¿Qué es la homoplutia?

Jorge Navarro[1]
Universidad de Murcia, Murcia, Spain.

[1]Supported by Ministerio de Ciencia e Innovación of Spain in the project PID2022-137396NB-I00, funded by MICIU/AEI/10.13039/501100011033 and by 'ERDF A way of making Europe'.

References

The talk is based on the following references:

- Capaldo M. and Navarro J. (2026). Homoploutia curves and indices. Submitted (revision 1).

References

The talk is based on the following references:

- Capaldo M. and Navarro J. (2026). Homoploutia curves and indices. Submitted (revision 1).
- Dr. Marco Capaldo, Postdoc position at Institute of Statistics, RWTH Aachen University, 52062 Aachen, Germany.

References

The talk is based on the following references:

- Capaldo M. and Navarro J. (2026). Homoploutia curves and indices. Submitted (revision 1).
- Dr. Marco Capaldo, Postdoc position at Institute of Statistics, RWTH Aachen University, 52062 Aachen, Germany.
- Milanovic, B. (2019). Capitalism, Alone. The Future of the System that Rules the World. Harvard University Press.

References

The talk is based on the following references:

- Capaldo M. and Navarro J. (2026). Homoploutia curves and indices. Submitted (revision 1).
- Dr. Marco Capaldo, Postdoc position at Institute of Statistics, RWTH Aachen University, 52062 Aachen, Germany.
- Milanovic, B. (2019). Capitalism, Alone. The Future of the System that Rules the World. Harvard University Press.
- Branko Milanovic is a Senior Scholar at the Stone Center on Socio-economic Inequality at the City University of New York.

Outline

Theoretical results

Introduction

- The term “homoploutia” can be traced back to Milanovic (2019) and measures the percentage of people rich in both capital and labor incomes.

Introduction

- The term “homoploutia” can be traced back to Milanovic (2019) and measures the percentage of people rich in both capital and labor incomes.
- **Homoploutia** is a neologism which comes from the Greek words “homo” for equal, and “ploutia” for wealth or “richness”.

Introduction

- The term "homoploutia" can be traced back to Milanovic (2019) and measures the percentage of people rich in both capital and labor incomes.
- **Homoploutia** is a neologism which comes from the Greek words "homo" for equal, and "ploutia" for wealth or "richness".
- If X_i represents the capital income and Y_i the labor income of individual i, for $i = 1, \ldots, n$, the top 10% homoploutia measure is defined (see equation (1) in Berman and Milanovic (2024)) as:

$$\widehat{H}_{10,10} = \frac{10}{n} \sum_{i=1}^{n} 1(X_i \in \text{top}10\% \; of \; X) \cdot 1(Y_i \in \text{top}10\% \; of \; Y), \quad (1.1)$$

where (X, Y) represents the theoretical values.

Introduction

- $\widehat{H}_{10,10}$ estimates the value of

$$H_{10,10} = 10 \Pr(X > q^X_{0.9}, Y > q^Y_{0.9}), \qquad (1.2)$$

where $q^X_{0.9}$ and $q^Y_{0.9}$ denote the 0.9 quantiles of X and Y.

Introduction

- $\widehat{H}_{10,10}$ estimates the value of

$$H_{10,10} = 10\,\Pr(X > q^X_{0.9}, Y > q^Y_{0.9}), \tag{1.2}$$

where $q^X_{0.9}$ and $q^Y_{0.9}$ denote the 0.9 quantiles of X and Y.

- This value can also be obtained as

$$H_{10,10} = \Pr(X > q^X_{0.9} | Y > q^Y_{0.9}) = \Pr(Y > q^Y_{0.9} | X > q^X_{0.9}),$$

that is, it represents the probability that an individual which belongs to the top 10% of X, is also in the top 10% of Y.

Introduction

- $\widehat{H}_{10,10}$ estimates the value of

$$H_{10,10} = 10\Pr(X > q^X_{0.9}, Y > q^Y_{0.9}), \tag{1.2}$$

where $q^X_{0.9}$ and $q^Y_{0.9}$ denote the 0.9 quantiles of X and Y.

- This value can also be obtained as

$$H_{10,10} = \Pr(X > q^X_{0.9} | Y > q^Y_{0.9}) = \Pr(Y > q^Y_{0.9} | X > q^X_{0.9}),$$

that is, it represents the probability that an individual which belongs to the top 10% of X, is also in the top 10% of Y.

- Therefore these measures show the connections between incomes from capital and labor.

Introduction

- $\widehat{H}_{10,10}$ estimates the value of

$$H_{10,10} = 10\,\Pr(X > q^X_{0.9}, Y > q^Y_{0.9}), \tag{1.2}$$

where $q^X_{0.9}$ and $q^Y_{0.9}$ denote the 0.9 quantiles of X and Y.

- This value can also be obtained as

$$H_{10,10} = \Pr(X > q^X_{0.9} | Y > q^Y_{0.9}) = \Pr(Y > q^Y_{0.9} | X > q^X_{0.9}),$$

that is, it represents the probability that an individual which belongs to the top 10% of X, is also in the top 10% of Y.

- Therefore these measures show the connections between incomes from capital and labor.
- In the "**classical capitalism**" one could expect that rich people have the main incomes from capital having a zero (no employment) or small salary (when compared with capital income).

Introduction

- However, recently, it has been observed that, in some countries, there is a **"modern capitalism"** with an upper class that has incomes from both capital and labor.

Introduction

- However, recently, it has been observed that, in some countries, there is a **“modern capitalism”** with an upper class that has incomes from both capital and labor.
- In this class we could have both owners of companies that also have relevant (well paid) executive positions within their companies or successful workers (relevant employees, artists, sport players, etc.) that do investment and so they also receive substantial incomes from their capital.

Introduction

- However, recently, it has been observed that, in some countries, there is a **“modern capitalism”** with an upper class that has incomes from both capital and labor.
- In this class we could have both owners of companies that also have relevant (well paid) executive positions within their companies or successful workers (relevant employees, artists, sport players, etc.) that do investment and so they also receive substantial incomes from their capital.
- In this sense, homoploutia is a key feature of modern capitalism that distinguishes it from classical capitalism.

Introduction

- However, recently, it has been observed that, in some countries, there is a **“modern capitalism”** with an upper class that has incomes from both capital and labor.
- In this class we could have both owners of companies that also have relevant (well paid) executive positions within their companies or successful workers (relevant employees, artists, sport players, etc.) that do investment and so they also receive substantial incomes from their capital.
- In this sense, homoploutia is a key feature of modern capitalism that distinguishes it from classical capitalism.
- Clearly, it is a measure of association that can be used in other areas.

Índice

Homoploutia curve

- From the copula theory (Sklar's theorem)

$$F(x,y) = \Pr(X \leq x, Y \leq y) = C(F_X(x), F_Y(y)), \text{ for all } x, y,$$

where $C : [0,1]^2 \to [0,1]$ is a copula and F_X, F_Y are the marginals. If they are continuous, then C is unique.

Homoploutia curve

- From the copula theory (Sklar's theorem)

$$F(x,y)=\Pr(X\le x, Y\le y)=C(F_X(x),F_Y(y)),\ \text{for all } x,y,$$

where $C:[0,1]^2\to[0,1]$ is a copula and F_X, F_Y are the marginals. If they are continuous, then C is unique.

- Analogously,

$$\bar{F}(x,y)=\Pr(X> x, Y> y)=\widehat{C}(\bar{F}_X(x),\bar{F}_Y(y)),\ \text{for all } x,y,$$

where $\widehat{C}$ is another copula function called "survival copula".

Homoploutia curve

- From the copula theory (Sklar's theorem)

$$F(x,y) = \Pr(X \le x, Y \le y) = C(F_X(x), F_Y(y)), \text{ for all } x, y,$$

where $C : [0,1]^2 \to [0,1]$ is a copula and F_X, F_Y are the marginals. If they are continuous, then C is unique.

- Analogously,

$$\bar{F}(x,y) = \Pr(X > x, Y > y) = \widehat{C}(\bar{F}_X(x), \bar{F}_Y(y)), \text{ for all } x, y,$$

where $\widehat{C}$ is another copula function called "survival copula".

- C determines $\widehat{C}$ (and vice versa) from

$$\widehat{C}(u,v) = u + v - 1 + C(1-u, 1-v), \ u, v \in [0,1].$$

Homoploutia curve

- We define the homoploutia curve of (X, Y) as

$$H(u) = \frac{\widehat{C}(u,u)}{u} = \frac{\widehat{\delta}(u)}{u}, \;\; u \in (0,1]$$

where $\widehat{\delta}(u) = \widehat{C}(u,u)$ is the diagonal section of $\widehat{C}$.

Homoploutia curve

- We define the homoploutia curve of (X,Y) as

$$H(u)=\frac{\widehat{C}(u,u)}{u}=\frac{\widehat{\delta}(u)}{u},\ \ u\in(0,1]$$

where $\widehat{\delta}(u)=\widehat{C}(u,u)$ is the diagonal section of $\widehat{C}$.

- Then it holds $H(1)=1$ and

$$H_{10,10}=10\bar{F}(q^X_{0.9},q^Y_{0.9})=10\widehat{C}(0.1,0.1)=\frac{\widehat{\delta}(0.1)}{0.1}=H(0.1).$$

Properties of the homoploutia curve

- (H1) H does not depend on the marginal distributions and

$$H(u) = \Pr(Y > q^Y_{1-u} | X > q^X_{1-u}) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-u}), \ u \in (0,1),$$

where q^X_α and q^Y_α represent the respective quantiles.

Properties of the homoploutia curve

- (H1) H does not depend on the marginal distributions and

$$H(u) = \Pr(Y > q^Y_{1-u} | X > q^X_{1-u}) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-u}),\ u \in (0,1),$$

where q^X_α and q^Y_α represent the respective quantiles.

- (H2) The Fréchet-Hoeffding bound copulas $M(u,v) = \min(u,v)$ and $W(u,v) = \max(u+v-1,0)$ lead to

$$H_W(u) \le H(u) \le 1 = H_M(u),\ \ u \in (0,1]$$

where $H_W(u) = 0$ for $u \in (0,0.5)$ and $H_W(u) = 2 - 1/u$ for $u \in [0.5,1]$. They represent the extreme cases where $U = V$ and $U = -V$, respectively.

Properties of the homoploutia curve

- (H1) H does not depend on the marginal distributions and

$$H(u) = \Pr(Y > q^Y_{1-u} | X > q^X_{1-u}) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-u}),\ u \in (0,1),$$

where q^X_α and q^Y_α represent the respective quantiles.

- (H2) The Fréchet-Hoeffding bound copulas $M(u,v) = \min(u,v)$ and $W(u,v) = \max(u+v-1,0)$ lead to

$$H_W(u) \le H(u) \le 1 = H_M(u),\ \ u \in (0,1]$$

where $H_W(u) = 0$ for $u \in (0,0.5)$ and $H_W(u) = 2 - 1/u$ for $u \in [0.5,1]$. They represent the extreme cases where $U = V$ and $U = -V$, respectively.

- The independence case is represented by the product copula $\Pi(u,v) = uv$ for $u,v \in [0,1]$ with $H(u) = u$ for $u \in [0,1]$.

Properties of the homoploutia curve

- (H3) If C is the CDF of (U, V), then

$$H(u) = \frac{\Pr(U > 1-u, V > 1-u)}{u} = \Pr(V > 1-u | U > 1-u),\ u \in (0, 1].$$

Properties of the homoploutia curve

- (H3) If C is the CDF of (U, V), then

$$H(u) = \frac{\Pr(U > 1-u, V > 1-u)}{u} = \Pr(V > 1-u | U > 1-u),\ u \in (0,1].$$

- For the copula M we get

$$\Pr(V > 1-u | U > 1-u) = \Pr(U > 1-u | V > 1-u) = 1,\ u \in (0,1],$$

complete "modern capitalism" for all the people.

Properties of the homoploutia curve

- (H3) If C is the CDF of (U, V), then

$$H(u)=\frac{\Pr(U>1-u,V>1-u)}{u}=\Pr(V>1-u|U>1-u),\ u\in(0,1].$$

- For the copula M we get

$$\Pr(V>1-u|U>1-u)=\Pr(U>1-u|V>1-u)=1,\ u\in(0,1],$$

  complete "modern capitalism" for all the people.
- For the copula W, we get

$$\Pr(V>1-u|U>1-u)=\Pr(U>1-u|V>1-u)=0,\ u\in(0,0.5]$$

  complete "classical capitalism" for the people above the median.

Properties of the homoploutia curve

- (H4) An homoploutia curve may refer to different copulas since it only depends on the diagonal section $\widehat{\delta}$.

Properties of the homoploutia curve

- (H4) An homoploutia curve may refer to different copulas since it only depends on the diagonal section $\widehat{\delta}$.
- (H5) If (X, Y) is PQD (NQD), then $H(u) \geq u$ $(\leq)$ for all u.

Properties of the homoploutia curve

- (H4) An homoploutia curve may refer to different copulas since it only depends on the diagonal section $\widehat{\delta}$.
- (H5) If (X, Y) is PQD (NQD), then $H(u) \geq u$ $(\leq)$ for all u.
- (H6) If (X, Y) is more PQD than $(X^\star, Y^\star)$, that is, the respective survival copula functions satisfy $\widehat{C} \geq \widehat{C}^\star$ (which is also called concordance order), then $H(u) \geq H^\star(u)$ for all $u \in (0, 1]$.

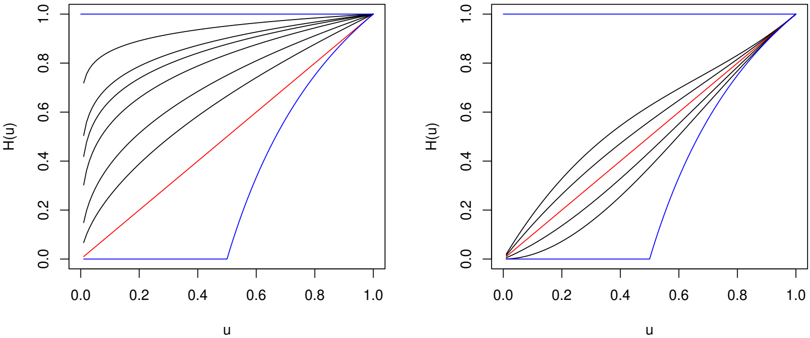

Figure: Homoploutia curves for the Gumbel (left) survival copula with $\theta = 1, 1.5, 2, 3, 4, 5, 10$ and the FGM (right) survival copula with $\theta = -1, 0.5, 0, 0.5, 1$. The red curve represents the IND case and the blue curves the extremes FH bound cases.

Properties of the homoploutia curve

- (H7) The homoploutia curve can be obtained from C as

$$H(u)=\frac{2u-1+\delta(1-u)}{u},\ \ u\in(0,1],$$

where $\delta(u)=C(u,u)$ is the diagonal section of C.

Properties of the homoploutia curve

- (H7) The homoploutia curve can be obtained from C as

$$H(u)=\frac{2u-1+\delta(1-u)}{u},\ \ u\in(0,1],$$

where $\delta(u)=C(u,u)$ is the diagonal section of C.

- (H8) If λ_U is the upper tail dependence parameter of C and $\widehat{\lambda}_L$ is the lower tail dependence parameter of $\widehat{C}$ (see Nelsen (2006), p. 214 for their definitions), then

$$\lambda_U=\widehat{\lambda}_L=\lim_{u\to 0^+}H(u).$$

Properties of the homoploutia curve

- (H7) The homoploutia curve can be obtained from C as

$$H(u)=\frac{2u-1+\delta(1-u)}{u},\ \ u\in(0,1],$$

where $\delta(u)=C(u,u)$ is the diagonal section of C.

- (H8) If λ_U is the upper tail dependence parameter of C and $\widehat{\lambda}_L$ is the lower tail dependence parameter of $\widehat{C}$ (see Nelsen (2006), p. 214 for their definitions), then

$$\lambda_U=\widehat{\lambda}_L=\lim_{u\to 0^+}H(u).$$

- For the Gumbel survival copula we get $\widehat{\lambda}_L=0$ while if C belong to this family (not $\widehat{C}$), then $\widehat{\lambda}_L=2-2^{1/\theta}\in[0,1)$.

Properties of the homoploutia curve

- (H7) The homoploutia curve can be obtained from C as
$$H(u)=\frac{2u-1+\delta(1-u)}{u},\ \ u\in(0,1],$$
where $\delta(u)=C(u,u)$ is the diagonal section of C.
- (H8) If λ_U is the upper tail dependence parameter of C and $\widehat{\lambda}_L$ is the lower tail dependence parameter of $\widehat{C}$ (see Nelsen (2006), p. 214 for their definitions), then
$$\lambda_U=\widehat{\lambda}_L=\lim_{u\to 0^+}H(u).$$
- For the Gumbel survival copula we get $\widehat{\lambda}_L=0$ while if C belong to this family (not $\widehat{C}$), then $\widehat{\lambda}_L=2-2^{1/\theta}\in[0,1)$.
- For the FGM copula we have $C=\widehat{C}$ and $\widehat{\lambda}_L=0$.

Properties of the homoploutia curve

- (H9) If (X, Y) has exponential marginals, then

$$Gini(X/\mu_X, Y/\mu_Y) = \int_0^1 [1 - H(u)]\mathrm{d}u, \tag{1.3}$$

where the bivariate Gini index is defined in Capaldo and Navarro (2025).

Properties of the homoploutia curve

- (H9) If (X, Y) has exponential marginals, then

$$Gini(X/\mu_X, Y/\mu_Y) = \int_0^1 [1 - H(u)]\mathrm{d}u, \qquad (1.3)$$

where the bivariate Gini index is defined in Capaldo and Navarro (2025).

- The bivariate Gini mean difference of (X, Y) is defined as

$$GMD(X, Y) = E|X - Y|.$$

Properties of the homoploutia curve

- (H9) If (X, Y) has exponential marginals, then

$$Gini(X/\mu_X, Y/\mu_Y) = \int_0^1 [1 - H(u)]\mathrm{d}u, \tag{1.3}$$

where the bivariate Gini index is defined in Capaldo and Navarro (2025).

- The bivariate Gini mean difference of (X, Y) is defined as

$$GMD(X, Y) = E|X - Y|.$$

- The bivariate Gini index of (X, Y) was defined as

$$Gini(X, Y) = \frac{GMD(X, Y)}{\mu_X + \mu_Y} \in [0, 1].$$

Properties of the homoploutia curve

- (H9) If (X, Y) has exponential marginals, then
$$Gini(X/\mu_X, Y/\mu_Y) = \int_0^1 [1 - H(u)]\mathrm{d}u, \qquad (1.3)$$
where the bivariate Gini index is defined in Capaldo and Navarro (2025).
- The bivariate Gini mean difference of (X, Y) is defined as
$$GMD(X, Y) = E|X - Y|.$$
- The bivariate Gini index of (X, Y) was defined as
$$Gini(X, Y) = \frac{GMD(X, Y)}{\mu_X + \mu_Y} \in [0, 1].$$
- Hence
$$Gini(X/\mu_X, Y/\mu_Y) = \frac{1}{2} E\left|\frac{X}{\mu_X} - \frac{Y}{\mu_Y}\right|.$$

Índice

Homoploutia index

- In practice it might be interesting to get a number that summarizes all the homoploutia indices (i.e. at any quantile).

Homoploutia index

- In practice it might be interesting to get a number that summarizes all the homoploutia indices (i.e. at any quantile).
- The same happen with the Gini index and the Lorenz curve.

Homoploutia index

- In practice it might be interesting to get a number that summarizes all the homoploutia indices (i.e. at any quantile).
- The same happen with the Gini index and the Lorenz curve.
- To compare different populations (e.g. countries or years), the people prefer to compare Gini indices than Lorenz curves.

Homoploutia index

- In practice it might be interesting to get a number that summarizes all the homoploutia indices (i.e. at any quantile).
- The same happen with the Gini index and the Lorenz curve.
- To compare different populations (e.g. countries or years), the people prefer to compare Gini indices than Lorenz curves.
- From property (H9) we can measure the global homoploutia with the following index

$$I_H = \int_0^1 [1 - H(u)] \mathrm{d}u = \int_0^1 \left[1 - \frac{\widehat{\delta}(u)}{u} \right] \mathrm{d}u.$$

Properties of the homoploutia index

- (I1) The homoploutia index does not depend on marginal distributions and satisfies $I_H \in [0, \log(2)]$.

Properties of the homoploutia index

- (I1) The homoploutia index does not depend on marginal distributions and satisfies $I_H \in [0, \log(2)]$.
- $I_H = 0$ represents the extreme modern capitalism and $I_H = \log(2) = 0.6931472$ the extreme classical capitalism.

Properties of the homoploutia index

- (I1) The homoploutia index does not depend on marginal distributions and satisfies $I_H \in [0, \log(2)]$.
- $I_H = 0$ represents the extreme modern capitalism and $I_H = \log(2) = 0.6931472$ the extreme classical capitalism.
- In the independence case we get $I_H = 0.5$.

Properties of the homoploutia index

- (I1) The homoploutia index does not depend on marginal distributions and satisfies $I_H \in [0, \log(2)]$.
- $I_H = 0$ represents the extreme modern capitalism and $I_H = \log(2) = 0.6931472$ the extreme classical capitalism.
- In the independence case we get $I_H = 0.5$.
- For the Gumbel survival (PQD) copula we get

θ	1	1.5	2	3	4	5	10
I_H	0.5	0.37004	0.29289	0.20629	0.15910	0.12945	0.06696

- For the FGM copulas we get:

θ	-1	-0.5	0	0.5	1
I_H	0.5833333	0.5416667	0.5	0.4583333	0.4166667

Extensions

- In the paper we also consider:

Extensions

- In the paper we also consider:
- Homoploutia curves at different quantile levels as

$$H_{1|2}(u|v) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-v}) = \frac{\widehat{C}(u,v)}{v}.$$

Extensions

- In the paper we also consider:
- Homoploutia curves at different quantile levels as

$$H_{1|2}(u|v) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-v}) = \frac{\widehat{C}(u,v)}{v}.$$

- $RTI(X|Y)$ holds iff $H_{1|2}(u|v)$ is decreasing in v for all u.

Extensions

- In the paper we also consider:
- Homoploutia curves at different quantile levels as

$$H_{1|2}(u|v) = \Pr(X > q^X_{1-u}|Y > q^Y_{1-v}) = \frac{\widehat{C}(u,v)}{v}.$$

- $RTI(X|Y)$ holds iff $H_{1|2}(u|v)$ is decreasing in v for all u.
- The "homophtocheia" (phtocheia is poverty in Greek) curve

$$\tilde{H}(u) = \Pr(X \leq q^X_u|Y \leq q^Y_u) = \frac{C(u,u)}{u} = \frac{\delta(u)}{u}, \ u \in (0,1].$$

Extensions

- In the paper we also consider:
- Homoploutia curves at different quantile levels as

$$H_{1|2}(u|v) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-v}) = \frac{\widehat{C}(u, v)}{v}.$$

- $RTI(X|Y)$ holds iff $H_{1|2}(u|v)$ is decreasing in v for all u.
- The "homophtocheia" (phtocheia is poverty in Greek) curve

$$\tilde{H}(u) = \Pr(X \le q^X_u | Y \le q^Y_u) = \frac{C(u, u)}{u} = \frac{\delta(u)}{u}, \ u \in (0, 1].$$

- It is also defined at different quantiles $\tilde{H}_{1|2}(u|v) = C(u, v)/v$.

Extensions

- In the paper we also consider:
- Homoploutia curves at different quantile levels as

$$H_{1|2}(u|v) = \Pr(X > q^X_{1-u} | Y > q^Y_{1-v}) = \frac{\widehat{C}(u,v)}{v}.$$

- $RTI(X|Y)$ holds iff $H_{1|2}(u|v)$ is decreasing in v for all u.
- The "homophtocheia" (phtocheia is poverty in Greek) curve

$$\tilde{H}(u) = \Pr(X \le q^X_u | Y \le q^Y_u) = \frac{C(u,u)}{u} = \frac{\delta(u)}{u}, \ u \in (0,1].$$

- It is also defined at different quantiles $\tilde{H}_{1|2}(u|v) = C(u,v)/v$.
- $LTD(X|Y)$ holds iff $\tilde{H}_{1|2}(u|v)$ is decreasing in v for all u.

Inference and examples

Índice

Example 1

- Let us consider the vector (U, V) with a uniform distribution on the support $S = A_1 \cup A_2 \cup A_3 \cup A_4$ with $A_1 = (0, 1/4) \times (0, 1/4)$, $A_2 = (1/4, 1/2) \times (1/2, 3/4)$, $A_3 = (1/2, 3/4) \times (1/4, 1/2)$, and $A_4 = (3/4, 1) \times (3/4, 1)$.

Example 1

- Let us consider the vector (U, V) with a uniform distribution on the support $S = A_1 \cup A_2 \cup A_3 \cup A_4$ with $A_1 = (0, 1/4) \times (0, 1/4)$, $A_2 = (1/4, 1/2) \times (1/2, 3/4)$, $A_3 = (1/2, 3/4) \times (1/4, 1/2)$, and $A_4 = (3/4, 1) \times (3/4, 1)$.
- The marginals are uniform and so its CDF C is a copula.

Example 1

- Let us consider the vector (U, V) with a uniform distribution on the support $S = A_1 \cup A_2 \cup A_3 \cup A_4$ with $A_1 = (0, 1/4) \times (0, 1/4)$, $A_2 = (1/4, 1/2) \times (1/2, 3/4)$, $A_3 = (1/2, 3/4) \times (1/4, 1/2)$, and $A_4 = (3/4, 1) \times (3/4, 1)$.
- The marginals are uniform and so its CDF C is a copula.
- Its survival copula $\widehat{C}$ coincides with C.

Example 1

- Let us consider the vector (U, V) with a uniform distribution on the support $S = A_1 \cup A_2 \cup A_3 \cup A_4$ with $A_1 = (0, 1/4) \times (0, 1/4)$, $A_2 = (1/4, 1/2) \times (1/2, 3/4)$, $A_3 = (1/2, 3/4) \times (1/4, 1/2)$, and $A_4 = (3/4, 1) \times (3/4, 1)$.
- The marginals are uniform and so its CDF C is a copula.
- Its survival copula $\widehat{C}$ coincides with C.
- Therefore, the homoploutia curves coincide.

Example 1

- Let us consider the vector (U, V) with a uniform distribution on the support $S = A_1 \cup A_2 \cup A_3 \cup A_4$ with $A_1 = (0, 1/4) \times (0, 1/4)$, $A_2 = (1/4, 1/2) \times (1/2, 3/4)$, $A_3 = (1/2, 3/4) \times (1/4, 1/2)$, and $A_4 = (3/4, 1) \times (3/4, 1)$.
- The marginals are uniform and so its CDF C is a copula.
- Its survival copula $\widehat{C}$ coincides with C.
- Therefore, the homoploutia curves coincide.
- A straightforward calculation shows that

$$H(u) = \begin{cases} 4u, & \text{for } u \in [0, 1/4); \\ 1/(4u), & \text{for } u \in [1/4, 1/2); \\ 2 - 3/(4u), & \text{for } u \in [1/2, 3/4); \\ 4u - 6 + 3/u, & \text{for } u \in [3/4, 1]. \end{cases}$$

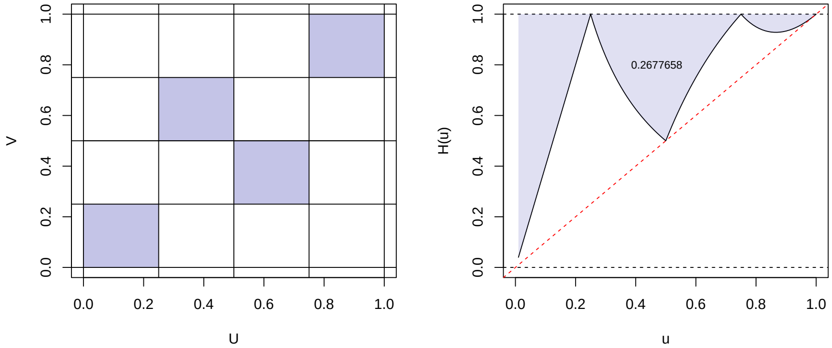

Figure: Support (left) and homoploutia curve and index (right) for the copula in Example 1. Note that H is not increasing.

Example 2

- Let us consider (U, V) with a uniform distribution on $S = A_1 \cup A_2 \cup A_3$ with $A_1 = (0, 1/2) \times (0, 1/4)$, $A_2 = (0, 1/2) \times (3/4, 1)$, and $A_3 = (1/2, 1) \times (1/4, 3/4)$.

Example 2

- Let us consider (U,V) with a uniform distribution on $S = A_1 \cup A_2 \cup A_3$ with $A_1 = (0,1/2) \times (0,1/4)$, $A_2 = (0,1/2) \times (3/4,1)$, and $A_3 = (1/2,1) \times (1/4,3/4)$.
- The marginals are uniform and so it determines a copula $\widehat{C}$.

Example 2

- Let us consider (U,V) with a uniform distribution on $S = A_1 \cup A_2 \cup A_3$ with $A_1 = (0,1/2) \times (0,1/4)$, $A_2 = (0,1/2) \times (3/4,1)$, and $A_3 = (1/2,1) \times (1/4,3/4)$.
- The marginals are uniform and so it determines a copula $\widehat{C}$.
- The homoploutia curve is

$$H(u) = \frac{\widehat{C}(u,u)}{u} = \begin{cases} 0, & \text{for } u \in [0,1/4); \\ 2u - 1/2, & \text{for } u \in [1/4,1/2); \\ 3/2 - 1/(2u), & \text{for } u \in [1/2,3/4); \\ 2u - 2 + 1/u, & \text{for } u \in [3/4,1]. \end{cases}$$

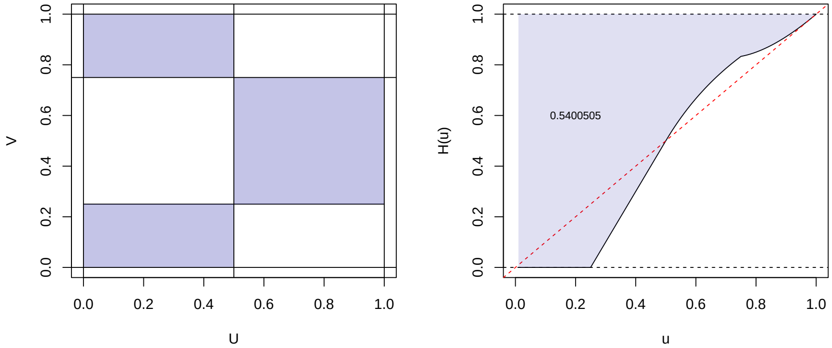

Figure: Support (left) and homoploutia curve and index (right) for the copula in Example 2.

Example 2

- Note that $H(0.25) = 0$, that is, there is no homoploutia for the 25% of the top richest people.

Example 2

- Note that $H(0.25) = 0$, that is, there is no homoploutia for the 25% of the top richest people.
- The homoploutia index $I_H = 0.5400505 > 0.5$.

Example 2

- Note that $H(0.25) = 0$, that is, there is no homoploutia for the 25% of the top richest people.
- The homoploutia index $I_H = 0.5400505 > 0.5$.
- (U, V) is neither PQD nor NQD.

Example 2

- Note that $H(0.25) = 0$, that is, there is no homoploutia for the 25% of the top richest people.
- The homoploutia index $I_H = 0.5400505 > 0.5$.
- (U, V) is neither PQD nor NQD.
- The Spearman's rho correlation coefficient is $\rho_S = 0$ and the Kendall's tau coefficient $\tau_K = 0$.

Example 2

- Note that $H(0.25)=0$, that is, there is no homoploutia for the 25% of the top richest people.
- The homoploutia index $I_H=0.5400505>0.5$.
- (U,V) is neither PQD nor NQD.
- The Spearman's rho correlation coefficient is $\rho_S=0$ and the Kendall's tau coefficient $\tau_K=0$.
- The Gini measure of association is $\gamma_C=0$ where

$$\begin{aligned}\gamma_C &= 4\int_0^1 [C(u,u)+C(u,1-u)-u]\,\mathrm{d}u\\ &= 4\int_0^1 [\Pr(U>u,V>u)-\Pr(U>u,V<1-u)]\,\mathrm{d}u.\end{aligned}$$

Índice

Inference

- Clearly, from (1.1), $H(j/n)$ can be estimated with

$$
\begin{aligned}
\widehat{H}(j/n) &= \frac{1}{j}\sum_{i=1}^{n} 1(X_i > X_{n-j:n})\cdot 1(Y_i > Y_{n-j:n}) \qquad (2.1)\\
&= \frac{1}{n}\sum_{i=1}^{n} \frac{1(X_i > X_{n-j:n})\cdot 1(Y_i > Y_{n-j:n})}{j/n}\\
&= \frac{\sum_{i=1}^{n} 1(X_i > X_{n-j:n})\cdot 1(Y_i > Y_{n-j:n})}{\sum_{i=1}^{n} 1(X_i > X_{n-j:n})},\ j=1,\dots,n.
\end{aligned}
$$

Inference

- Clearly, from (1.1), $H(j/n)$ can be estimated with

$$\widehat{H}(j/n) = \frac{1}{j}\sum_{i=1}^{n} 1(X_i > X_{n-j:n}) \cdot 1(Y_i > Y_{n-j:n}) \tag{2.1}$$

$$= \frac{1}{n}\sum_{i=1}^{n} \frac{1(X_i > X_{n-j:n}) \cdot 1(Y_i > Y_{n-j:n})}{j/n}$$

$$= \frac{\sum_{i=1}^{n} 1(X_i > X_{n-j:n}) \cdot 1(Y_i > Y_{n-j:n})}{\sum_{i=1}^{n} 1(X_i > X_{n-j:n})},\ j = 1, \dots, n.$$

- It represents the ratio of the number of data points in the rectangle $(X_{n-j:n}, X_{n:n}] \times (Y_{n-j:n}, Y_{n:n}]$ versus the data points in the rectangle $(X_{n-j:n}, X_{n:n}] \times [0, Y_{n:n}]$.

Inference

- The homoploutia index $I_H = \int_0^1 (1 - H(u))du$ can be estimated with

$$\widehat{I}_H = \frac{1}{n} \sum_{j=1}^{n-1} (1 - \widehat{H}(j/n)). \tag{2.2}$$

Inference

- The homoploutia index $I_H=\int_0^1(1-H(u))du$ can be estimated with

$$\widehat{I}_H=\frac{1}{n}\sum_{j=1}^{n-1}(1-\widehat{H}(j/n)). \tag{2.2}$$

- Clearly, for a fixed j, the sum in (2.1) follows a Binomial distribution and so the estimator $\widehat{H}(j/n)$ has very good asymptotic properties when $n\to\infty$ (unbiased, asymptotic normality, etc.).

Inference

- The homoploutia index $I_H = \int_0^1 (1 - H(u))du$ can be estimated with

$$\widehat{I}_H = \frac{1}{n} \sum_{j=1}^{n-1} (1 - \widehat{H}(j/n)). \tag{2.2}$$

- Clearly, for a fixed j, the sum in (2.1) follows a Binomial distribution and so the estimator $\widehat{H}(j/n)$ has very good asymptotic properties when $n \to \infty$ (unbiased, asymptotic normality, etc.).
- However we may have bad estimations for small values of j.

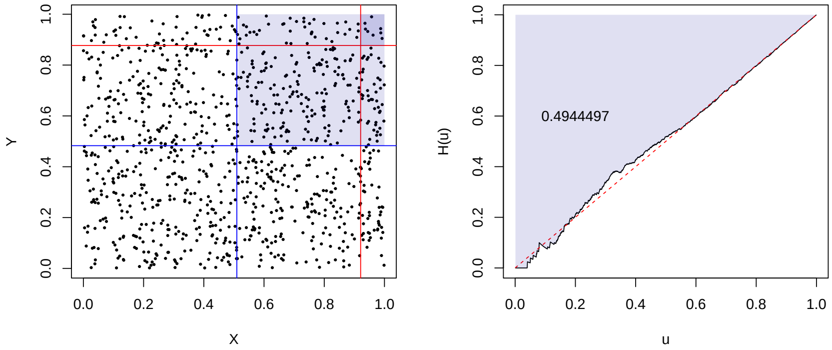

Figure: Data points (left) from a product copula with $n = 1000$ and rectangles obtained with the top 10% (dark grey) and top 50% (light grey). Estimation of $H(u)$ (right, black) jointly with the theoretical one (red dashed line) and estimation of I_H (grey area).

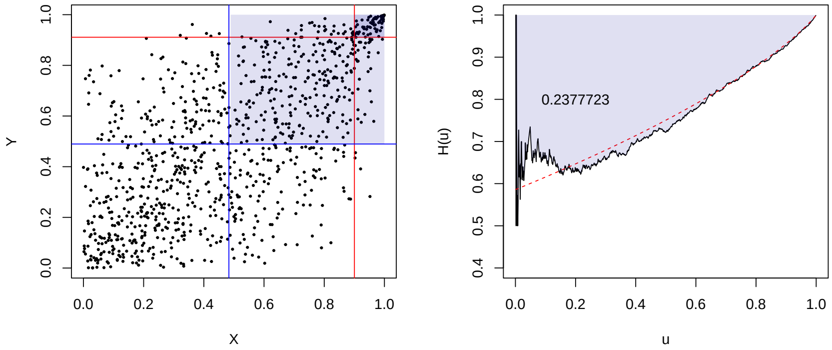

Figure: Data points (left) from Gumbel copula with parameter $\theta = 2$ and $n = 1000$ and rectangles obtained with the top 10% (dark grey) and top 50% (light grey). Estimation of $H(u)$ (right, black) jointly with the theoretical one (red dashed line) and estimation of I_H (grey area).

Índice

Multivariate versions and applications to other sciences

- The evolution of homoploutia curves in the United States from 1950 to 2020 based on three different data sources can be seen in Figure 1 of Berman and Milanovic (2024).

Multivariate versions and applications to other sciences

- The evolution of homoploutia curves in the United States from 1950 to 2020 based on three different data sources can be seen in Figure 1 of Berman and Milanovic (2024).
- An empirical analysis of the homoploutia in Germany, Spain and Italy over the period 2000-2017 can be seen in Giangregorion and Villani (2025).

Multivariate versions and applications to other sciences

- The evolution of homoploutia curves in the United States from 1950 to 2020 based on three different data sources can be seen in Figure 1 of Berman and Milanovic (2024).
- An empirical analysis of the homoploutia in Germany, Spain and Italy over the period 2000-2017 can be seen in Giangregorion and Villani (2025).
- Here we want also to remark that they could have interest to study related variables in other fields.

Real data example 1

- For example, they can be used in different sports to detect the players (or teams) that are at the same time in the top of two (or more) characteristics.

Real data example 1

- For example, they can be used in different sports to detect the players (or teams) that are at the same time in the top of two (or more) characteristics.
- Thus, we can study which NBA players are at the top10 in the list of points (PTS) and assistances (AST) per games.

Real data example 1

- For example, they can be used in different sports to detect the players (or teams) that are at the same time in the top of two (or more) characteristics.
- Thus, we can study which NBA players are at the top10 in the list of points (PTS) and assistances (AST) per games.
- With the data collected on February 11, 2026, from the NBA website (www.nba.com) we obtain the following plot.

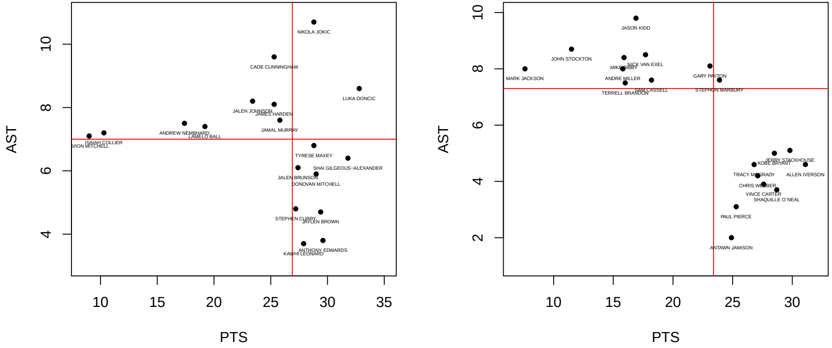

Figure: NBA players at the top ten lists of points (PTS) and assistances (AST) per games in the current season (left, Feb. 11, 2026) and in the 2000/01 complete regular season (right).

Real data example 1

- As there are two players (Luka Doncic, LAL, and Nikola Jokic, DEN) in both lists, the top10 homoploutia between these two variables is 0.2.

Real data example 1

- As there are two players (Luka Doncic, LAL, and Nikola Jokic, DEN) in both lists, the top10 homoploutia between these two variables is 0.2.
- In the 2000/01 complete regular season (right plot) only one player (Stephon Marbury, NJN) belongs to both lists and so the homoploutia is 0.1.

Real data example 1

- As there are two players (Luka Doncic, LAL, and Nikola Jokic, DEN) in both lists, the top10 homoploutia between these two variables is 0.2.
- In the 2000/01 complete regular season (right plot) only one player (Stephon Marbury, NJN) belongs to both lists and so the homoploutia is 0.1.
- There is a player (Gary Payton, SEA) close to enter to both top10 lists but that the others are far away (this is not the case in the current season).

Real data example 1

- As there are two players (Luka Doncic, LAL, and Nikola Jokic, DEN) in both lists, the top10 homoploutia between these two variables is 0.2.
- In the 2000/01 complete regular season (right plot) only one player (Stephon Marbury, NJN) belongs to both lists and so the homoploutia is 0.1.
- There is a player (Gary Payton, SEA) close to enter to both top10 lists but that the others are far away (this is not the case in the current season).
- So we could say that nowadays the players are more complete (at least in these two characteristics) than in 2000/01.

Real data example 1

- As there are two players (Luka Doncic, LAL, and Nikola Jokic, DEN) in both lists, the top10 homoploutia between these two variables is 0.2.
- In the 2000/01 complete regular season (right plot) only one player (Stephon Marbury, NJN) belongs to both lists and so the homoploutia is 0.1.
- There is a player (Gary Payton, SEA) close to enter to both top10 lists but that the others are far away (this is not the case in the current season).
- So we could say that nowadays the players are more complete (at least in these two characteristics) than in 2000/01.
- By studying these indices we can see the evolution of these characteristics.

Real data example 2

- We can consider multivariate homoploutia curves.

Real data example 2

- We can consider multivariate homoploutia curves.
- For example, the homoploutia curve of (X, Y, Z) is

$$\begin{aligned} H(u) &= \Pr(Y > q^Y_{1-u}, Z > q^Z_{1-u} | X > q^X_{1-u}) \\ &= \Pr(X > q^X_{1-u}, Z > q^Z_{1-u} | Y > q^Y_{1-u}) \\ &= \Pr(X > q^X_{1-u}, Y > q^Y_{1-u} | Z > q^Z_{1-u}) \\ &= \frac{\widehat{C}(u,u,u)}{u} = \frac{\widehat{\delta}(u)}{u}, \ \ u \in (0,1]. \end{aligned}$$

Real data example 2

- We can consider multivariate homoploutia curves.
- For example, the homoploutia curve of (X, Y, Z) is

$$
\begin{aligned}
H(u) &= \Pr(Y > q^Y_{1-u}, Z > q^Z_{1-u} | X > q^X_{1-u}) \\
&= \Pr(X > q^X_{1-u}, Z > q^Z_{1-u} | Y > q^Y_{1-u}) \\
&= \Pr(X > q^X_{1-u}, Y > q^Y_{1-u} | Z > q^Z_{1-u}) \\
&= \frac{\widehat{C}(u,u,u)}{u} = \frac{\widehat{\delta}(u)}{u}, \quad u \in (0,1].
\end{aligned}
$$

- The associated homoploutia index is

$$I_H = \int_0^1 [1 - H(u)] \mathrm{d}u = \int_0^1 \left[1 - \frac{\widehat{\delta}(u)}{u}\right] \mathrm{d}u.$$

Real data example 2

- We can consider multivariate homoploutia curves.
- For example, the homoploutia curve of (X, Y, Z) is

$$
\begin{aligned}
H(u) &= \Pr(Y > q^Y_{1-u}, Z > q^Z_{1-u} | X > q^X_{1-u}) \\
&= \Pr(X > q^X_{1-u}, Z > q^Z_{1-u} | Y > q^Y_{1-u}) \\
&= \Pr(X > q^X_{1-u}, Y > q^Y_{1-u} | Z > q^Z_{1-u}) \\
&= \frac{\widehat{C}(u,u,u)}{u} = \frac{\widehat{\delta}(u)}{u}, \quad u \in (0,1].
\end{aligned}
$$

- The associated homoploutia index is

$$
I_H = \int_0^1 [1 - H(u)] \mathrm{d}u = \int_0^1 \left[1 - \frac{\widehat{\delta}(u)}{u}\right] \mathrm{d}u.
$$

- As $\widehat{C}(u,u,u) \le \widehat{C}(u,u,1)$, the curve with three variables is smaller than the curves with two variables.

Real data example 2

- To illustrate this case we choose the data of Table 1.2.1 in the book Mardia et al. (1997), p. 3, with the marks in Mechanics, Vectors, Algebra, Analysis, Statistics of 88 students.

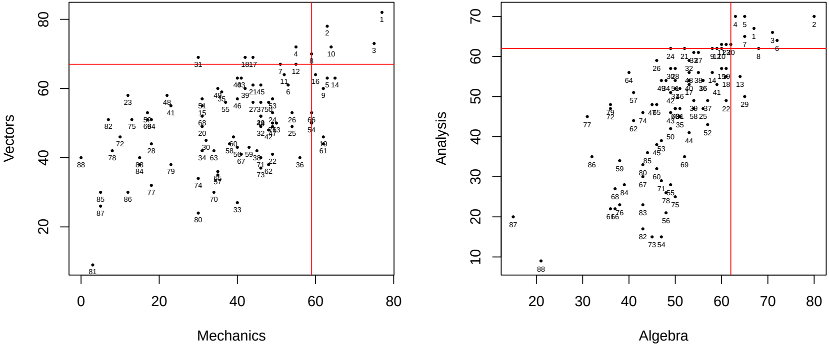

Figure: Students at the top10 lists in Mechanics and Vectors (left) and in Algebra and Analysis (right).

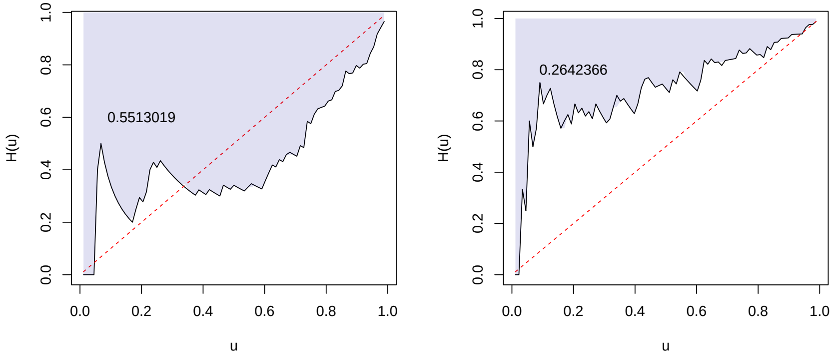

Figure: Homoploutia curves and indices for the students' data with all the subjects together (left) and just with Algebra and Analysis (right).

Real data example 3

- The “homophtocheia” (phtocheia is poverty in Greek) curve

$$\tilde{H}(u) = \Pr(X \leq q_u^X | Y \leq q_u^Y) = \frac{C(u,u)}{u} = \frac{\delta(u)}{u}, \ u \in (0,1].$$

Real data example 3

- The "homophtocheia" (phtocheia is poverty in Greek) curve

$$\tilde{H}(u) = \Pr(X \leq q_u^X | Y \leq q_u^Y) = \frac{C(u,u)}{u} = \frac{\delta(u)}{u}, \ \ u \in (0,1].$$

- It is applied to measure inequality and risk indices of extreme poverty in Spain (Encuesta Financiera de las Familias, EFF, Banco de España, 2022).

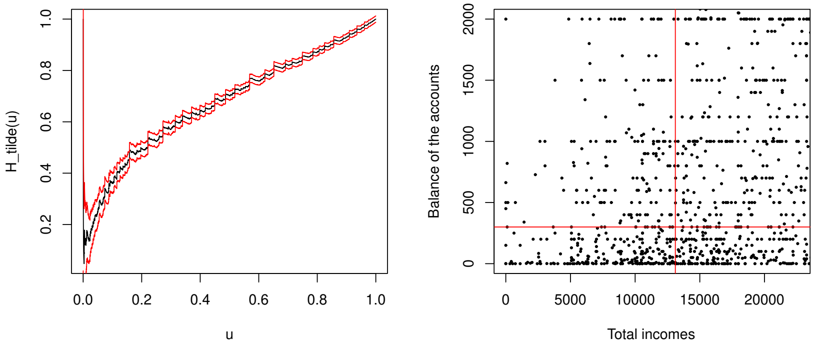

Figure: Homophtocheia curve and 95% confidence bands (left) for total incomes X and the balance of the bank accounts Y in Spain in 2021. In the right plot we can see the values of the 837 poorest families (below both first quartiles). The red lines represent the 10th sample percentiles.

Real data example 3

- The estimations with $n = 6377$ families are

$$\tilde{H}(637/n) = \tilde{H}(0.09989023) \approx \frac{231}{637} = 0.3626374 = \tilde{H}_n(637/n),$$

that is, for the 637 families below the horizontal red line, 231 of them are also to the left of the vertical red line.

Real data example 3

- The estimations with $n = 6377$ families are

$$\tilde{H}(637/n) = \tilde{H}(0.09989023) \approx \frac{231}{637} = 0.3626374 = \tilde{H}_n(637/n),$$

that is, for the 637 families below the horizontal red line, 231 of them are also to the left of the vertical red line.

- Analogously we see that

$$\tilde{H}(1594/n) = \tilde{H}(0.2499608) \approx \frac{837}{1594} = 0.5250941 = \tilde{H}_n(1594/n).$$

These 837 points are the ones showed in the right plot.

References

References

Berman, Y., Milanovic, B. (2024). Homoploutia: Top labor and capital incomes in the United States, 1950-2020. Review of Income and Wealth 70(3), 766–784.

Capaldo, M., Navarro, J. (2025). New multivariate Gini's indices. Journal of Multivariate Analysis 206, 105394.

Giangregorio, L., Villani, D. (2025). Functional distribution, personal income inequality, and top shares of income: do social classes still matter? The Journal of Economic Inequality 23, 143–176.

Mardia, K.V., Kent, J.T., Bibby, J.M. (1997). Multivariate Analysis. Academic Press, San Diego, United States.

Milanovic, B. (2019). Capitalism, Alone: The Future of the System that Rules the World. Harvard University Press.

Nelsen, R.B. (2006). An Introduction to Copulas. Springer.

Final slide

- The slides and more references are in my web page

https://webs.um.es/jorgenav/miwiki/doku.php

Final slide

- The slides and more references are in my web page

  https://webs.um.es/jorgenav/miwiki/doku.php

- Or type jorgenav in Google.

Final slide

- The slides and more references are in my web page

  https://webs.um.es/jorgenav/miwiki/doku.php

- Or type jorgenav in Google.
- That's all. Thank you for your attention!!

Final slide

- The slides and more references are in my web page

  https://webs.um.es/jorgenav/miwiki/doku.php

- Or type jorgenav in Google.
- That's all. Thank you for your attention!!
- Questions?