Manifestly T-dual formulation of AdS space

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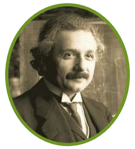
ARXIVE:1701.0671

REFERENCES: ARXIVE:1403.3887, 1411.2206,1507.0361

I. Introduction & results

T-duality

◆ Beyond Einstein gravity



⇒ Low energy effective theory of string

<u>Gravity with T-duality</u>

T-duality

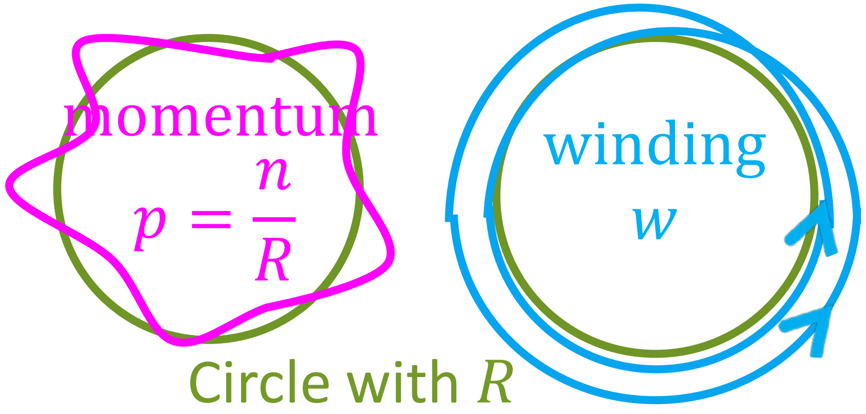
- momentum p ↔ winding w

⇒ $R \leftrightarrow 1/R$ (α' omitted) for circle

⇒ $G_{mn} \leftrightarrow 1/G_{mn}$ in general

i.e. Duality of short / long distance

◆ No initial singularity of our early universe !



$$\mathcal{H}_{0\ \mathrm{mode}} = \frac{n^2}{R^2} + w^2 R^2$$

$$\mathcal{H} = \frac{1}{2}(p, \partial_\sigma x) \begin{pmatrix} G^{-1} & GB \\ -BG & G - BG^{-1}B \end{pmatrix} \begin{pmatrix} p \\ \partial_\sigma x \end{pmatrix}$$

Global O(d,d) T-duality symmetry

Two vielbein

$$(G+B)_{mn} \quad = \quad e_m{}^a (e')_{an}$$

Linear transf. under O(d,d)

$$\begin{pmatrix} e' & e^{-1} \end{pmatrix} \quad \to \quad \begin{pmatrix} e' & e^{-1} \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$= \quad \begin{pmatrix} e'a + e^{-1}b & e'c + e^{-1}d \end{pmatrix}$$

Fractional transf. under O(d,d)

$$(G+B) \quad \to \quad \frac{e'a + e^{-1}b}{e'c + e^{-1}d} = \frac{(G+B)a + b}{(G+B)c + d}$$

Doubled vielbein

$$e^{\mathrm{T}} \hat{\eta} e = \begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

Double Lorentz inv.

$$\hat{\eta} \quad = \quad \begin{pmatrix} & \mathbf{1} \\ \mathbf{1} & \end{pmatrix}$$

Linear transf. under O(d,d)

$$e \to eA$$

$\leftrightarrow$ O(d,d) is symmetry of $\mathcal{H}$

$$\#(G,B) = \# \frac{\mathrm{O(d,d)}}{\mathrm{SO(d-1,1)}^2} = \mathrm{d}^2$$

Background

◆T-duality: '84 Kikkawa & Yamazaki, '86 Sakai & Senda,…'97 Buscher,….., Doubled coordinates: '89 Duff, '90 Tseytlin,…

◆Non-abelian T-duality: '93 de la Ossa & Quevedo,Gasperini, Ricci & Veneziano, '94 Giveon & Rocek, Alvarez, Alvarez-Gaume, Barbon, Lozano, Itsios, Nunez,Sfetsos,Thompson,O Colgain, Hassler, Lust,…

⇒ New description of stringy gravity & new geometry by T-duality

◆Double field theory: '93 Siegel, '09 Hull & Zwiebach, '10 Hohm, Kwaw, Jeon, Lee, Park, Thompson, Berman,'13 Polacek, Siegel, '14 Kamimura, Siegel, M.H.,… , Sakatani, Uehara, Rey, Dibitetto, Fernandez-Melgarejo,

◆Generalized geometry: '02 Hitchin, '04 Gualtieri,'07 Hull, '08 Pacchoco & Waldram,'09 Grana, Lous, sim, '10 Berman & Perry, '11 Coimbra, Strickland-Constable, Exotic: de Bohr, Shigemori, Kimura, yata, Sasaki, Ikeda, Watamura, Heller

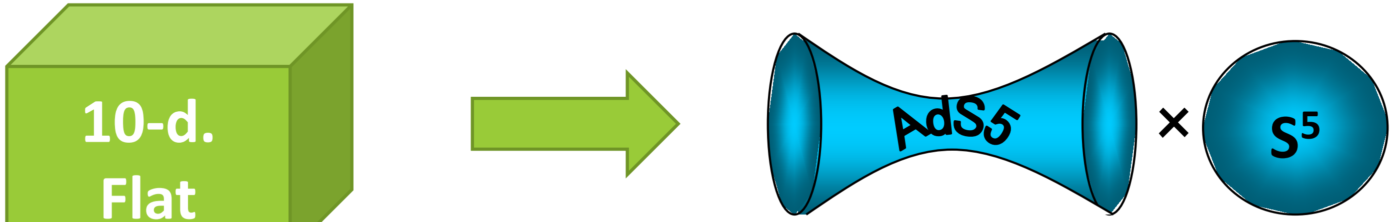
⇒ New aspects of AdS/CFT duality

Our approach : AdS in the doubled space

◆Integrability & T-duality: '02 Klimcik, '06 Mizoguchi & M.H, '07 Ricci, Tseytlin & Wolf ,….'16 Hoare & Tseytlin, Thompson, Borsato, Wulff, Lozano, Macpherson, Montero, Nunez , Sakamoto , Yoshida, …

Highlights: Questions on AdS w/ T-duality

Q1. What is the T-duality coset of AdS which is S.S.B by RR flux?



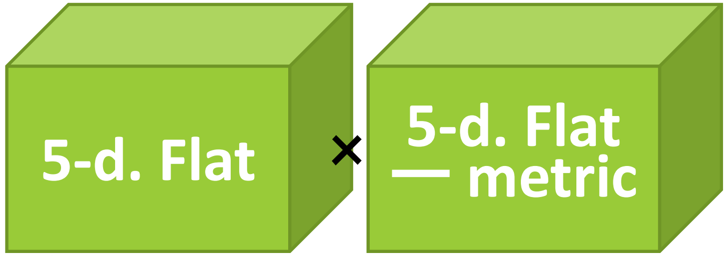
Lorentz group : SO(9,1) $\quad$ SO(4,1) × SO(5)

T-duality coset: $\frac{O(10,10)}{SO(9,1)^2}$ $\quad$ $\frac{O(10,10)}{SO(4,1)^2 SO(5)^2}$?

G_{mn}&B_{mn} are parameters of the coset $\quad$ i.e. $\#(G_{mn}$ & $B_{mn}) \overset{?}{=}$ dim. of the coset

Highlights: Questions on AdS w/ T-duality

Q2. What is the T-dual space of AdS ?



Doubled space metric

$$\eta_{\underline{mn}} = (\eta_{mn};\eta_{m'n'}) = (-1,1,\cdots,1;\underline{1,-1,\cdots,-1})$$

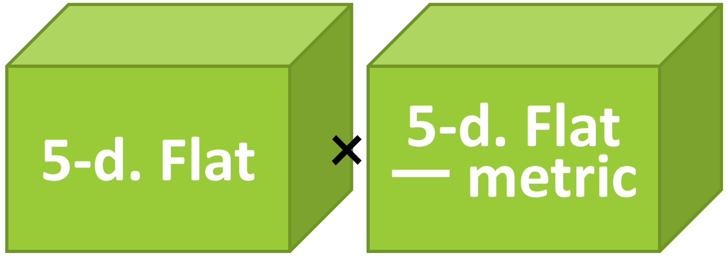
$$\eta_{\underline{mn}} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix} \to \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

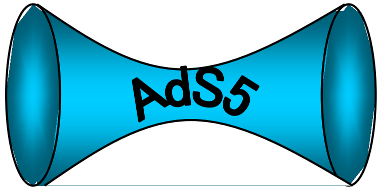
$$\left\{ \begin{array}{l} p_m \\ \partial_\sigma x^m \end{array} \right. \to \left\{ \begin{array}{lcl} P_m & = & p_m + \partial_\sigma x^m \\ P_{m'} & = & p_m - \partial_\sigma x^m \end{array} \right.$$

Left / **Right**

Highlights: Questions on AdS w/ T-duality

Q2. What is the T-dual space of AdS ?





× ?

Doubled space metric

$$\eta_{\underline{mn}} = (\eta_{mn};\eta_{m'n'}) = (-1,1,\cdots,1;\underline{1,-1,\cdots,-1})$$

AdS embedded in the space

$$(\eta_{\natural\natural};\eta_{mn}) = (-1;-1,1,\cdots,1)$$

Doubled space metric

$$(\eta_{mn};\eta_{m'n'}) = (-1,1,\cdots,1;\underline{1,-1,\cdots,-1})$$

How O(d,d) T-dual symmetry coexist with AdS isometry ?

What is T-dual space metric ?

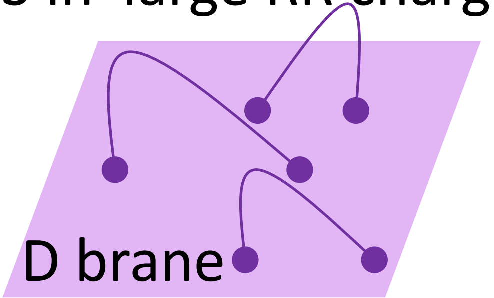
$$(\eta_{\natural\natural};\eta_{m'n'}) = \ ?$$

Highlights: Questions on AdS w/ T-duality

Q3. Do the left and right momenta mix by the RR charge ?

- N=2 Superalgebra with the RR charge becomes super-AdS in large RR charge.

Left	$\{\tilde{D}_\mu, \tilde{D}_\nu\}$	$=$	$\tilde{P}_{\mu\nu}$
Left-right mixed	$\{\tilde{D}_\mu, \tilde{D}_{\nu'}\}$	$=$	$\tilde{\Upsilon}_{\mu\nu'}$
Right	$\{\tilde{D}_{\mu'}, \tilde{D}_{\nu'}\}$	$=$	$\tilde{P}_{\mu'\nu'}$



- How about its bosonic part ? i.e. doubled AdS algebra ?

Left	$[\tilde{P}_m, \tilde{P}_n]$	$=$	$\tilde{S}_{mn}$
Left-right mixed	$[\tilde{P}_m, \tilde{P}_{n'}]$	$=$	?
Right	$[\tilde{P}_{m'}, \tilde{P}_{n'}]$	$=$	$\tilde{S}_{m'n'}$

Highlights: Questions on AdS w/ T-duality

Q4. How to reduce a half of doubled momenta to get physical momentum?

- Total momentum

$$\tilde{P}_{\text{total};m} = \tilde{P}_m + \tilde{P}_{m'}$$

Flat algebra $[\tilde{P}_{\text{total}}, \tilde{P}_{\text{total}}] \approx 0$

- Dimensional reduction constraint

$$\phi_m = \tilde{P}_m - \tilde{P}_{m'} = 0$$

1st class $[\phi_m, \phi_m] \approx 0$

OK! It preserves T-geometry.

For flat
'14 Kamimura, Siegel & M.H.

- Total momentum

$$\tilde{P}_{\text{total};m} \overset{?}{=} \tilde{P}_m + \tilde{P}_{m'}$$

AdS algebra $[\tilde{P}_{\text{total}}, \tilde{P}_{\text{total}}] \approx \tilde{S}_{\text{total}}$

- Dimensional reduction constraint

$$\phi_m \overset{?}{=} \tilde{P}_m - \tilde{P}_{m'} = 0$$

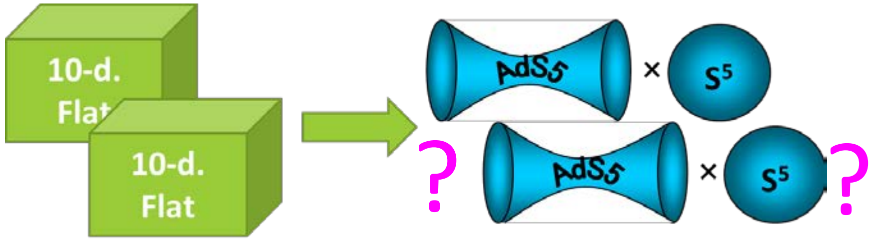
1st class ? $[\phi, \phi] \approx \tilde{S}_{\text{total}} \neq 0$?

as long as $[\tilde{P}_m, \tilde{P}_{n'}] = 0$

Highlights: An answer is “doubled” AdS

Q1. What is the T-duality coset of AdS which is S.S.B by RR flux?

Doubled Lorentz sym.



Left	$SO(9,1)$	
Right		$SO(1,9)$

⇒

Left AdS	$SO(4,1)$			
Left S		$SO(5)$		
Right AdS			$SO(1,4)$	
Right S				$SO(5)$

=

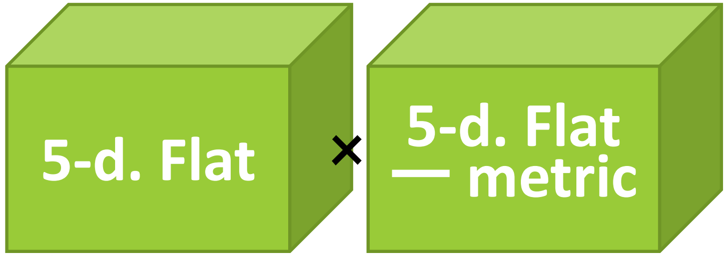
Left AdS	$SO(4,1)$	Mixed		
Right AdS	Mixed	$SO(1,4)$		
Left S			$SO(5)$	Mixed
Right S			Mixed	$SO(5)$

A1. T-duality coset: $\dfrac{O(10,10)}{SO(5,5)^2}$

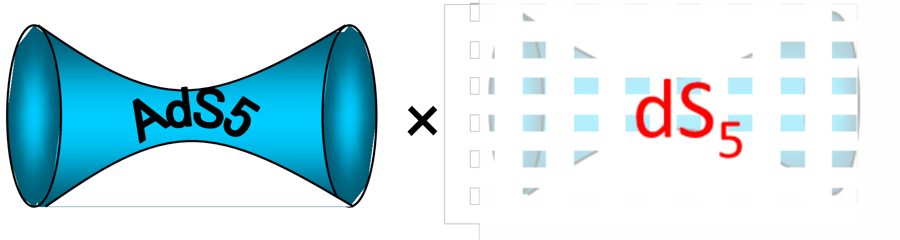
∴ $\#(G_{mn}$ & $B_{mn})$ = dim. of the coset !

Highlights: An answer is "doubled" AdS

Q2. What is the T-dual space of AdS ?



AdS$_5$ part is focused.



Doubled space metric

$$\eta_{\underline{mn}} = (\eta_{mn}; \eta_{m'n'}) = (-1, 1, \cdots, 1; 1, -1, \cdots, -1)$$

AdS embedded in the space

$$(\eta_{\natural\natural}; \eta_{mn}) = (\underline{-1}; -1, 1, \cdots, 1)$$

$(\natural)$	p_n	$p_{n'}$
p_m	$\mathrm{SO}(4,1)$	Mixed
$p_{m'}$	Mixed	$\mathrm{SO}(1,4)$

$\natural$ direction is common

Doubled space metric

$$(\eta_{mn}; \eta_{m'n'}) = (-1, 1, \cdots, 1; \underline{1, -1, \cdots, -1})$$

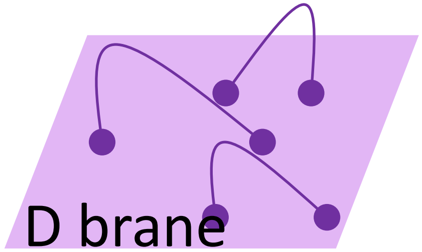
A2. T-dual space is "dS" metric

$$(\eta_{\natural\natural}; \eta_{m'n'}) = (\underline{-1}; \underline{1, -1, \cdots, -1})$$

Highlights: An answer is "doubled" AdS

Q3. Do the left and right momenta mix by the RR charge ?

- N=2 Superalgebra with the RR charge & super-AdS algebra



Left	$\{\tilde{D}_\mu, \tilde{D}_\nu\}$	$=$	$\tilde{P}_{\mu\nu}$	
Left-right mixed	$\{\tilde{D}_\mu, \tilde{D}_{\nu'}\}$	$=$	$\tilde{\Upsilon}_{\mu\nu'}$	$\Rightarrow \tilde{S}_{\mu\nu'}$
Right	$\{\tilde{D}_{\mu'}, \tilde{D}_{\nu'}\}$	$=$	$\tilde{P}_{\mu'\nu'}$	

A3. Yes, L & R mix.
SO(d,d+1)

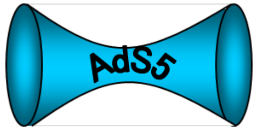
- How about doubled AdS algebra?

Left	$[\tilde{P}_m, \tilde{P}_n]$	$=$	$\tilde{S}_{mn}$
Left-right mixed	$[\tilde{P}_m, \tilde{P}_{n'}]$	$=$	$\tilde{S}_{mn'}$
Right	$[\tilde{P}_{m'}, \tilde{P}_{n'}]$	$=$	$\tilde{S}_{m'n'}$

	$\tilde{P}_n$	$\tilde{P}_{n'}$
$\tilde{P}_m$	$\tilde{S}_{mn}$	$\tilde{S}_{mn'}$
$\tilde{P}_{m'}$	$\tilde{S}_{m'n}$	$\tilde{S}_{m'n'}$

Highlights: An answer is "doubled" AdS

Q4. How to reduce a half of doubled momenta to get physical momentum?



× ?

- AdS Left & Right mixing leads to

$$[\tilde{P}_m, \tilde{P}_{n'}] = \tilde{S}_{mn'}$$

$$\Rightarrow \quad [\tilde{P}_{\rm total}, \tilde{P}_{\rm total}] \neq [\phi_m, \phi_n]$$

- Total momentum

$$\tilde{P}_{{\rm total};m} \overset{?}{=} \tilde{P}_m + \tilde{P}_{m'}$$

AdS algebra $[\tilde{P}_{\rm total}, \tilde{P}_{\rm total}] \approx \tilde{S}_{\rm total} \neq 0$

- Dimensional reduction constraint

$$\phi_m \overset{?}{=} \tilde{P}_m - \tilde{P}_{m'} = 0$$

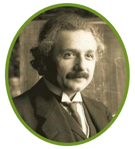
1st class ? $[\phi, \phi] \approx 0$

A4. $\phi_m = 0$ 1st class to keep T-duality geometry！
$\tilde{P}_{\rm total}$ makes physical AdS algebra！

Plan

I. Introduction and results

'17 Kamimura, Siegel & M.H.

II. Manifestly T-dual formulation
   1. Gravity as gauge theory 
   2. Stringy extension 
   3. Characteristics
   4. String action 

Review based on '93 Siegel

'14,15 Kamimura, Siegel & M.H.

III. Group manifold with T-duality

IV. AdS space with T-duality

'17 Kamimura, Siegel & M.H.

II. Manifestly T-dual formulation

1. Gravity as gauge theory

A_m

- Gauge theory
  - Gauge field & gauge algebra

$$A_m{}^I \qquad [G_I, G_J] = i f_{IJ}{}^K G_K$$

  - **Covariant derivative**

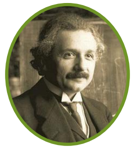
$$p_m = \frac{1}{i}\partial_m \quad \rightarrow \quad \nabla_m = p_m + A_m{}^I G_I$$

  - Gauge transformation

$$\delta_\lambda \nabla_m \quad = \quad \delta_\lambda A_m{}^I G_I = i[\nabla_m,\ \lambda^I G_I] \Rightarrow \delta_\lambda A_m{}^I = \partial_m \lambda^I - A_m{}^J \lambda^K f_{JK}{}^I$$

  - Field strength

$$[\nabla_m, \nabla_n] \quad = \quad i F_{mn}{}^I G_I \Rightarrow F_{mn}{}^I = \partial_{[m} A_{n]}{}^I - A_m{}^J A_n{}^K f_{JK}{}^I$$

1. Gravity as gauge theory 

$e_a{}^m$

◆ Gravity theory

- Gauge field & gauge algebra

$$e_a{}^m \qquad [p_m, p_n] = 0, \ [s_{mn}, s_{lk}] = i\eta_{[k|[m}s_{n]|l]}, \ [s_{mn}, p_l] = ip_{[m}\eta_{n]l}$$

- **Covariant derivative**

$$p_m = \tfrac{1}{i}\partial_m \quad \rightarrow \quad \nabla_a = e_a{}^m p_m + \tfrac{1}{2}\omega_a{}^{mn}s_{mn}$$

- Gauge transformation

$$\delta_\xi \nabla_a = i[\nabla_a, \ \xi^m p_m + \tfrac{1}{2}\lambda^{mn}s_{mn}]$$

$$\Rightarrow \begin{cases} \delta_\xi e_a{}^m = e_a{}^n\partial_n\xi^m - \xi^n\partial_n e_a{}^m - \omega_a{}^{ml}\xi_l + \lambda^m{}_n e_a{}^n \\ \delta_\xi \omega_a{}^{mn} = e_a{}^l\partial_l\lambda^{mn} - \xi^l\partial_l\omega_a{}^{mn} - \lambda^{[m|}{}_l\omega_a{}^{l|n]} \end{cases}$$

- Field strength

$$[\nabla_a, \nabla_b] = -iT_{ab}{}^c\nabla_c - iR_{ab}{}^{cd}s_{cd} \Rightarrow \begin{cases} R_{ab}{}^{cd} = e_a{}^m e_b{}^n(\partial_{[m}\omega_{n]}{}^{cd} + \omega_{[m}{}^{ce}\omega_{n]e}{}^d) \\ T_{ab}{}^c = \omega_{[ab]}{}^c + e_{[a}{}^n(\partial_n e_{b]}{}^m)e_m{}^c = 0 \end{cases}$$

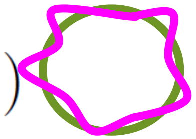
2. Stringy gravity

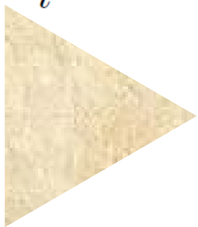
'93 Siegel

◆Stringy extension

- **Affine extension of covariant derivative**

$$p_m = \frac{1}{i}\partial_m \quad \rightarrow \quad \overset{\square}{\triangleright}_{\underline{m}}(\sigma) = (p_m, \partial_\sigma x^m)$$





$$[\overset{\square}{\triangleright}_{\underline{m}}(\sigma), \overset{\square}{\triangleright}_{\underline{n}}(0)] = i\eta_{\underline{mn}}\partial_\sigma\delta(\sigma)$$

O(d,d) invariant metric

$$\eta_{\underline{mn}} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$$

- **Affine nondegenerate Poincare covariant derivative**

$$\rightarrow \quad \overset{\square}{\triangleright}_{\underline{M}}(\sigma) = (S_{\underline{mn}}, P_{\underline{m}}, \Sigma^{\underline{mn}})$$

$$[\overset{\square}{\triangleright}_{\underline{M}}(\sigma), \overset{\square}{\triangleright}_{\underline{N}}(0)] = -i\overset{\square}{f}_{\underline{MN}}{}^{\underline{K}}\overset{\square}{\triangleright}_{\underline{K}}\delta(\sigma) + i\eta_{\underline{MN}}\partial_\sigma\delta(\sigma)$$

Jacobi identity ⇒
η_{MN} is nondegenerate &
f_{MNL} is totally antisymmetric !

Nondegenerate algebras

- Lorentz: $[s_{mn}, p_l] = -ip_{[m}\eta_{n]l} \Leftrightarrow f_{sp}{}^p = f_{spp}$

Totally antisymmetricity: $f_{spp} = f_{pps}$

Nondegeneracy: ${}^\exists\eta_{s\sigma} = 1$

Nondegenerate algebra! $\Rightarrow {}^\exists f_{pp}{}^\sigma \Leftrightarrow [p_m, p_n] = -i\sigma_{mn}$

'01 Kamimura & M.H.
'09 Gomis, Kamimura, Lukierski

- Susy: $\{d_\mu, d_\nu\} = -ip_{\mu\nu} \Leftrightarrow f_{dd}{}^p = f_{ddp}$

Totally antisymmetricity: $f_{ddp} = f_{dpd}$

Nondegeneracy: ${}^\exists\eta_{d\omega} = 1$

Nondegenerate algebra! $\Rightarrow {}^\exists f_{dp}{}^\omega \Leftrightarrow [d_\mu, p_n] = (\gamma_n)_{\mu\nu}\omega^\nu$

'85 Siegel, '89 Green

2. Stringy gravity

G_{mn}, B_{mn}

$= e_{\underline{a}}{}^{\underline{m}}$

- Stringy gravity theory
  - Gauge fields & gauge algebra "affine doubled translation algebra"

$$(G_{mn}, B_{mn}) \qquad e_{\underline{a}}{}^{\underline{m}} \in \frac{\mathrm{O(d,d)}}{\mathrm{O(d)}^2} \qquad e_{\underline{a}}{}^{\underline{m}} \eta^{\underline{ab}} e_{\underline{b}}{}^{\underline{n}} = \eta^{\underline{mn}}$$

Orthogonal condition

  - **Covariant derivative**

$$\overset{\square}{\triangleright}_{\underline{m}} \quad \rightarrow \quad \triangleright_{\underline{a}} = e_{\underline{a}}{}^{\underline{m}} \overset{\square}{\triangleright}_{\underline{m}}$$

  - Gauge transformation

$$\delta_\xi \triangleright_{\underline{a}} \quad = \quad i[\triangleright_{\underline{a}}, \ \xi^{\underline{m}} \overset{\square}{\triangleright}_{\underline{m}}]|_{\text{regular part}}$$

$$\begin{cases} \delta_\xi G_{mn} = \xi^l \partial_l G_{mn} + \partial_{(m|} \xi^l G_{l|n)} \\ \delta_\xi B_{mn} = \xi^l \partial_l B_{mn} + \partial_{[m|} \xi^l B_{l|n]} + \partial_{[m} \xi_{n]} \end{cases}$$

2. Stringy gravity

$E_{\underline{A}}{}^{\underline{M}}$

◆Stringy gravity theory

- Gauge field & gauge algebra “affine doubled super-Poincare algebra”

$$E_{\underline{A}}{}^{\underline{M}} \qquad E_{\underline{A}}{}^{\underline{M}}\eta^{\underline{AB}}E_{\underline{B}}{}^{\underline{N}} = \eta^{\underline{MN}}$$

Orthogonal condition

- **Covariant derivative**

$$\overset{\square}{\triangleright}_{\underline{M}} \quad \rightarrow \quad \triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}}\overset{\square}{\triangleright}_{\underline{M}}$$

- Gauge transformation

$$\delta_\xi E_{\underline{A}}{}^{\underline{M}}\overset{\square}{\triangleright}_{\underline{M}} \quad = \quad i[\triangleright_{\underline{A}},\ \xi^{\underline{M}}\overset{\square}{\triangleright}_{\underline{M}}]|_{\text{regular part}}$$

$$[\Lambda_1{}^{\underline{M}}\overset{\square}{\triangleright}_{\underline{M}}(\sigma),\ \Lambda_2{}^{\underline{N}}\overset{\square}{\triangleright}_{\underline{N}}(0)] \quad = \quad -i\Lambda_{12}{}^{\underline{M}}\overset{\square}{\triangleright}_{\underline{M}}\delta(\sigma)$$
$$-i\tfrac{1}{2}\left(\Lambda_1\cdot\Lambda_2(\sigma)-\Lambda_1\cdot\Lambda_2(0)\right)\partial_\sigma\delta(\sigma)$$

$$\Lambda_{12}{}^{\underline{M}} \quad = \quad \Lambda_{[1}{}^{\underline{K}}\partial_{\underline{K}}\Lambda_{2]}{}^{\underline{M}} - \tfrac{1}{2}\Lambda_{[1}{}^{\underline{K}}\partial^{\underline{M}}\Lambda_{2]\underline{K}} + \Lambda_1{}^{\underline{J}}\Lambda_2{}^{\underline{K}}f_{\underline{JK}}{}^{\underline{M}}$$

Stringy effect

Supergravity fields

gravitino, dilatino

Lorentz connection, NSNS field strength

Supersymmetric B-fields

$$E_{\underline{AM}} = \begin{array}{c|ccccc} & D_\beta & P_b & \mathsf{F}^{\beta\beta'} & \Omega^\beta & \Sigma^{bc} \\ \hline D_\alpha & B_{\alpha\beta} & B_{\alpha b} & C_\alpha^{\beta\beta'} & e_\alpha{}^\beta & \omega_\alpha^{bc} \\ P_a & & e_{ab}, B_{ab} & C_{\mathrm{RR}} & \psi, \lambda & \omega, F_{\mathrm{NS}} \\ \mathsf{F}^{\alpha\alpha'} & & & \phi & \lambda & F_{\mathrm{RR}} \\ \Omega^\alpha & & & & F_{\mathrm{RR}}^{\alpha\beta'}, F_{\mathrm{NS}}^{\alpha\beta} & \omega \\ \Sigma^{ab} & & & & & r \end{array}$$

graviton B-field

Orthogonality

RR gauge-fields

NSNS, RR gauge-field strength

Generators of the N=2 nondegenerate super-Poincare

$$(S_{\underline{ab}},\ \Sigma^{\underline{ab}}) \ ,\ (P_{\underline{a}})$$

$$(D_{\underline{\alpha}},\ \Omega^{\underline{\alpha}}) \ ,\ (\Upsilon_{\alpha\beta'},\ \mathsf{F}^{\alpha\beta'})$$

‘14 Kamimura, Siegel & M.H.

2. Stringy gravity

$E_{\underline{A}}{}^{\underline{M}}$

◆Stringy gravity theory

- **Covariant derivative** algebra

$$[\triangleright_{\underline{A}}(\sigma), \triangleright_{\underline{B}}(0)] = -\tfrac{i}{2} T_{\underline{AB}}{}^{\underline{C}} \triangleright_{\underline{C}} \delta(\sigma) - i\eta_{\underline{AB}} \partial_\sigma \delta(\sigma)$$

- Torsion

$$T_{\underline{ABC}} = (D_{[\underline{A}} E_{\underline{B}}{}^{\underline{M}}) E_{\underline{C}]\underline{M}} + E_{\underline{A}}{}^{\underline{M}} E_{\underline{B}}{}^{\underline{N}} E_{\underline{C}}{}^{\underline{L}} f_{\underline{MNL}}$$

- Bianchi identity

$$0 = D_{[\underline{M}} T_{\underline{NLK}]} + \tfrac{3}{4} T_{[\underline{MN}}{}^{\underline{I}} T_{\underline{LK}]\underline{I}}$$

- Gravity action

$$\mathcal{S} = \int \Phi^2 R \qquad \Phi\text{:dilaton}$$

Torsions include curvature tensor

$$T_{\underline{PP}}{}^{\underline{S}} = R_{\underline{ab}}{}^{\underline{cd}}$$

Torsion constraints & Bianchi identity give field eq.

3. Characteristics: Doubling

USUAL WZW CONSTRUCTION:

G→G$_L$ × G$_R$ $\ni \underline{g} = gg' \neq g'g$

	Left	Right
Covariant derivative	$\underline{g}^{-1}\partial_-\underline{g}$	NON
Symmetry generator	NON	$\partial_+\underline{gg}^{-1}$

OUR DOUBLING: ‘13 Polacek & Siegel

G→G × G’ $\ni \underline{g} = gg' = g'g$ in flat

	Left	Right	
Covariant derivative	$\underline{g}^{-1}\partial_-\underline{g}$	$\underline{g}^{-1}\partial_+\underline{g}$	Local SUSY
Symmetry generator	$\partial_-\underline{gg}^{-1}$	$\partial_+\underline{gg}^{-1}$	Global SUSY

Needed for N=2 SUSY,
so better for doubled !

3. Characteristics: From double to single

SECTION CONDITION & GAUGE FIX

$$\mathcal{H}_\sigma = p_m \partial_\sigma x^m = \frac{\partial}{\partial x^m}\frac{\partial}{\partial y^m}$$

Imposing on fields

$$\langle \Psi | \mathcal{H}_\sigma | \chi \rangle = 0$$

$$\Leftrightarrow \frac{\partial}{\partial x^m}\frac{\partial}{\partial y^m}\chi = 0 = \frac{\partial}{\partial x^m}\Psi \frac{\partial}{\partial y^m}\chi$$

Non-doubled coordinate
breaks manifest T-duality

Gauge fix:

$$\frac{\partial}{\partial y^m}\Psi(x) = \frac{\partial}{\partial y^m}\chi(x) = 0$$

1ST CLASS CONSTRAINT

'14 Kamimura, Siegel & M.H.

$$\begin{aligned}
\mathcal{H}_\sigma &= \frac{1}{2}({P_m}^2 - {P_{m'}}^2) && \textbf{Covariant derivatives}\\
&= \frac{1}{2}({\tilde{P}_m}^2 - {\tilde{P}_{m'}}^2) && \textbf{Symmetry generators}\\
&= \frac{1}{2}(\tilde{P}_m + \tilde{P}_{m'})\boxed{(\tilde{P}^m - \tilde{P}^{m'})}\\
&= \frac{1}{2}\tilde{P}_{\text{total};m}\phi^m = 0 && \text{in flat}
\end{aligned}$$

Keep doubled coordinates for T-duality !

1st class constraint: $\phi^m = 0$

4. String action

Hamiltonian for a string in the flat doubled space

‘15 Kamimura, Siegel & M.H.

$$\begin{aligned}\mathcal{H} \;&=\; \lambda_\tau \mathcal{H}_\tau + \lambda_\sigma \mathcal{H}_\sigma \\ &+\mu^{mn} S_{mn} + \tilde{\mu}_{mn}\tilde{\Sigma}^{mn} + \tilde{\mu}^{m}(\tilde{P}_m - \tilde{P}_{m'}) \\ &+\text{right sectors}\end{aligned}$$

2-dim. Metric
⇒ 2-dim. Diffeo.

Local Lorenz, dimensional reduction constraints

- Virasoro constraints

$$\begin{cases} \mathcal{H}_\tau = \frac{1}{2}\overset{\Box}{\triangleright}_{\underline{M}}\hat{\eta}^{\underline{MN}}\overset{\Box}{\triangleright}_{\underline{N}} \\ \mathcal{H}_\sigma = \frac{1}{2}\overset{\Box}{\triangleright}_{\underline{M}}\eta^{\underline{MN}}\overset{\Box}{\triangleright}_{\underline{N}} \end{cases}$$

$$\hat{\eta}^{\underline{MN}} = \begin{pmatrix} \mathbf{1} & \\ & \mathbf{1} \end{pmatrix} \qquad \eta^{\underline{MN}} = \begin{pmatrix} \mathbf{1} & \\ & -\mathbf{1} \end{pmatrix}$$

$$\overset{\Box}{\triangleright}_{\underline{M}} = (S_{mn}, P_m, \Sigma^{mn}; S_{m'n'}, P_{m'}, \Sigma^{m'n'} \cdots)$$

$$S_{mn} = \overset{\Box}{\nabla}_S,\;\; P_m = \overset{\Box}{\nabla}_P + \overset{\Box}{J}_P,\;\; \Sigma^{mn} = \overset{\Box}{\nabla}_\Sigma + 2\overset{\Box}{J}_\Sigma$$

$$S_{m'n'} = \overset{\Box}{\nabla}_{S'},\;\; P_{m'} = \overset{\Box}{\nabla}_{P'} - \overset{\Box}{J}_{P'},\;\; \Sigma^{m'n'} = \overset{\Box}{\nabla}_{\Sigma'} - 2\overset{\Box}{J}_{\Sigma'}$$

- In usual gauge: Green-Schwarz action
- In bosonic simple gauge: $\mathcal{L} = dX \wedge *dX - dX \wedge \underline{dY} + B$

Dual coordinate !

III. Group manifold with T-duality

Affine Lie algebra & B-field

- **String covariant derivative with B-field** $$\triangleright_I = \nabla_I + J^K(\eta_{KI} + B_{KI})$$

(LI currents $g^{-1}dg = iJ^I G_I$, $J^I = dZ^M R_M{}^I$, $\nabla_I = (R^{-1})_I{}^M \frac{1}{i}\partial_M$)

- Affine Lie algebra $$[\triangleright_I(\sigma), \triangleright_J(\sigma')] = -if_{IJ}{}^K \triangleright_K \delta(\sigma - \sigma') - i\eta_{IJ}\partial_\sigma \delta(\sigma - \sigma')$$

⇒Condition on B-field !

$$i\nabla_{[I}B_{JK]} - f_{[IJ|}{}^L B_{L|K]} = 2f_{IJK}$$

'15 Kamimura, Siegel & M.H.

Local universality of H in doubled space

- Curved space→Nonabelian group manifold: $T_{\underline{ABC}} \to f_{\underline{ABC}}$
  - Covariant derivative & vielbein — Orthogonality in the doubled space

$$\triangleright_{\underline{A}} = E_{\underline{A}}{}^{\underline{M}} \overset{\square}{\triangleright}_{\underline{M}} \quad , \quad E_{\underline{A}}{}^{\underline{M}} \eta_{\underline{MN}} E_{\underline{B}}{}^{\underline{N}} = \eta_{\underline{AB}}$$

  - "Curved" vs. "Flat"

$$f_{\underline{ABC}} = (D_{[\underline{A}} E_{\underline{B}}{}^{\underline{M}}) E_{\underline{C}]\underline{M}} + E_{\underline{A}}{}^{\underline{M}} E_{\underline{B}}{}^{\underline{N}} E_{\underline{C}}{}^{\underline{L}} \overset{\square}{f}_{\underline{MNL}}$$

⇒The three forms are the same !

$$H = \overset{\square}{H}$$

$$\begin{cases} \overset{\square}{H} = d\overset{\square}{B} = \frac{1}{3!} \overset{\square}{J}{}^{\underline{M}} \wedge \overset{\square}{J}{}^{\underline{N}} \wedge \overset{\square}{J}{}^{\underline{L}} \overset{\square}{f}_{\underline{MNL}} \\ H = dB = \frac{1}{3!} J^{\underline{M}} \wedge J^{\underline{N}} \wedge J^{\underline{L}} f_{\underline{MNL}} \end{cases}$$

Local universality of the three form in the doubled space !

IV. AdS space with T-duality

Affine nondegenerate doubled AdS algebra

- **Left**

$$\begin{cases} [P_a(\sigma), P_b(0)] &= -i(\frac{1}{r_{\mathrm{AdS}}^2} S_{ab} + \Sigma_{ab})\delta(\sigma) + i\eta_{ab}\partial_\sigma\delta(\sigma) \\ [S_{ab}(\sigma), \Sigma_{cd}(0)] &= -i\eta_{[d|[a}\Sigma_{b]|c]}\delta(\sigma) + i\eta_{d|[a}\eta_{b]|c}\partial_\sigma\delta(\sigma) \end{cases}$$

$$\eta_{a'b'} = -\eta_{ab}$$

- **Right**

$$\begin{cases} [P_{a'}(\sigma), P_{b'}(0)] &= -i(\frac{1}{r_{\mathrm{AdS}}^2} S_{a'b'} + \Sigma_{a'b'})\delta(\sigma) + i\eta_{a'b'}\partial_\sigma\delta(\sigma) \\ [S_{a'b'}(\sigma), \Sigma_{c'd'}(0)] &= -i\eta_{[d'|[a'}\Sigma_{b']|c']}\delta(\sigma) + i\eta_{d'|[a'}\eta_{b']|c'}\partial_\sigma\delta(\sigma) \end{cases}$$

- **Mixed**

$$[P_a(\sigma), P_{b'}(0)] = -i(\frac{1}{r_{\mathrm{AdS}}^2} S_{ab'} + \Sigma_{ab'})\delta(\sigma)$$

Consistent algebra to define the AdS space with manifest T-duality!

Physical AdS algebra

- AdS algebra

$$\begin{cases} [\mathcal{P}_{\text{total};a}, \mathcal{P}_{\text{total};b}] &= \frac{2i}{r^2}\mathcal{S}_{\text{total};ab} \\ [\mathcal{S}_{\text{total};ab}, \mathcal{S}_{\text{total};cd}] &= \eta_{[d|[a}\mathcal{S}_{\text{total};b]|c]} \\ [\mathcal{S}_{\text{total};ab}, \mathcal{P}_{\text{total};c}] &= i\mathcal{P}_{\text{total};[a}\eta_{b]c} \end{cases}$$

- Generators

$$\mathcal{P}_{\text{total};a} = \int [\tfrac{1}{2}(\tilde{P}_a + \tilde{P}_{a'}) + \tfrac{1}{2}\phi_a]$$

$$\mathcal{S}_{\text{total};ab} = \int [\tfrac{1}{2}(\tilde{S}_{ab} - \tilde{S}_{a'b'} + \tilde{S}_{ab'} - \tilde{S}_{ba'}) + \tfrac{1}{4}(\psi_{ab} - \varphi_{ab})]$$

L&R coordinates are still included !

- Dimensional reduction constraints

Left: AdS & Right: dS
Constraints relates them.

$$\phi_a = \tilde{P}_a - \tilde{P}_{a'} = 0$$

$$\psi_{ab} = (\tilde{S} + r^2\tilde{\Sigma})_{\text{L}} + (\tilde{S} + r^2\tilde{\Sigma})_{\text{R}} - (\tilde{S} + r^2\tilde{\Sigma})_{\text{Mixed}} = 0$$

$$\varphi_{ab} = (\tilde{S} - r^2\tilde{\Sigma})_{\text{L}} - (\tilde{S} - r^2\tilde{\Sigma})_{\text{R}} + (\tilde{S} - r^2\tilde{\Sigma})_{\text{Mixed}} = 0$$

V. Summary

- Manifestly T-dual formulation of AdS space is proposed.
  - The AdS space is defined by "affine nondegenerate doubled AdS algebra".
  - Left& right sectors mix.
  - Left is in AdS & Right is in dS .
  - Dimensional reduction constraints & the physical AdS algebra preserve all coordinates for T-duality.

- Applied to group manifolds
  - 3 form H=dB is locally universal in the doubled space.