Weak Dependence Notions

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References

The conference is based on the following references:

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Outline

Preliminary results

Notation

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- $F_X(t) = \Pr(X \leq t)$ and $F_Y(t) = \Pr(Y \leq t)$ cumulative distribution functions (CDF).

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- $m_X(t) = E(X - t|X > t)$ and $m_Y(t) = E(Y - t|Y > t)$ mean residual (MRL) life functions.

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- $f_X = F_X'$ and $f_Y = F_Y'$ probability density functions (PDF).
- $h_X = f_X/\bar{F}_X$ and $h_Y = f_Y/\bar{F}_Y$ hazard rate (HR) functions.
- $m_X(t) = E(X - t|X > t)$ and $m_Y(t) = E(Y - t|Y > t)$ mean residual (MRL) life functions.
- $\bar{m}_X(t) = E(t - X|X < t)$ and $\bar{m}_Y(t) = E(t - Y|Y < t)$ mean inactivity time (MIT) functions.

Main stochastic orders

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- Mean residual life order: $X \leq_{MRL} Y \Leftrightarrow m_X \leq m_Y$.
- Mean inactivity time order: $X \leq_{MIT} Y \Leftrightarrow \bar{m}_X \geq \bar{m}_Y$.
- Likelihood ratio order: $X \leq_{LR} Y \Leftrightarrow f_Y/f_X$ increases.
- Increasing concave order $X \leq_{ICV} Y \Leftrightarrow E(\phi(X)) \leq E(\phi(X))$ for all increasing and concave functions ϕ.

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- Mean inactivity time order: $X \leq_{MIT} Y \Leftrightarrow \bar{m}_X \geq \bar{m}_Y$.
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- Increasing concave order $X \leq_{ICV} Y \Leftrightarrow E(\phi(X)) \leq E(\phi(X))$ for all increasing and concave functions ϕ.
- Increasing convex order $X \leq_{ICX} Y \Leftrightarrow E(\phi(X)) \leq E(\phi(X))$ for all increasing and convex functions ϕ.

Main stochastic orders

- Relationships:

$$\begin{array}{ccccc}
X \leq_{LR} Y & \Rightarrow & X \leq_{HR} Y & \Rightarrow & X \leq_{MRL} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{RHR} Y & \Rightarrow & X \leq_{ST} Y & \Rightarrow & X \leq_{ICX} Y \\
\Downarrow & & \Downarrow & & \Downarrow \\
X \leq_{MIT} Y & \Rightarrow & X \leq_{ICV} Y & \Rightarrow & E(X) \leq E(Y)
\end{array}$$

Properties based on residual lifetimes

Proposition

If $X_t = (X - t|X > t)$ *and* $Y_t = (Y - t|Y > t)$, *the following conditions are equivalent:*

i) $X \leq_{HR} Y$;

ii) $X_t \leq_{HR} Y_t$ *for all* t;

iii) $X_t \leq_{ST} Y_t$ *for all* t;

iv) $h_X \geq h_Y$ *(abs. cont. case).*

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i) $X \leq_{LR} Y$;

ii) $X_t \leq_{LR} Y_t$ *for all* t;

iii) $X_t \leq_{RHR} Y_t$ *for all* t;

iv) $(X|s < X < t) \leq_{ST} (Y|s < Y < t)$ *for all* $0 < s < t$.

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Proposition

The following conditions are equivalent:

i) $X \leq_{MRL} Y$;

ii) $X_t \leq_{MRL} Y_t$ for all t;

iii) $X_t \leq_{ICX} Y_t$ for all t.

Properties based on inactivity times

Proposition

If ${}_tX = (t - X|X < t)$ *and* ${}_tY = (t - Y|Y < t)$, *the following conditions are equivalent:*

i) $X \leq_{RHR} Y$;

ii) ${}_tX \geq_{HR} {}_tY$ *for all* $t > 0$;

iii) ${}_tX \geq_{ST} {}_tY$ *for all* $t > 0$.

Properties based on inactivity times

Proposition

The following conditions are equivalent:

i) $X \leq_{LR} Y$;

ii) ${}_tX \geq_{LR} {}_tY$ *for all* $t > 0$;

iii) ${}_tX \geq_{RHR} {}_tY$ *for all* $t > 0$;

iv) $(X|s < X < t) \leq_{ST} (Y|s < Y < t)$ *for all* $0 < s < t$.

Properties based on inactivity times

Proposition

The following conditions are equivalent:

i) $X \leq_{MIT} Y$;

ii) ${}_tX \geq_{MRL} {}_tY$ for all $t > 0$;

iii) ${}_tX \geq_{ICX} {}_tY$ for all $t > 0$.

Dependence notions

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- $RTI(X|Y)$ (Right Tail Increasing) iff $(X|Y > t)$ ST-increasing.

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- $LTI(X|Y)$ (Left Tail Increasing) iff $(X|Y \leq t)$ is ST-decreasing.
- $SI(X|Y)$ (Stochastically Increasing) iff $(X|Y = t)$ is ST-increasing.

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Dependence notions

- *RCSI* (Right Corner Set Increasing) iff $\Pr(X > x, Y > y | X > s, Y > t)$ is increasing in s and t for all x, y or, equivalently, if $(X|Y > t)$ is HR-increasing in t.

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- *LCSD* (Left Corner Set Decreasing) iff $\Pr(X \leq x, Y \leq y | X \leq s, Y \leq t)$ is decreasing in s and t for all x, y or, equivalently if $(X|Y \leq t)$ is RHR-increasing in t.

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- *RCSD* (Right Corner Set Decreasing) iff $\Pr(X > x, Y > y | X > s, Y > t)$ is decreasing in s and t for all x, y or, equivalently, if $(X|Y > t)$ is HR-decreasing in t.
- *LCSD* (Left Corner Set Decreasing) iff $\Pr(X \leq x, Y \leq y | X \leq s, Y \leq t)$ is decreasing in s and t for all x, y or, equivalently if $(X|Y \leq t)$ is RHR-increasing in t.
- *LCSI* (Left Corner Set Increasing) iff $\Pr(X \leq x, Y \leq y | X \leq s, Y \leq t)$ is increasing in s and t for all x, y or, equivalently if $(X|Y \leq t)$ is RHR-decreasing in t.

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Dependence notions

- *SIRL*$(X|Y)$ (Stochastically Increasing in Residual Life) iff $(X|Y=t)$ is HR-increasing in t.
- *SDRL*$(X|Y)$ (Stochastically Decreasing in Residual Life) iff $(X|Y=t)$ is HR-decreasing in t.
- *PRLD* (Positive Likelihood Ratio Dependent) iff its joint density function is TP_2 or, equivalently, if $(X|Y=t)$ LR-increasing in t.

Dependence notions

- $SIRL(X|Y)$ (Stochastically Increasing in Residual Life) iff $(X|Y=t)$ is HR-increasing in t.
- $SDRL(X|Y)$ (Stochastically Decreasing in Residual Life) iff $(X|Y=t)$ is HR-decreasing in t.
- $PRLD$ (Positive Likelihood Ratio Dependent) iff its joint density function is TP_2 or, equivalently, if $(X|Y=t)$ LR-increasing in t.
- $NRLD$ (Negative Likelihood Ratio Dependent) iff its joint density function is RR_2 or, equivalently, if $(X|Y=t)$ LR-decreasing in t.

Copula representations

- (X, Y) non-negative random vector.

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- The joint distribution function can be written as

$$F(x, y) = \Pr(X \le x, Y \le y) = C(F_X(x), F_Y(y))$$

for all $x, y \in \mathbb{R}$, where $C : [0,1]^2 \to [0,1]$ is a copula function.

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for all $x, y \in \mathbb{R}$, where $C : [0, 1]^2 \to [0, 1]$ is a copula function.

- The joint survival (or reliability) function can be written as

$$\bar{F}(x, y) = \Pr(X > x, Y > y) = \widehat{C}(\bar{F}_X(x), \bar{F}_Y(y))$$

for all $x, y \in \mathbb{R}$, where $\widehat{C} : [0, 1]^2 \to [0, 1]$ is a copula function called **survival copula**.

Properties

Proposition

The following conditions are equivalent:

i) $X \leq_{ST} (X|Y > s)$ *for all* s *and all* $\bar{F}_X, \bar{F}_Y$;

ii) $Y \leq_{ST} (Y|X > t)$ *for all* t *and all* $\bar{F}_X, \bar{F}_Y$;

iii) $\widehat{C}(u, v) \geq uv$ *for all* $u, v \in (0, 1)$;

iv) $Cov(\phi_1(X), \phi_2(Y)) \geq 0$ *for all increasing functions* ϕ_1 *and* ϕ_2;

iv) (X, Y) *is PQD.*

Properties

Proposition

If $X_{t,s} = (X - t|X > t, Y > s)$ and $Y_{t,s} = (Y - s|X > t, Y > s)$, for continuous $\bar{F}_X, \bar{F}_Y$ the following conditions are equivalent:

i) $X \leq_{HR} (X|Y > s)$ *for all* s *and all* $\bar{F}_X, \bar{F}_Y$*;*

ii) $X_t \leq_{HR} X_{t,s}$ *for all* t, s *and all* $\bar{F}_X, \bar{F}_Y$*;*

iii) $X_t \leq_{ST} X_{t,s}$ *for all* t, s *and all* $\bar{F}_X, \bar{F}_Y$*;*

iv) $\widehat{C}(u, v)/u$ *is decreasing in* u *for all* $v \in (0, 1)$*;*

v) $RTI(Y|X)$.

Properties

Proposition

The following conditions are equivalent:

i) $X \leq_{LR} (X|Y > s)$ *for all* s *and all* $\bar{F}_X, \bar{F}_Y$;

ii) $X_t \leq_{LR} X_{t,s}$ *for all* t, s *and all* $\bar{F}_X, \bar{F}_Y$;

iii) $X_t \leq_{RHR} X_{t,s}$ *for all* t, s *and all* $\bar{F}_X, \bar{F}_Y$;

iv) $\widehat{C}(u,v)$ *is concave in* u *(or* $\partial_1 \widehat{C}(u,v)$ *is decreasing in* u*) for all* $v \in (0,1)$;

v) $SI(Y|X)$.

Properties

Proposition

The following conditions are equivalent:

i) $(X|Y > s_1) \leq_{HR} (X|Y > s_2)$ *for all* $s_1 \leq s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
ii) $(Y|X > t_1) \leq_{HR} (Y|X > t_2)$ *for all* $t_1 \leq t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
iii) $X_{t,s_1} \leq_{HR} X_{t,s_2}$ *for all* t, *all* $s_1 \leq s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
iv) $X_{t,s_1} \leq_{ST} X_{t,s_2}$ *for all* t, *all* $s_1 \leq s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
v) $Y_{t_1,s} \leq_{HR} Y_{t_2,s}$ *for all* s, *all* $t_1 \leq t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
vi) $Y_{t_1,s} \leq_{ST} Y_{t_2,s}$ *for all* s, *all* $t_1 \leq t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;
vii) $\widehat{C}(u,v)$ *is* TP_2;
viii) (X,Y) *is RCSI*.

Properties

Proposition

The following conditions are equivalent:

i) $(X|Y > s_1) \leq_{LR} (X|Y > s_2)$ for all $s_1 < s_2$ and all $\bar{F}_X, \bar{F}_Y$;

ii) $(X_t|Y > s_1) \leq_{LR} (X_t|Y > s_2)$ for all t, $s_1 < s_2$ and all $\bar{F}_X, \bar{F}_Y$;

iii) $(X_t|Y > s_1) \leq_{RHR} (X_t|Y > s_1)$ for all t, $s_1 < s_2$ and all $\bar{F}_X, \bar{F}_Y$;

iv) $\partial_1 \widehat{C}(u, v)$ is TP_2;

v) $SIRL(Y|X)$.

Properties

Proposition

The following conditions are equivalent:

i) $(Y|X = t_1) \leq_{LR} (Y|X = t_2)$ *for all* $t_1 < t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

ii) $(X|Y = s_1) \leq_{LR} (X|Y = s_2)$ *for all* $s_1 < s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

iii) $(X_t|Y = s_1) \leq_{LR} (X_t|Y = s_2)$ *for all* t, $s_1 < s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

iv) $(X_t|Y = s_1) \leq_{RHR} (X_t|Y = s_2)$ *for all* t, $s_1 < s_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

v) $(Y_s|X = t_1) \leq_{LR} (Y_s|X = t_2)$ *for all* s, $t_1 < t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

vi) $(Y_s|X = t_1) \leq_{RHR} (Y_s|X = t_2)$ *for all* s, $t_1 < t_2$ *and all* $\bar{F}_X, \bar{F}_Y$;

vii) $\partial^2_{1,2} C(u, v)$ *is* TP_2;

viii) *PLRD.*

New dependence notions

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- We say that (X, Y) is Stochastically Increasing (Decreasing) in the order ORD, denoted $SI_{ORD}(X|Y)$ ($SD_{ORD}(X|Y)$) if

$$(X|Y = t_1) \leq_{ORD} (X|Y = t_2) \quad (\geq_{ORD})$$

holds for all $t_1 < t_2$ for a given stochastic order ORD.

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holds for all $t_1 < t_2$ for a given stochastic order ORD.
- With this definition $SI(X|Y) = SI_{ST}(X|Y)$, $SIRL(X|Y) = SI_{HR}(X|Y)$ and $PLRD = SI_{LR}(Y|X)$.

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holds for all $t_1 < t_2$ for a given stochastic order ORD.

- With this definition $SI(X|Y) = SI_{ST}(X|Y)$, $SIRL(X|Y) = SI_{HR}(X|Y)$ and $PLRD = SI_{LR}(Y|X)$.
- Note that $SI_{LR}(Y|X)$ is equivalent to $SI_{LR}(X|Y)$ and so we can just write SI_{LR}.

New dependence notions

- We say that (X, Y) is Right Tail Increasing (Decreasing) in the order ORD, denoted $RTI_{ORD}(X|Y)$ $(RTD_{ORD}(X|Y))$, if

$$(X|Y > t_1) \leq_{ORD} (X|Y > t_2) \quad (\geq_{ORD})$$

holds for all $t_1 < t_2$ for a given stochastic order ORD.

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- Then $RTI(X|Y) = RTI_{ST}(X|Y)$,
$RCSI = RTI_{HR}(X|Y) = RTI_{HR}(Y|X)$ and
$SIRL(Y|X) = RTI_{LR}(X|Y)$.

New dependence notions

- We say that (X, Y) is Right Tail Increasing (Decreasing) at zero in the order ORD, denoted $RTI^0_{ORD}(X|Y)$ ($RTD^0_{ORD}(X|Y)$), if

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- The proofs of these equivalences can be found in Navarro and Sordo (2018) and Longobardi and Pellerey (2019).
- Some of these notions are actually equivalent, like, e.g., $RTI^0_{LR}(Y|X)$ and $SI_{ST}(X|Y)$, or $RTI^0_{HR}(Y|X)$ and $RTI_{ST}(X|Y)$ (see Foschi and Spizzichino (2013) for details).

Relationships

$$
\begin{array}{ccccc}
PQD(X,Y) & \Leftarrow & RTI(Y|X) & \Leftarrow & SI(Y|X) \\
\Uparrow & & \Uparrow & & \Uparrow \\
RTI(X|Y) & \Leftarrow & RCSI(X,Y) & \Leftarrow & SIRL(Y|X) \\
\Uparrow & & \Uparrow & & \Uparrow \\
SI(X|Y) & \Leftarrow & SIRL(X|Y) & \Leftarrow & PLRD(X,Y)
\end{array}
$$

Table: Relationships among positive dependence properties.

Relationships

$$\begin{array}{ccccc}
SI(Y|X) & \Rightarrow & LTD(Y|X) & \Rightarrow & PQD(X,Y) \\
\Uparrow & & \Uparrow & & \Uparrow \\
SI_{RHR}(Y|X) & \Rightarrow & LCSD(X,Y) & \Rightarrow & LTD(X|Y) \\
\Uparrow & & \Uparrow & & \Uparrow \\
PLRD(X,Y) & \Rightarrow & SI_{RHR}(X|Y) & \Rightarrow & SI(X|Y)
\end{array}$$

Table: Relationships among reversed positive dependence properties.

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- However, they imply non-negativity of the linear Pearson's correlation coefficient $r_{X,Y}$.
- For that reason, they can be considered as "weak positive dependence notions".
- The negative dependence properties are defined in a similar way.

Weak dependence notions

Proposition

The following conditions are equivalent:

i) $X \leq_{ICX} (X|Y > y)$ *for all* y *(i.e.* $RTI^0_{ICX}(X|Y)$*);*

ii) *The survival copula* $\widehat{C}$ *satisfies*

$$\int_0^z [\widehat{C}(u,v) - uv]\, d\bar{F}_X^{-1}(u) \leq 0,\ \forall z \in [0,1],\ \forall v \in [0,1]. \tag{2.1}$$

Weak dependence notions

Proposition

The following conditions are equivalent:

i) $X \leq_{MRL} (X|Y > y)$ *for all* y *(i.e.* $RTI^0_{MRL}(X|Y)$*);*

ii) $X_t \leq_{MRL} X_{t,s}$ *for all* t, s*;*

iii) $X_t \leq_{ICX} X_{t,s}$ *for all* t, s*;*

iv) *The survival copula* $\widehat{C}$ *satisfies*

$$\int_0^z [z\widehat{C}(u,v) - u\widehat{C}(z,v))\, d\bar{F}_X^{-1}(u) \leq 0 \;\forall z, v \in [0,1] \quad (2.2)$$

Weak dependence notions

- The Positive Quadrant Dependence in Expectation property ($PQDE$) was defined in Balakrishnan and Lai (2009) with $PQDE(X|Y)$ iff $E(X) \leq E(X|Y > y)$ for all $y \geq 0$.

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$$\int_0^1 (\widehat{C}(u,v) - uv)\, d\bar{F}_X^{-1}(u) \leq 0, \ \forall t \in [0,1], \ \forall v \in [0,1]. \quad (2.3)$$

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- Then $PQD \Rightarrow PQDE(X|Y) \Rightarrow r_{X,Y} \geq 0$.

Relationships between weak dependence notions

$$
\begin{array}{ccccc}
r_{X,Y} \geq 0 & & & & \\
\Uparrow & & & & \\
PQDE(X|Y) & \Leftarrow & RTI^0_{ICX}(X|Y) & \Leftarrow & RTI^0_{MRL}(X|Y) \\
\Uparrow & & \Uparrow & & \Uparrow \\
RTI^0_{ICX}(Y|X) & \Leftarrow & PQD(X,Y) & \not\Leftarrow & RTI_{MRL}(X|Y) \\
\Uparrow & & \not\Uparrow & & \Uparrow \\
RTI^0_{MRL}(Y|X) & \Leftarrow & RTI_{MRL}(Y|X) & \Leftarrow & RCSI(X,Y)
\end{array}
$$

Table: Relationships among weak positive dependence properties.

Relationships between weak dependence notions

$$
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\Uparrow & & & & \\
PQDE(X|Y) & \Leftarrow & RTI^0_{ICX}(X|Y) & \Leftarrow & RTI^0_{MRL}(X|Y) \\
\Uparrow & & \Uparrow & & \Uparrow \\
RTI^0_{ICX}(X|Y) & \Leftarrow & RTI_{ICX}(X|Y) & \Leftarrow & RTI_{MRL}(X|Y) \\
\Uparrow & & \Uparrow & & \Uparrow \\
\mathrm{PQD(X,Y)} & \Leftarrow & RTI_{ST}(X|Y) & \Leftarrow & RCSI(X,Y)
\end{array}
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Table: Relationships among weak positive dependence properties.

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Other dependence properties

- All the properties mentioned above are not independent on the marginal distributions.
- However, interesting properties of the survival copula of the vector are introduced when we assume uniform marginals.
- They can be used to define new weak dependence properties that do not depend on the marginals.
- In fact, letting P denote the property

$$\int_0^z [\widehat{C}(u,v) - uv]\, du \geq 0, \forall z \in [0,1], \forall v \in [0,1],$$

and letting $\widetilde{P}$ denote the property

$$X \leq_{ICX} (X|Y > y)\ \forall y$$

one immediately observes that both are positive dependence properties weaker than PQD.

Other dependence properties

- Property P satisfies

$$PQD \Rightarrow P \Rightarrow \rho_{X,Y} \geq 0,$$

where

$$\rho_{X,Y} = 12 \int_0^1 \int_0^1 C(u,v) dudv - 3$$

is the Spearman's rho coefficient for X and Y (for the formula of $\rho_{X,Y}$, see (5.1.15c) in Nelsen (2006)), and the first implication follows from (2.1).

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- Property $\widetilde{P}$ satisfies

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- Property $\widetilde{P}$ satisfies

$$PQD \Rightarrow \widetilde{P} \Rightarrow r_{X,Y} \geq 0. \tag{2.4}$$

- One can define weak dependence properties as it has been done for P, by letting the margins to be uniformly distributed on $(0,1)$ in the definitions above.

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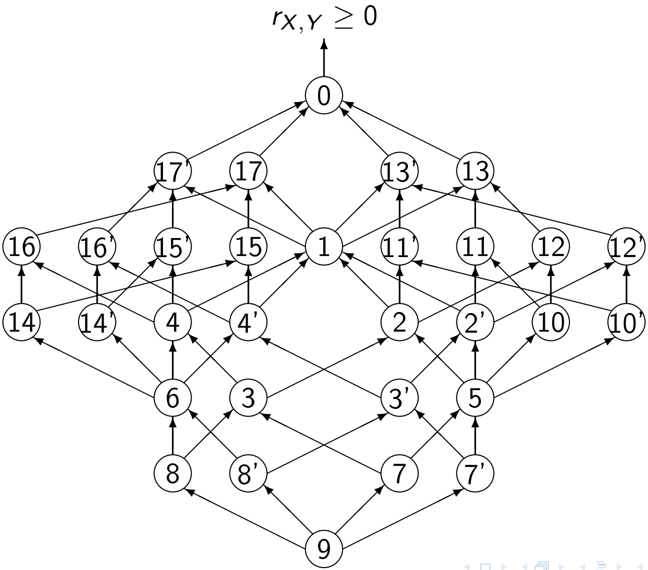
- In the paper we also include counterexamples showing that these classes are different.
- Finally, we conclude with a diagram that connect all these properties.


$r_{X,Y} \geq 0$
0
17'
17
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16
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1
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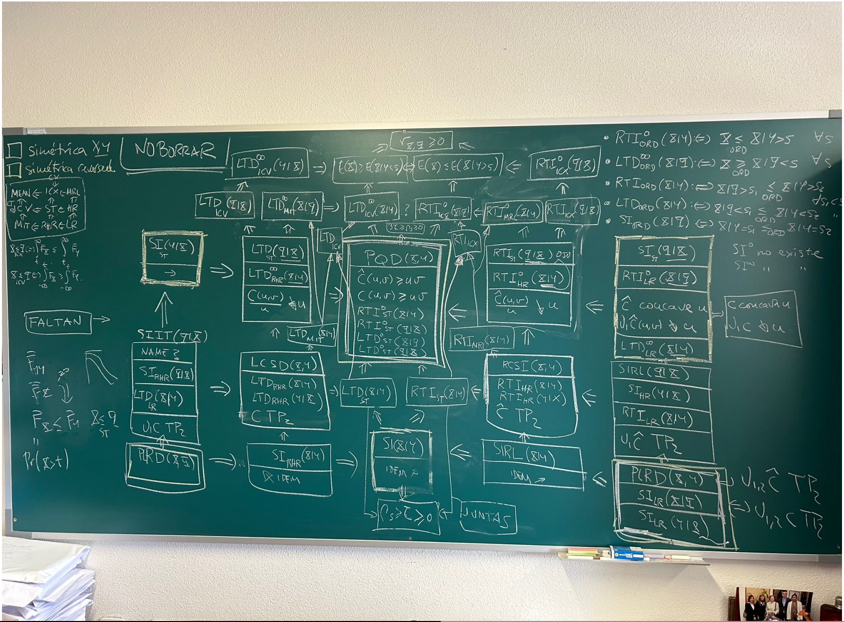
4'
2
2'
10
10'
6
3
3'
5
8
8'
7
7'
9

N	Name		
0	$PQDE$	$E(X) \leq E(X\|Y > s)$	$E(X) \geq E(X\|Y \leq s)$
1	PQD	$RTI^0_{ST}(Y\|X)$	$LTD^{\infty}_{ST}(Y\|X)$
2	$RTI(Y\|X)$	$RTI_{ST}(Y\|X)$	$RTI^0_{HR}(X\|Y)$
2'	$RTI(X\|Y)$	$RTI_{ST}(X\|Y)$	$RTI^0_{HR}(Y\|X)$
3	$SI(Y\|X)$	$SI_{ST}(Y\|X)$ or $RTI^0_{LR}(X\|Y)$	$LTD^{\infty}_{LR}(X\|Y)$
3'	$SI(X\|Y)$	$SI_{ST}(X\|Y)$ or $RTI^0_{LR}(Y\|X)$	$LTD^{\infty}_{LR}(Y\|X)$
4	$LTD(Y\|X)$	$LTD_{ST}(Y\|X)$	$LTD^{\infty}_{RHR}(X\|Y)$
4'	$LTD(X\|Y)$	$LTD_{ST}(X\|Y)$	$LTD^{\infty}_{RHR}(Y\|X)$
5	$RCSI$	$RTI_{HR}(Y\|X)$	$RTI_{HR}(X\|Y)$
6	$LCSD$	$LTD_{RHR}(Y\|X)$	$LTD_{RHR}(X\|Y)$
7	$SIRL(Y\|X)$	$SI_{HR}(Y\|X)$	$RTI_{LR}(X\|Y)$
7'	$SIRL(X\|Y)$	$SI_{HR}(X\|Y)$	$RTI_{LR}(Y\|X)$
8		$SI_{RHR}(Y\|X)$	$LTD_{LR}(X\|Y)$
8'		$SI_{RHR}(X\|Y)$	$LTD_{LR}(Y\|X)$
9	$PLRD$	$SI_{LR}(Y\|X)$	$SI_{LR}(X\|Y)$

N	Name	New dependence notions
10	$RTI_{MRL}(Y\|X)$	$(Y\|X>s) \leq_{MRL} (Y\|X>t)$ for all $s<t$
10'	$RTI_{MRL}(X\|Y)$	$(X\|Y>s) \leq_{MRL} (X\|Y>t)$ for all $s<t$
11	$RTI^0_{MRL}(Y\|X)$	$Y \leq_{MRL} (Y\|X>t)$ for all $0<t$
11'	$RTI^0_{MRL}(X\|Y)$	$X \leq_{MRL} (X\|Y>t)$ for all $0<t$
12	$RTI_{ICX}(Y\|X)$	$(Y\|X>s) \leq_{ICX} (Y\|X>t)$ for all $s<t$
12'	$RTI_{ICX}(X\|Y)$	$(X\|Y>s) \leq_{ICX} (X\|Y>t)$ for all $s<t$
13	$RTI^0_{ICX}(Y\|X)$	$Y \leq_{ICX} (Y\|X>t)$ for all $0<t$
13'	$RTI^0_{ICX}(X\|Y)$	$X \leq_{ICX} (X\|Y>t)$ for all $0<t$
14	$LTD_{MIT}(Y\|X)$	$(Y\|X\leq s) \leq_{MIT} (Y\|X\leq t)$ for all $s<t$
14'	$LTD_{MIT}(X\|Y)$	$(X\|Y\leq s) \leq_{MIT} (X\|Y\leq t)$ for all $s<t$
15	$LTD^{\infty}_{MIT}(Y\|X)$	$(Y\|X\leq s) \leq_{MIT} Y$ for all s
15'	$LTD^{\infty}_{MIT}(X\|Y)$	$(X\|Y\leq s) \leq_{MIT} X$ for all s
16	$LTD_{ICV}(Y\|X)$	$(Y\|X\leq s) \leq_{ICV} (Y\|X\leq t)$ for all $s<t$
16'	$LTD_{ICV}(X\|Y)$	$(X\|Y\leq s) \leq_{ICV} (X\|Y\leq t)$ for all $s<t$
17	$LTD^{\infty}_{ICV}(Y\|X)$	$(Y\|X\leq s) \leq_{ICV} Y$ for all s
17'	$LTD^{\infty}_{ICV}(X\|Y)$	$(X\|Y\leq s) \leq_{ICV} X$ for all s


NO BORRAR
FALTAN
NAME?
PQD(X,Y)
LCSD(X,Y)
RCSI(X,Y)
SIRL(X|Y)
JUNTAS
SI° no existe

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- Similar negative dependence can be proposed as well.
- The main disadvantage of the new dependence notions proposed here is that they depend on the marginal distributions (as the Pearson's correlation coefficient).
- This problem can be solved by replacing them with the respective copula properties obtained by assuming uniform marginals.
- They can also be related with properties of coherent systems (see Navarro, Durante and Fernández-Sánchez (2021) and Navarro (2022)).

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- That's all. Thank you for your attention!!

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- That's all. Thank you for your attention!!
- Questions?