $\overline{F}_Q = 1 - F_Q$ and $\overline{Q}(u_1, \ldots, u_n) = 1 - Q(1 - u_1, \ldots, 1 - u_n)$ is the *multivariate dual distortion function.*
- $\overline{Q}$ is also a multivariate distortion function.

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2. Examples
   - Proportional hazard rate model
   - Order statistics
   - Coherent systems
   - Other examples
3. Preservation results
   - Stochastic orders
   - Stochastic aging classes
   - Parrondo's paradox
4. Inference results from DD

Proportional hazard rate (PHR) model

- The PHR (Cox) model associated to a RF $\overline{F}$ is

$$\overline{F}_\alpha(t) = \left(\overline{F}(t)\right)^\alpha = \overline{q}\left(\overline{F}(t)\right)$$

for $\alpha > 0$. $\overline{F}_\alpha$ a DD with $\overline{q}(u) = u^\alpha$ and $q(u) = 1 - (1-u)^\alpha$.

- The hazard (failure) rate function is defined by $h(t) = f(t)/\overline{F}(t)$ where f is the pdf.
- Under the PHR model, $h_\alpha(t) = \alpha h(t)$.
- The proportional reversed hazard rate PRHR model associated to a DF F is

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Order statistics (OS)

- $X_1, \dots, X_n$ IID$\sim F$ random variables.
- $X_1, \dots, X_n$ exchangeable (EXC), i.e., for any permutation σ

$$(X_1, \dots, X_n) =_{ST} (X_{\sigma(1)}, \dots, X_{\sigma(n)}).$$

- $(X_1, \dots, X_n)$ an arbitrary random vector with

$$\mathbf{F}(x_1, \dots, x_n) = \Pr(X_1 \leq x_1, \dots, X_n \leq x_n)$$

$$\overline{\mathbf{F}}(x_1, \dots, x_n) = \Pr(X_1 > x_1, \dots, X_n > x_n).$$

- Let $X_{1:n}, \dots, X_{n:n}$ be the associated OS.
- Let $F_{i:n}(t) = \Pr(X_{i:n} \leq t)$ be the DF.
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Distorted Distribution Representation- IID case

- In the IID case, we have

$$F_{i:n}(t) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} F_{j:j}(t) = q_{i:n}(F(t)), \quad (2.1)$$

(see David and Nagaraja 2003, p. 46) where

$$F_{j:j}(t) = \Pr(X_{j:j} \leq t) = \Pr(\max(X_1, \ldots, X_j) \leq t) = F^j(t)$$

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$$q_{i:n}(u) = \sum_{j=i}^{n} (-1)^{j-i} \binom{n}{j} \binom{j-1}{i-1} u^j$$

is a strictly increasing polinomial in $[0, 1]$.

- Both $F_{j:j}$ and $F_{i:n}$ are DD from F.

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Distorted Distribution Representation- IID case

- The upper OS $X_{j:j}$ (lifetime of the parallel system) satisfies the RPHR model with $\alpha = j$ since

$$F_{j:j}(t) = \Pr(X_{j:j} \leq t) = \Pr(\max(X_1, \ldots, X_j) \leq t) = (F(t))^j .$$

- The lower OS $X_{1:j}$ (lifetime of the series system) satisfies the PHR model

$$\overline{F}_{1:j}(t) = \Pr(X_{1:j} \leq t) = \Pr(\min(X_1, \ldots, X_j) > t) = (\overline{F}(t))^j .$$

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Distorted Distribution Representation- EXC case

- In the EXC case the left hand side of (2.1) holds with

$$F_{j:j}(t) = \Pr(\max(X_1,\dots,X_j) \le t) = \mathbf{F}(\underbrace{t,\dots,t}_{j},\underbrace{\infty,\dots,\infty}_{n-j}).$$

- The copula representation for $\mathbf{F}$ is

$$\mathbf{F}(x_1,\dots,x_n) = C(F_1(x_1),\dots,F_n(x_n)), \qquad (2.2)$$

where $F_i(t) = \Pr(X_i \le t)$ and C is the copula.

- In the EXC case, $F_1(t) = \cdots = F_n(t) = F(t)$,

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Distorted Distribution Representation- GENERAL case

- In the general case

$$F_{i:n}(t) = \Pr(X_{i:n} \le t) = \Pr\left(\bigcup_{j=1}^{r} \{X^{C_j} \le t\}\right)$$

where $X^{C_j} = \max_{k \in C_j} X_k$ and $|C_j| = i$, $j = 1, \dots, r$, $r = \binom{n}{i}$.

- Then

$$F_{i:n}(t) = \sum_{j=1}^{r} \Pr(X^{C_j} \le t) - \sum_{j \ne k} \Pr(X^{C_j \cup C_k} \le t) + \dots \pm \Pr(X^{C_1 \cup \dots \cup C_r} \le t).$$

- By using the copula representation (2.2)

$$F^A(t) = \Pr(X^A \le t) = \Pr(\max_{j \in A} X_j \le t) = C(F_1(x_1^A), \dots, F_n(x_n^A)),$$

where $x_i^A = t$ if $i \in A$ and $x_i^A = \infty$ if $i \notin A$, $A \subseteq \{1, \dots, n\}$.

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$$F_{i:n}(t) = \sum_{j=1}^{r} \Pr(X^{C_j} \le t) - \sum_{j \ne k} \Pr(X^{C_j \cup C_k} \le t) + \ldots \pm \Pr(X^{C_1 \cup \cdots \cup C_r} \le t).$$

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$$F^A(t) = \Pr(X^A \le t) = \Pr(\max_{j \in A} X_j \le t) = C(F_1(x_1^A), \ldots, F_n(x_n^A)),$$

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Distorted Distribution Representation- GENERAL case

- In the general case

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Distorted Distribution Representation- GENERAL case

- Therefore
$$F^A(t) = Q_A^C(F_1(t), \dots, F_n(t))$$
for all $A \subseteq \{1, \dots, n\}$, where $Q_A^C(u_1, \dots, u_n) = C(u_1^A, \dots, u_n^A)$ and $u_i^A = u_i$ if $i \in A$ and $u_i^A = 1$ if $i \notin A$.
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$$+ \cdots \pm Q_{C_1 \cup \cdots \cup C_r}^C(F_1(t), \dots, F_n(t))$$
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An example-General case

- Let us consider $X_{2:3}$, then $C_1 = \{1,2\}$, $C_2 = \{1,3\}$, $C_3 = \{2,3\}$

$$F_{2:3}(t) = \Pr\left(\{X^{\{1,2\}} \le t\} \cup \{X^{\{1,3\}} \le t\} \cup \{X^{\{2,3\}} \le t\}\right)$$
$$= \Pr\left(X^{\{1,2\}} \le t\right) + \Pr\left(X^{\{1,3\}} \le t\right) + \Pr\left(X^{\{2,3\}} \le t\right)$$
$$- 2\Pr\left(X^{\{1,2,3\}} \le t\right)$$
$$= \mathbf{F}(t,t,\infty) + \mathbf{F}(t,\infty,t) + \mathbf{F}(\infty,t,t) - 2\mathbf{F}(t,t,t)$$

- Then, by using the copula representation, we get

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$$- 2C(F_1(t),F_2(t),F_3(t)) = Q^C_{2:3}(F_1(t),F_2(t),F_3(t)),$$

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An example-Particular cases

- In the EXC case, we get

$$\begin{aligned} F_{2:3}(t) &= C(F(t),F(t),1) + C(F(t),1,F(t)) + C(1,F(t),F(t)) \\ &\quad - 2C(F(t),F(t),F(t)) \\ &= 3C(F(t),F(t),1) - 2C(F(t),F(t),F(t)) = q^C_{2:3}(F(t)), \end{aligned}$$

where $q^C_{2:3}(u) = 3C(u,u,1) - 2C(u,u,u)$.

- In the IID case, we get

$$F_{2:3}(t) = F^2(t) - 3F^3(t) = q_{2:3}(F(t)),$$

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- In the INID case, we get

$$\begin{aligned} F_{2:3}(t) &= F_1(t)F_2(t) + F_1(t)F_3(t) + F_2(t)F_3(t) - 2F_1(t)F_2(t)F_3(t) \\ &= Q_{2:3}(F_1(t),F_2(t),F_3(t)), \end{aligned}$$

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Coherent systems

- A coherent system is

$$\phi = \phi(x_1, \ldots, x_n) : \{0,1\}^n \to \{0,1\},$$

where $x_i \in \{0,1\}$ represents the state of the ith component and where ϕ (which represents the state of the system) is nondecreasing and strictly increasing in x_i for at least one point $(x_1, \ldots, x_n)$, for $i = 1, \ldots, n$.

- If $X_1, \ldots, X_n$ are the component lifetimes, then there exists ψ such that the system lifetime $T = \psi(X_1, \ldots, X_n)$.
- $X_{1:n}, \ldots, X_{n:n}$ are the lifetimes of k-out-of-n systems.
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Coherent systems- IID and EXC case

- Samaniego (IEEE TR, 1985), IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} p_i \overline{F}_{i:n}(t), \quad (2.4)$$

where $p_i = \Pr(T = X_{i:n})$.

- $\mathbf{p} = (p_1, \ldots, p_n)$ is the signature of the system.
- IID case: p_i only depends on ϕ

$$p_i = \frac{\left|\{\sigma : \phi(x_1, \ldots, x_n) = x_{i:n}, \text{ when } x_{\sigma(1)} < \ldots < x_{\sigma(n)}\}\right|}{n!} \quad (2.5)$$

- Navarro, Samaniego, Balakrishnan and Bhathacharya (NRL, 2008), (2.4) holds for EXC r.v. when $\mathbf{p}$ is given by (2.5).
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Generalized mixture representations

- Navarro, Ruiz and Sandoval (CSTM, 2007), EXC case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t). \quad (2.6)$$

- $\mathbf{a} = (a_1, \ldots, a_n)$ is the minimal signature of T.
- a_i only depends on ϕ but can be negative and so (2.6) is called a generalized mixture.
- In the IID case:

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}^i(t) = \overline{q}_\phi(\overline{F}(t)), \quad (2.7)$$

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Coherent systems-General case

- A **path set** of T is a set $P \subseteq \{1, \ldots, n\}$ such that if all the components in P work, then the system works.
- A **minimal path set** of T is a path set which does not contains other path sets.
- If $P_1, \ldots, P_r$ are the minimal path sets of T, then $T = \max_{j=1,\ldots,r} X_{P_j}$, where $X_P = \min_{i \in P} X_i$.

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$$= \sum_{i=1}^{r}\overline{F}_{P_i}(t) - \sum_{i \neq j}\overline{F}_{P_i \cup P_j}(t) + \cdots \pm \overline{F}_{P_1 \cup \cdots \cup P_r}(t). \tag{2.8}$$

where $\overline{F}_P(t) = \Pr(X_P > t)$.

Coherent systems-General case

- The copula representation for the RF of $(X_1,\dots,X_n)$ is

$$\overline{\mathbf{F}}(x_1,\dots,x_n)=K(\overline{F}_1(x_1),\dots,\overline{F}(x_n)),$$

where $\overline{F}_i(t)=\Pr(X_i>t)$ and K is the survival copula.

- Then

$$\overline{F}_P(t)=Q_{P,K}(\overline{F}_1(t),\dots,\overline{F}_n(t))$$

where $Q_{P,K}(u_1^P,\dots,u_n^P)$ and $u_i^P=u_i$ for $i\in P$ and $u_i^P=1$ for $i\notin P$.

- Therefore, from (2.8), we get

$$\overline{F}_T(t)=Q_{\phi,K}(\overline{F}_1(t),\dots,\overline{F}_n(t)).$$

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Example

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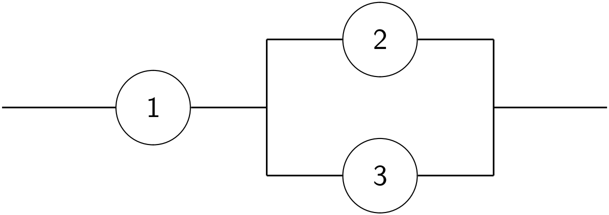
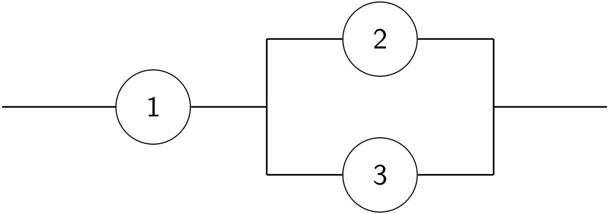


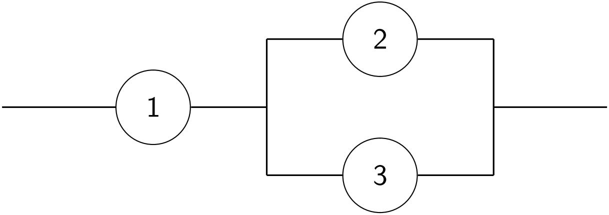
Coherent system lifetime $T = \min(X_1, \max(X_2, X_3))$.

Example



$3! = 6$ permutations.

Example



$$X_1 < X_2 < X_3 \Rightarrow T = X_1 = X_{1:3}$$

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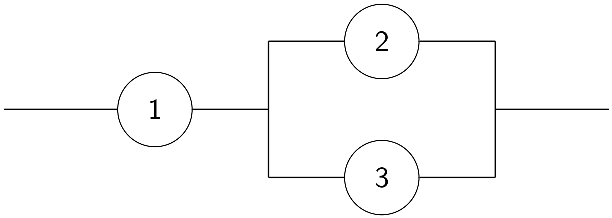

$$X_1 < X_3 < X_2 \Rightarrow T = X_1 = X_{1:3}$$

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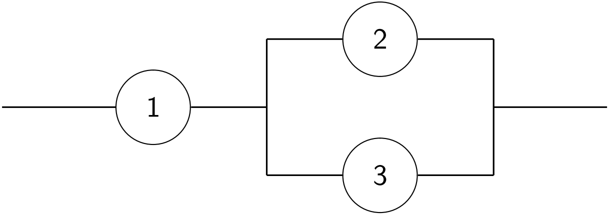
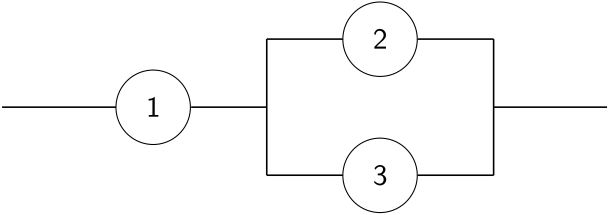


$$X_2 < X_1 < X_3 \Rightarrow T = X_1 = X_{2:3}$$

Example



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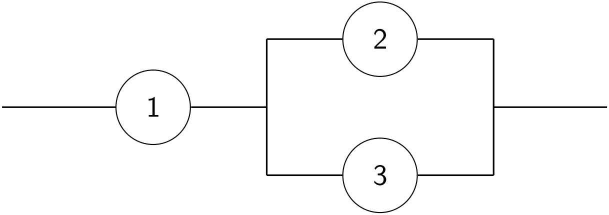

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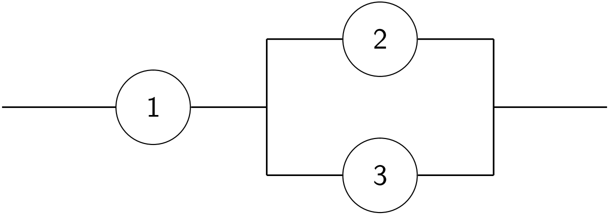

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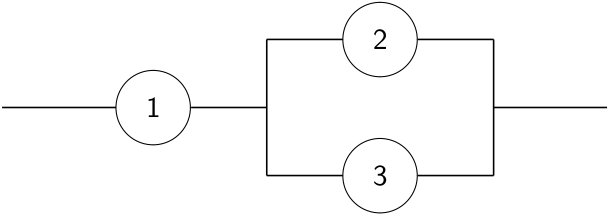
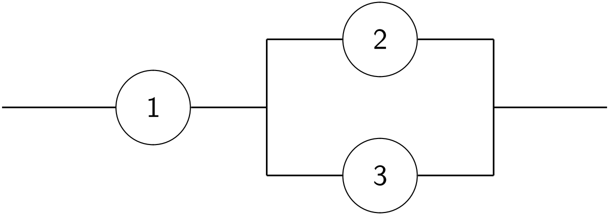


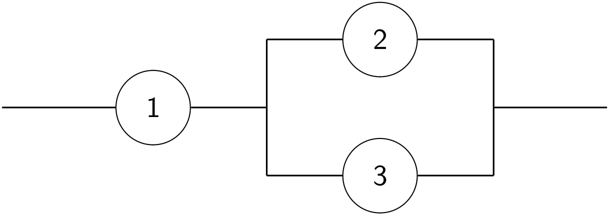
IID $\overline{F}$ cont.: $\mathbf{p} = (2/6, 4/6, 0) = (1/3, 2/3, 0)$.

Example



IID $\overline{F}$ cont.: $\overline{F}_T(t) = \frac{1}{3}\overline{F}_{1:3}(t) + \frac{2}{3}\overline{F}_{2:3}(t)$.

Example-general case



Coherent system lifetime $T = \max(\min(X_1, X_2), \min(X_1, X_3))$.

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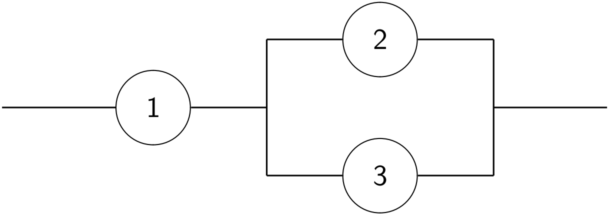

Minimal path sets $P_1 = \{1, 2\}$ and $P_1 = \{1, 3\}$.

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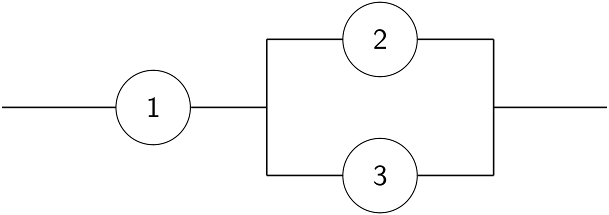

$$\overline{F}_T(t) = \Pr(\{X_{\{1,3\}} > t\} \cup \{X_{\{1,2\}} > t\})$$
$$= \overline{F}_{\{1,2\}}(t) + \overline{F}_{\{1,3\}}(t) - \overline{F}_{\{1,2,3\}}(t).$$

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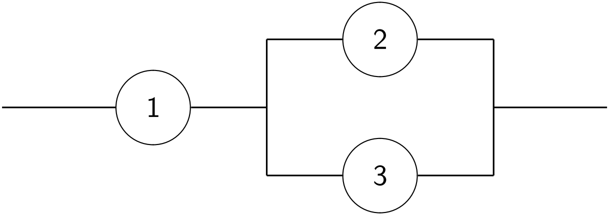
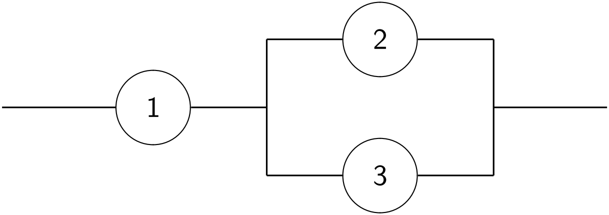


$\overline{F}_{\{1,2\}}(t) = \overline{\mathbf{F}}(t,t,0) = K(\overline{F}_1(t),\overline{F}_2(t),1),\ldots$
$\overline{F}_T(t) = Q_{\phi,K}(\overline{F}_1(t),\overline{F}_2(t),\overline{F}_3(t))$ where
$Q_{\phi,K}(u_1,u_2,u_3) = K(u_1,u_2,1) + K(u_1,1,u_3) - K(u_1,u_2,u_3).$

Example-general case



EXC: $\overline{F}_T(t) = 2\overline{F}_{1:2}(t) - \overline{F}_{1:3}(t) = q_{\phi,K}(\overline{F}(t))$ where $q_{\phi,K}(u) = 2K(u,u,1) - K(u,u,u)$.

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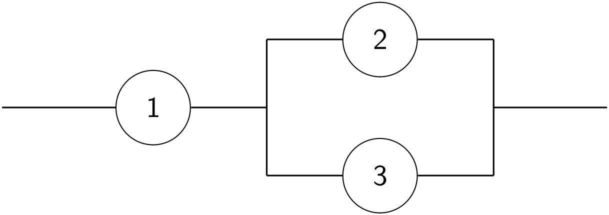
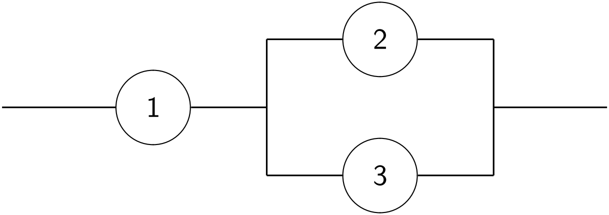


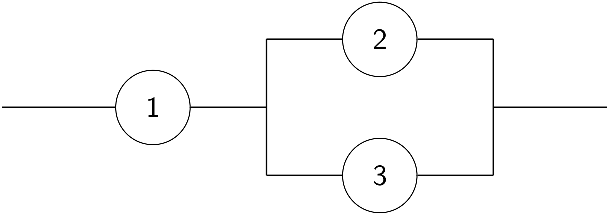
Minimal signature $\mathbf{a} = (0, 2, -1)$.

Example-general case



IID: $\overline{F}_T(t) = 2\overline{F}^2(t) - \overline{F}^3(t) = q_\phi(\overline{F}(t))$ where $q_\phi(u) = 2u^2 - u^3$.

Example-general case



The minimal signatures for coherent systems with $n \leq 5$ can be seen in:
Navarro and Rubio (2010, Comm Stat Simul Comp 39, 68–84).

Generalized Order statistics (GOS)

- For an arbitrary DF F, GOS $X_{1:n}^{GOS}, \ldots, X_{n:n}^{GOS}$ based on F can be obtained (Kamps, 1995, B. G. Teubner Stuttgart, p.49) via the quantile transformation

$$X_{r:n}^{GOS} = F^{-1}(U_{r:n}^{GOS}), \quad r = 1, \ldots, n,$$

where $(U_{1:n}^{*}, \ldots, U_{n:n}^{*})$ has the joint PDF

$$g^{GOS}(u_1, \ldots, u_n) = k \left(\prod_{j=1}^{n-1} \gamma_j \right) \left(\prod_{i=1}^{n-1} (1 - u_i)^{m_i} \right) (1-u_n)^{k-1}$$

for $0 \leq u_1 \leq \ldots \leq u_n < 1$, $n \geq 2$, $k \geq 1$, $\gamma_1, \ldots, \gamma_n > 0$ and $m_i = \gamma_i - \gamma_{i+1} - 1$.

Generalized Order statistics (GOS)

- If $\gamma_1, \ldots, \gamma_n$ are pairwise different, then

$$F_{r:n}^{GOS}(t) = 1 - c_{r-1} \sum_{i=1}^{r} \frac{a_{i,r}}{\gamma_i} (1 - F(t))^{\gamma_i} = q_{r:n}^{GOS}(F(t))$$

with the constants

$$c_{r-1} = \prod_{j=1}^{r} \gamma_j, \quad a_{i,r} = \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{\gamma_j - \gamma_i}, \quad 1 \leq i \leq r \leq n$$

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- Then the GOS are DD from F.

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Particular cases of GOS

- The GOS include:
- OS, IID case ($m_1 = \cdots = m_{n-1} = 0$ and $k = 1$).
- kRV, k-th record values ($m_1 = \cdots = m_{n-1} = -1$ and $k = 1, 2, \ldots$).
- RV, record values ($m_1 = \cdots = m_{n-1} = -1$ and $k = 1$).
- SOS, Sequential Order Statistics under the Proportional Hazard Rate (PHR) model, i.e., with $\overline{F}_r = \overline{F}^{\alpha_r}$ for $r = 1, \ldots, n$ ($\gamma_r = (n - r + 1)\alpha_r$ and $k = \alpha_n$).
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Preservation results

- If q_1 and q_2 are two distorted functions,

$$q_1(F) \leq_{ord} q_2(F) \text{ for all } F?$$

- If q is a distorted function,

$$F \leq_{ord} G \Rightarrow q(F) \leq_{ord} q(G)?$$

- If Q is a multivariate distorted function,

$$F_i \leq_{ord} G_i, i = 1, \ldots, n, \Rightarrow Q(F_1, \ldots, F_n) \leq_{ord} Q(G_1, \ldots, G_n)?$$

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Main stochastic orderings

- $X \leq_{ST} Y \Leftrightarrow \overline{F}_X(t) \leq \overline{F}_Y(t)$, stochastic order.
- $X \leq_{HR} Y \Leftrightarrow h_X(t) \geq h_Y(t)$, hazard rate order.
- $X \leq_{HR} Y \Leftrightarrow (X - t|X > t) \leq_{ST} (Y - t|Y > t)$ for all t.
- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
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- $X \leq_{LR} Y \Leftrightarrow f_Y(t)/f_X(t)$ is nondecreasing, likelihood ratio order.
- $X \leq_{LR} Y \Leftrightarrow (X|s < X < t) \leq_{ST} (Y|s < Y < t)$ for $s < t$.
- $X \leq_{LR} Y \Rightarrow X \leq_{HR} Y \Rightarrow X \leq_{ST} Y$

Preservation of stochastic orders

- If T_i has the DD $q_i(F(t))$, $i = 1, 2$, then:
- $T_1 \leq_{ST} T_2$ for all F if and only if $q_1(u)/q_2(u) \geq 1$ in $(0, 1)$.
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Preservation results of aging classes

- Let $\mathcal{C}$ be an aging class.
- If q is a distorted function,

$$F \in \mathcal{C} \Rightarrow q(F) \in \mathcal{C}?$$

- If Q is a multivariate distorted function,

$$F_i \in \mathcal{C}, i = 1, \ldots, n, \Rightarrow Q(F_1, \ldots, F_n) \in \mathcal{C}?$$

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Stochastic aging classes

- X is Increasing (Decreasing) Hazard rate IHR (DHR) if h is increasing (decreasing).
- X is IHR $\Leftrightarrow (X - s|X > s) \geq_{ST} (X - t|X > t)$ for all $s < t$.
- X is New Better (Worse) than Used NBU (NWU) if $X \geq_{ST} (X - t|X > t)$ $(\leq_{ST})$ for all $t > 0$.
- X is Increasing (Decreasing) Likelihood Ratio ILR (DLR) if f is log-concave (log-convex).
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- $ILR \Rightarrow IHR \Rightarrow NBU$.

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- In the IID case:
- The IHR class and the HR order are preserved for $X_{i:n}$ since $\alpha_{i:n}(u)$ is decreasing (Esary and Proschan 1963, Tech.).
- The DHR class is not necessarily preserved for $X_{i:n}$! It is only preserved for $X_{1:n}$ since $\alpha_{1:n}(u)$ is constant.
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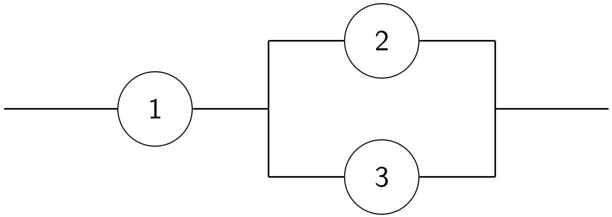

- Coherent system lifetime $T = \min(X_1, \max(X_2, X_3))$.
- In the IID case: $q(u) = u + u^2 - u^3$ and $\overline{q}(u) = 2u^2 - 3u^3$.
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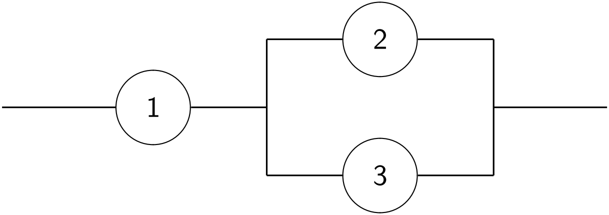

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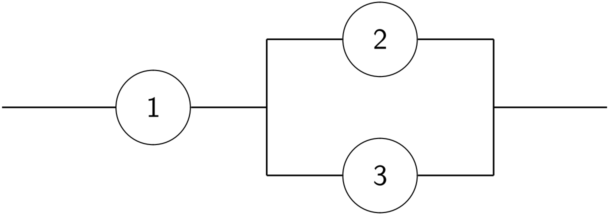

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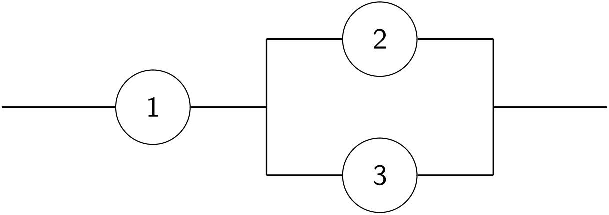

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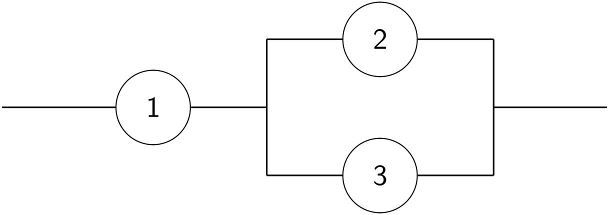

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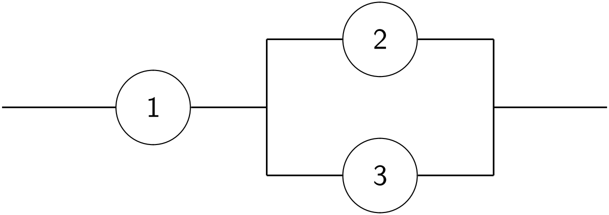

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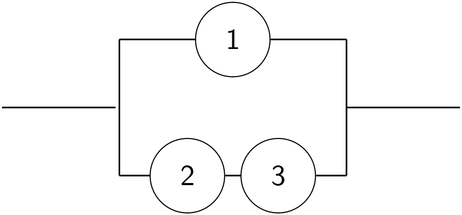

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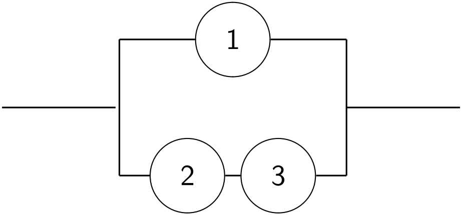

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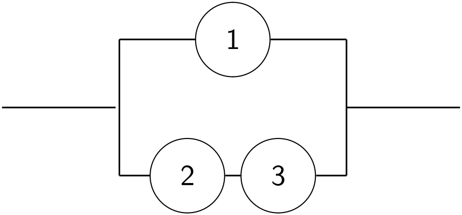

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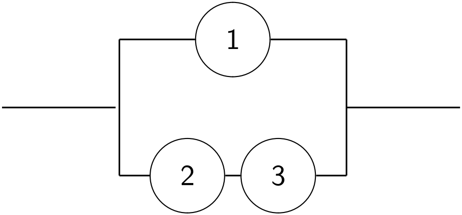

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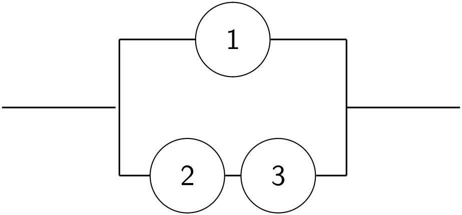

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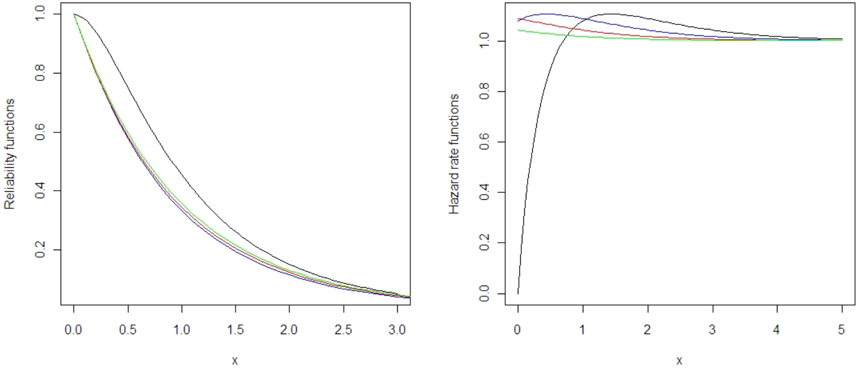

Figure: Reliability functions (left) of residual lifetimes $(T - t|T > t)$ of the system $T = \max(X_1, \min(X_2, X_3))$ when X_i are IID$\sim$ $Exp(\mu = 1)$ with $t = 0, 1, 2, 3$ (black, blue, red, green) and hazard rate function (right).

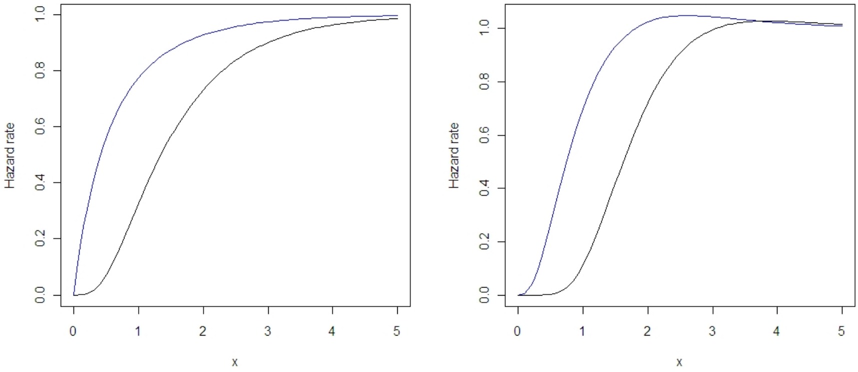

Figure: Hazard rate functions of X_1 (left) and $T = \max(X_1, \min(X_2, X_3))$ (right) when X_i are IID with reliability $\overline{F}(t) = 1 - (1 - e^{-t})^a$ for $t > 0$ and $a = 2, 5$ (blue, black).

Example-DID case

- Series system $X_{1:n} = \min(X_1, \ldots, X_n)$ with ID components having a Clayton-Oakes survival copula

$$K(u_1, \ldots, u_n) = \left(\sum_{i=1}^{n} u_i^{1-\theta} - (n-1) \right)^{1/(1-\theta)}, \quad \theta > 1.$$

- Then

$$\overline{q}(u) = K(u, \ldots, u) = (nu^{1-\theta} - n + 1)^{1/(1-\theta)}.$$

- As $\alpha_q(u) = \frac{n}{n-(n-1)u^{\theta-1}}$ is a strictly increasing function for all $\theta > 1$, the DHR class is preserved for all n.
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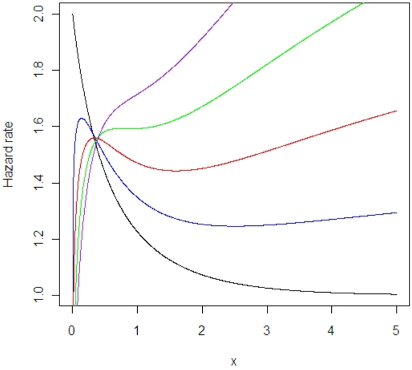

Figure: Hazard rate functions of the series system $T = \min(X_1, X_2)$ when (X_1, X_2) has a Clayton-Oakes survival copula with $\theta = 2$ and marginal reliability $\overline{F}(t) = \exp(-t^a)$, $t > 0$, with $a = 1$ (black, Exponential), $a = 1.1, 1.2, 1.3, 1.4$ (blue, red, green, purple, IHR Weibull).

Preservation of Stochastic aging classes DD

- Let $F_q = q(F)$ and let

$$\beta(u) = u\overline{q}''(u)/\overline{q}'(u),$$

and

$$\overline{\beta}(u) = (1-u)\overline{q}''(u)/\overline{q}'(u).$$

Then:

- If F is ILR and there exists $a \in [0,1]$ such that β is non-negative and decreasing in $(0,a)$ and $\overline{\beta}$ is non-positive and decreasing in $(a,1)$, then F_q is ILR.
- If F is DLR with support (l,∞) $(l \geq 0)$, β is non-negative and increasing in $(0,1)$, then F_q is DLR.

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Preservation of Stochastic aging classes GDD

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$$\alpha_i(u_1, \ldots, u_n) = \frac{u_i D_i \overline{Q}(u_1, \ldots, u_n)}{\overline{Q}(u_1, \ldots, u_n)},$$

where $D_i \overline{Q}(u_1, \ldots, u_n) = \frac{\partial}{\partial u_i} \overline{Q}(u_1, \ldots, u_n)$. Then:

- The IHR (DHR) class is preserved if α_i is decreasing (increasing) in $(0,1)^n$ for $i = 1, \ldots, n$.
- The NBU (NWU) class is preserved if

$$\overline{Q}(u_1 v_1, \ldots, u_n v_n) \leq \overline{Q}(u_1, \ldots, u_n)\overline{Q}(v_1, \ldots, v_n) \quad (\geq)$$

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Preservation of Stochastic aging classes GDD

- If $X_1, \ldots, X_n$ are independent, then:
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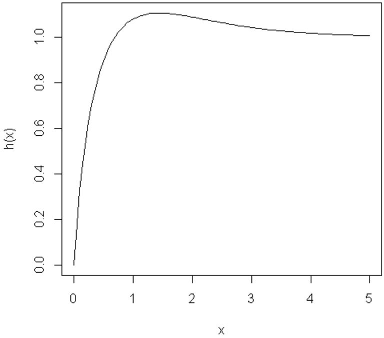

Figure: Hazard rate function of $X_{2:2} = \max(X_1, X_2)$ when the components are independent and $X_i \sim Exp(\mu = i)$, $i = 1, 2$. X_i are IHR and DHR but $X_{2:2}$ is neither IHR nor DHR.

Parrondo's paradox

- Parrondo's paradox shows (Game Theory) how, in some games, a random strategy might be better than any deterministic strategy.
- The same paradox holds for coherent systems.
- Let us assume that we have two kind of units with reliability functions $\overline{F}_1 < \overline{F}_2$ (in a similar number) to build series systems with two independent units.
- Let $T = \min(X_1, X_2)$ be the system obtained when $\overline{F}_i(t) = \Pr(X_i > t)$, $i = 1, 2$.
- Let S be the system obtained when the units are chosen randomly.
- Then $T \leq_{ST} S$ since

$$\overline{F}_T(t) = \overline{F}_1(t)\overline{F}_2(t) \leq (0.5\overline{F}_1(t) + 0.5\overline{F}_1(t))^2 = \overline{F}_S(t).$$

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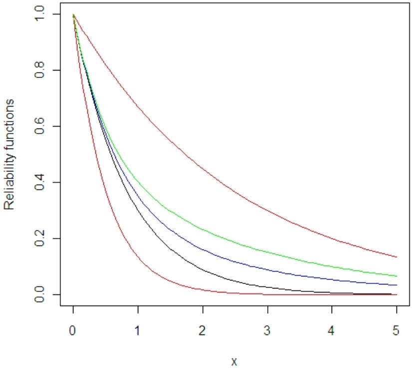

Figure: Reliability functions of systems T (black) and S (blue) when the units have exponential distributions with means 1 and 5. The red lines represent the reliability of systems with two good or bad units. Can you guess how to obtain the green line?

Parrondo's paradox in other systems

- The same happen with series system of size n with independent components.
- The ordering are reversed for parallel systems.
- A function $g : \mathbb{R}^n \to \mathbb{R}$ is weakly Schur-concave (convex) if

$$g(u_1, u_2, \dots, u_n) \le g(\overline{u}, \overline{u}, \dots, \overline{u}) \quad (\ge)$$

for all $(u_1, u_2, \dots, u_n)$, where $\overline{u} = (u_1 + u_2 + \dots + u_n)/n$.

Theorem (Navarro and Spizzichino, ASMBI 2010)

If $(X_1, X_2, \dots, X_n)$ and $(Y_1, Y_2, \dots, Y_n)$ have the same copula, $\overline{F}_i(t) = \Pr(X_i > t)$ and $\overline{G}(t) = (\overline{F}_1(t) + \dots + \overline{F}_n(t))/n = \Pr(Y_i > t)$ for $i = 1, \dots, n$, and $\overline{Q}_{\phi,K}$ is weakly Schur-concave (convex), then

$$T = \phi(X_1, \dots, X_n) \le_{ST} S = \phi(Y_1, \dots, Y_n) \quad (\ge_{ST}).$$

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Theorem (Navarro and Spizzichino, ASMBI 2010)

If $(X_1, X_2, \dots, X_n)$ and $(Y_1, Y_2, \dots, Y_n)$ have the same copula, $\overline{F}_i(t) = \Pr(X_i > t)$ and $\overline{G}(t) = (\overline{F}_1(t) + \dots + \overline{F}_n(t))/n = \Pr(Y_i > t)$ for $i = 1, \dots, n$, and $\overline{Q}_{\phi,K}$ is weakly Schur-concave (convex), then

$$T = \phi(X_1, \dots, X_n) \leq_{ST} S = \phi(Y_1, \dots, Y_n) \quad (\geq_{ST}).$$

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- The theorem can be applied to distorted distributions.
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Parametric estimation from DD under PHR

- We assume that $\overline{F}(t) = \overline{G}^{\alpha}(t)$, where $\overline{G}$ is a known reliability function with support $(0, \infty)$ and $\alpha > 0$ is unknown.
- We want to estimate α by using the sample $Y_1, \ldots, Y_m$ of IID$\sim F_q$, where $F_q = q(F)$, $\overline{F}_q = \overline{q}(\overline{F})$ and $\overline{q}(u) = \sum_{i=1}^{n} a_i u^i$ is a distorted polynomial.
- This property holds if we have a sample from a fixed OS $X_{i:n}$ or from a coherent system $T = \phi(X_1, \ldots, X_n)$ in the IID case.

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Method of moments estimator (MME)

- From the assumptions we have:

$$E(Y_j) = \int_0^\infty \overline{q}(\overline{F}(x))dx = \sum_{i=1}^{n} a_i \int_0^\infty [\overline{G}(x)]^{i\alpha} dx.$$

- Then for MME we need to solve:

$$\frac{1}{m}\sum_{j=1}^{m} Y_j = \sum_{i=1}^{n} a_i \int_0^\infty [\overline{G}(x)]^{i\alpha} dx. \tag{4.1}$$

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- Then the likelihood function is

$$L(\alpha) = \prod_{k=1}^{m} f_q(y_k) = \left\{\frac{\prod_{k=1}^{m} g(y_k)}{\prod_{k=1}^{m} \bar{G}(y_k)}\right\}\alpha^m \prod_{k=1}^{m}\sum_{i=1}^{n} i a_i \bar{G}^{i\alpha}(y_k).$$

- Then the log-likelihood function is

$$\ln L(\alpha) = c + m\ln\alpha + \sum_{k=1}^{m}\ln\left\{\sum_{i=1}^{n} i a_i \bar{G}^{i\alpha}(y_k)\right\},$$

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$$\frac{\partial \ln L(\alpha)}{\partial \alpha} = \frac{m}{\alpha} + \sum_{k=1}^{m} \left\{ \frac{\sum_{i=1}^{n} i^2 a_i \bar{G}^{i\alpha}(y_k)}{\sum_{i=1}^{n} i a_i \bar{G}^{i\alpha}(y_k)} \right\} \ln \bar{G}(y_k) = 0. \quad (4.2)$$

Proposition

If the function

$$\gamma(x) = \frac{\sum_{i=1}^{n} i^2 a_i x^i}{\sum_{i=1}^{n} i a_i x^i}$$

is strictly decreasing in $(0,1)$, then (4.2) has a unique positive solution $\widehat{\alpha}_{MLE}$ and $L(\alpha)$ attains a maximum at that point.

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Maximum likelihood estimator (MLE)

- The function $\gamma(x)$ is strictly decreasing for all the coherent systems with 4 or less IID components.
- Numerical methods are used to solve these equations.
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Thanks

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