Exotic Branes and Superconformal Field Theories

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Exotic branes are codim-2 objects in string theory compactifications.

They are related to standard branes via string duality.

Since D-branes play a significant role in investigating field theories

from the viewpoint of string theory,

it is natural to think that exotic branes should also contribute to them.

We would like to investigate brane configurations
in the presence of exotic branes.

By virtue of them,
we may (or may not) find new superconformal field theories
in lower dimensions.

This research is a work in progress.

Contents

What is exotic brane?

Elitzur, Giveon, Kutasov, Rabinovici: hep-th/9707217
Blau, O'Loughlin: hep-th/9712047
Obers and Pioline: hep-th/9809039
Eyras and Lozano: hep-th/9908094
de Boer and Shigemori: arXiv:1004.2521
Bergshoeff, Ortín, Riccioni: arXiv:1109.4484
etc..

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

$$M_{\mathrm{D7}} = \frac{R_1 R_2 \cdots R_7}{g_s \, \ell_s^8} \quad \begin{matrix} \overset{\mathrm{T}_7}{\nearrow} \\ \\ \underset{\mathrm{S}}{\searrow} \end{matrix}$$

$$\mathrm{T}_y \; : \quad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathrm{S} \; : \quad g_s \to \frac{1}{g_s}, \quad \ell_s^2 \to g_s \, \ell_s^2$$

R_y : compact radius of y-direction

g_s : string coupling constant

ℓ_s : string length

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$$M_{\text{D7}} = \frac{R_1 R_2 \cdots R_7}{g_s\, \ell_s^8} \quad \underset{\text{S}}{\searrow} \quad \frac{R_1 R_2 \cdots R_7}{g_s^3\, \ell_s^8} \quad \leftarrow \text{ exotic!}$$

$$\text{T}_y \; : \quad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\text{S} \; : \quad g_s \to \frac{1}{g_s}, \quad \ell_s^2 \to g_s\, \ell_s^2$$

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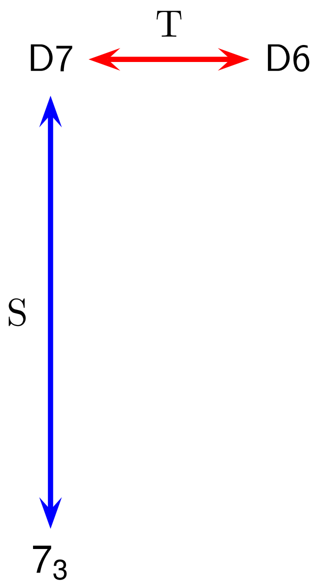

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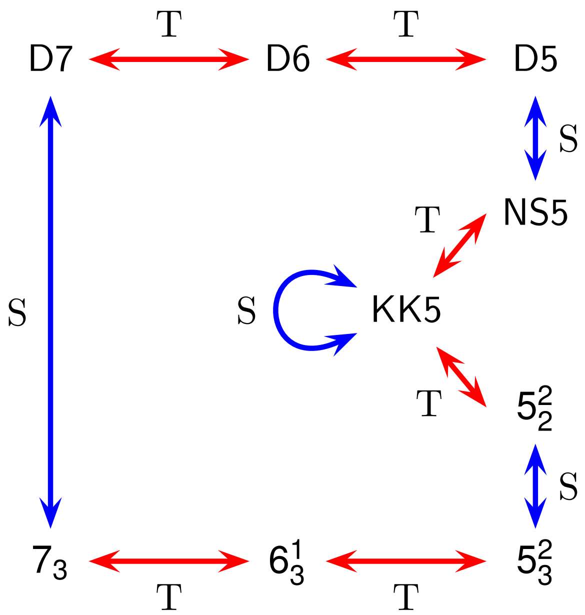

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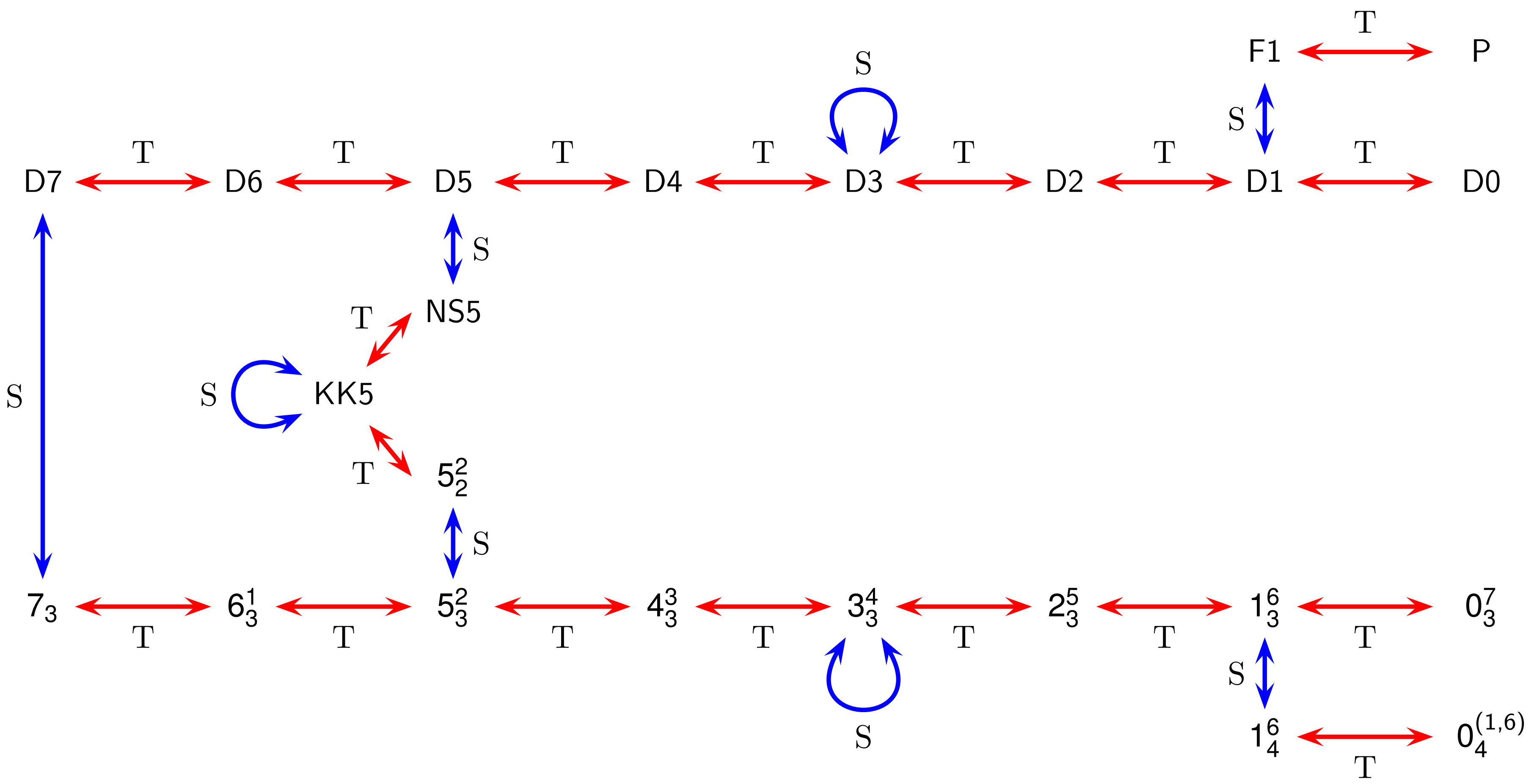

Consider charged particles in 3D maximal supergravity :

They are D7-brane wrapped on 7-torus and its dualized objects

D-dim	IIB				IIA			
3 (240)	D7 (1)	D5 (21)	D3 (35)	D1 (7)	D6 (7)	D4 (35)	D2 (21)	D0 (1)
	F1 (7)	P (7)	NS5 (21)	KK5 (42)	F1 (7)	P (7)	NS5 (21)	KK5 (42)
	1^6_4 (7)	$0^{(1,6)}_4$ (7)	5^2_2 (21)		1^6_4 (7)	$0^{(1,6)}_4$ (7)	5^2_2 (21)	
	7_3 (1)	5^2_3 (21)	3^4_3 (35)	1^6_3 (7)	6^1_3 (7)	4^3_3 (35)	2^5_3 (21)	0^7_3 (1)

b^c_n has mass (tension) $= \dfrac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2}{g_s^n \, \ell_s^{b+2c+1}}$

Consider charged particles in 3D maximal supergravity :

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D-dim	IIB				IIA			
3 (240)	D7 (1)	D5 (21)	D3 (35)	D1 (7)	D6 (7)	D4 (35)	D2 (21)	D0 (1)
	F1 (7)	P (7)	NS5 (21)	KK5 (42)	F1 (7)	P (7)	NS5 (21)	KK5 (42)
	1^6_4 (7)	$0^{(1,6)}_4$ (7)	5^2_2 (21)		1^6_4 (7)	$0^{(1,6)}_4$ (7)	5^2_2 (21)	
	7_3 (1)	5^2_3 (21)	3^4_3 (35)	1^6_3 (7)	6^1_3 (7)	4^3_3 (35)	2^5_3 (21)	0^7_3 (1)

eg.) 5^2_2-particle in 3D is uplifted to 5^2_2-brane in 8D($=5+3$) (as codim-2 object).

When exotic 5^2_2-brane in 8D is embedded into 10D,

this does not depend on $2=10-8$ transverse directions. (smeared)

necessary to keep aspects of codim-2 object

Exotic b_n^c-brane :

- charged particle in 3D, codim-2 object in $(b+3)$-dim
- pair with standard b-brane of codim-2 in $(b+3)$-dim
- c smeared transverse directions from 10D viewpoint
- tension proportional to g_s^{-n}

D7-brane (codim-2 object in 10D) has been studied for 20 years : F-theory

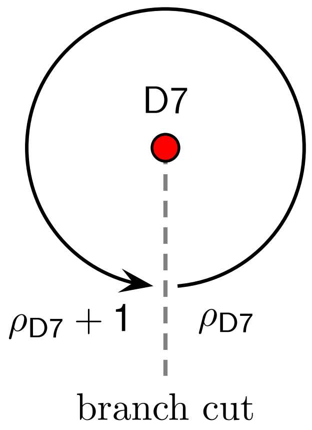
Vafa: hep-th/9602022

D7-brane localized in 89-plane :

$$\rho_{\text{D7}}(z) \equiv C + \mathrm{i}e^{-\phi} = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi}\log\frac{\Lambda}{r} \qquad (z = x^8 + \mathrm{i}x^9 = r\,e^{\mathrm{i}\theta})$$

When ρ_{D7} moves around D7-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" (monodromy) : $\rho_{\text{D7}} \to \rho_{\text{D7}} + 1$



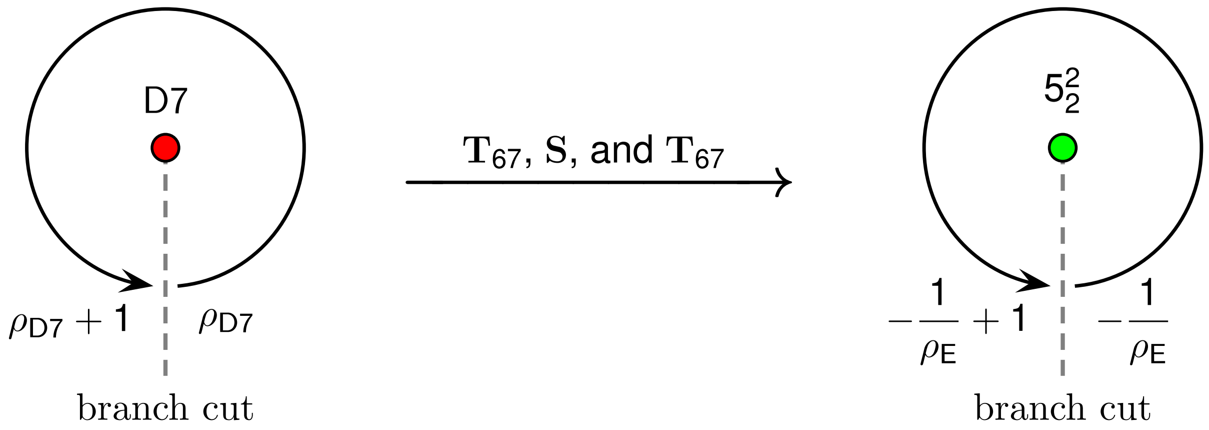
There exists a branch cut in 89-plane.

exotic 5^2_2-brane localized in 89-plane : (smeared along 67-directions)

$$\rho_{\rm E}(z) \;=\; B_{67} + {\rm i}e^{+2\phi} \;=\; -\Big[\frac{\theta}{2\pi} + \frac{\rm i}{2\pi}\log\frac{\Lambda}{r}\Big]^{-1} \qquad (z = x^8 + {\rm i}x^9 = r\,{\rm e}^{{\rm i}\theta})$$

When $\rho_{\rm E}$ moves around 5^2_2-brane counterclockwise $\theta \to \theta + 2\pi$,

it receives a magnetic "charge" (monodromy) : $-1/\rho_{\rm E} \to -1/\rho_{\rm E} + 1$



The monodromy originates from $SL(2,\mathbb{Z}) \subset SO(2,2;\mathbb{Z})$ (T_{67}-duality)

Motivated by

de Boer, Shigemori: arXiv:1004.2521 and Kikuchi, Okada, Sakatani: arXiv:1205.5549,

I have investigated

5^2_2-brane

in collaboration with S. Sasaki and M. Yata

string worldsheet : arXiv:1304.4061 1305.4439 1310.6163 1406.0087 1503.08635 1506.05005 1512.05548

worldvolume : 1404.5442 1601.05589

supergravity : 1410.8403 1411.3457 1601.02175

and many researchers, including guys in this audience

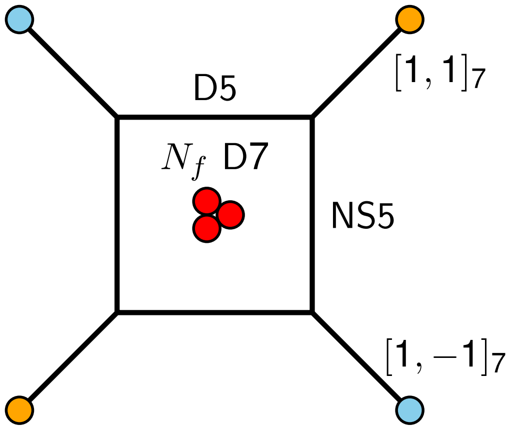
Next, what we should do is :

- Loot at brane configurations where exotic branes live.
- Focus on $SL(2,\mathbb{Z})$ monodromy of exotic branes.
- etc..

Brane configurations where exotic branes live

TK: arXiv:1602.08606

Example of brane configurations with 7-branes :



5D $\mathcal{N} = 1$ $SU(2)$ gauge theory on 5-brane web

with N_f D7-branes

$\rightarrow$ SCFT with E_{N_f+1} symmetry

Symmetry is purely determined by the singularity of fibers :

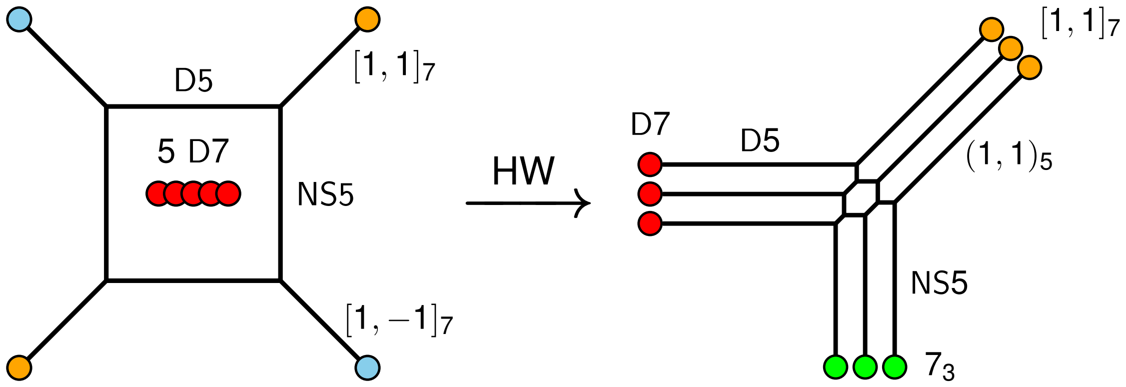
Kodaira classification (ADE-type singularity on K3 surface)

Seiberg: hep-th/9608111

Aharony, Hanany: hep-th/9704170

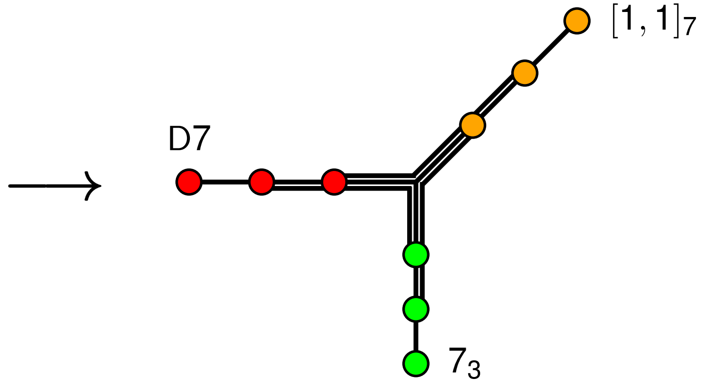
DeWolfe, Hanany, Iqbal, Katz: hep-th/9902179

5D theory on 5-branes with $N_f = 5$ D7-branes :



IIB	0	1	2	3	4	5	6	7	8	9
5 D7	–	–	–	–	–	–	–	–		
D5	–	–	–	–	–				–	
NS5	–	–	–	–	–					–
$(1,1)_5$	–	–	–	–	–				angle	

—— 5D theory —— (directions 0–4)

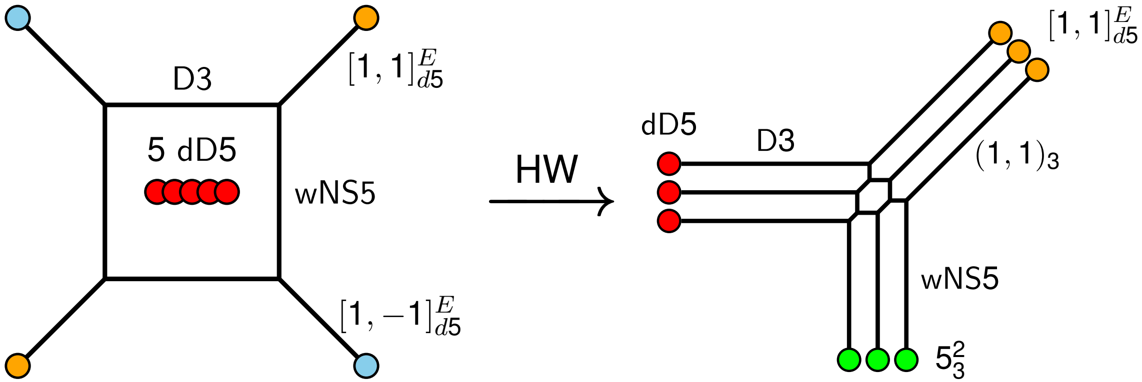


5D T_3 theory with E_6 symmetry

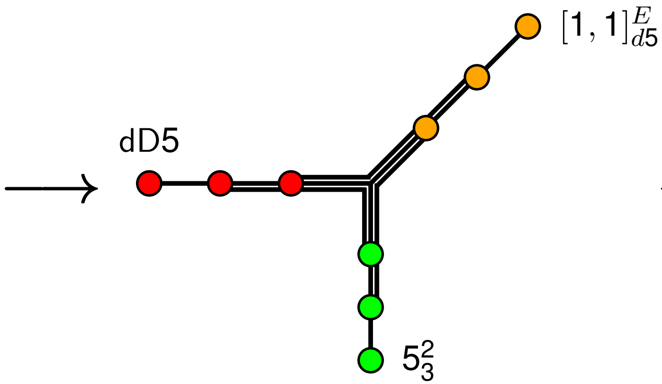
Gaiotto, Witten: arXiv:0804.2902, 0807.3720
Benini, Benvenuti, Tachikawa: arXiv:0906.0359

D7-branes and 7_3-branes are involved.

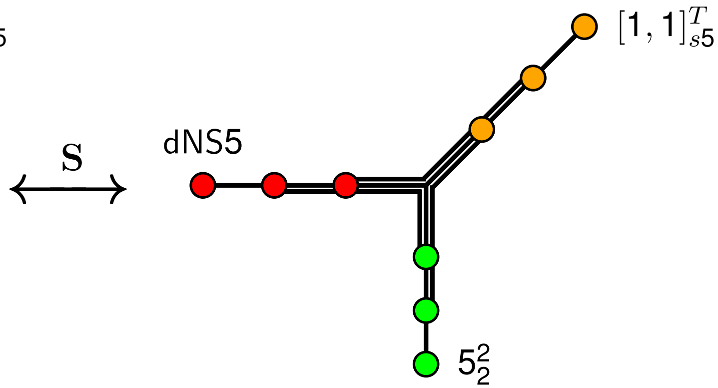
3D theory on 3-branes with 5 defect D5-branes :



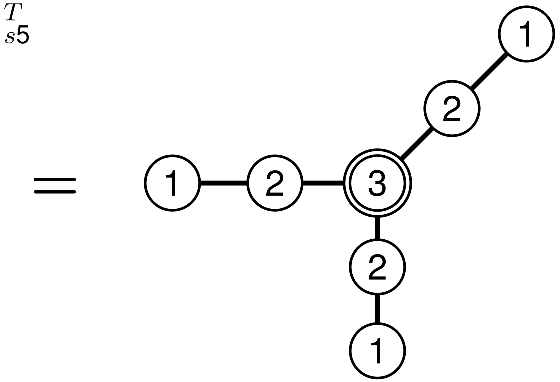
IIB	0	1	2	③	④	5	6	7	8	9
5 dD5	–	–	–			–	–	–		
D3	–	–	–						–	
wNS5	–	–	–	–	–					–
$(1,1)_3$	–	–	–						angle	
	– 3D theory –			(smeared)						



3D T_3 theory



mirror of 3D T_3 theory



star-shaped quiver

(dD5, 5^2_3) and (dNS5, 5^2_2) are involved.

Intriligator, Seiberg: hep-th/9609207
Benini, Tachikawa, Xie: arXiv:1007.0992

Monodromy of defect brane is given by string U-duality group in D-dim :

D	U-duality	#	IIA	IIB
10B	$SL(2,\mathbb{Z})$	$2 \subset 3$	–	D7$_{(1)}$ $7_3{}_{(1)}$
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	$2 \subset 3$	D6$_{(1)}$ $6^1_3{}_{(1)}$	D7$_{(1)}$ $7_3{}_{(1)}$
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	$6 \subset (8,1)$ $2 \subset (1,3)$	D6$_{(2)}$ $6^1_3{}_{(2)}$ KK5$_{(2)}$ NS5$_{(1)}$ $5^2_2{}_{(1)}$	D7$_{(1)}$ $7_3{}_{(1)}$ D5$_{(1)}$ $5^2_3{}_{(1)}$ NS5$_{(1)}$ $5^2_2{}_{(1)}$ KK5$_{(2)}$
7	$SL(5,\mathbb{Z})$	$20 \subset 24$	D6$_{(3)}$ $6^1_3{}_{(3)}$ NS5$_{(3)}$ $5^2_2{}_{(3)}$ KK5$_{(6)}$ D4$_{(1)}$ $4^3_3{}_{(1)}$	D7$_{(1)}$ $7_3{}_{(1)}$ D5$_{(3)}$ $5^2_3{}_{(3)}$ NS5$_{(3)}$ $5^2_2{}_{(3)}$ KK5$_{(6)}$
⋮	⋮	⋮	⋮	⋮

All branes are defect branes of codim-2.

Bergshoeff, Ortín, Riccioni: arXiv:1109.4484

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9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	$2 \subset 3$	D6(1) 6^1_3(1)	D7(1) 7_3(1)
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	$6 \subset (8,1)$ $2 \subset (1,3)$	D6(2) 6^1_3(2) KK5(2) NS5(1) 5^2_2(1)	D7(1) 7_3(1) D5(1) 5^2_3(1) NS5(1) 5^2_2(1) KK5(2)
7	$SL(5,\mathbb{Z})$	$20 \subset 24$	D6(3) 6^1_3(3) NS5(3) 5^2_2(3) KK5(6) D4(1) 4^3_3(1)	D7(1) 7_3(1) D5(3) 5^2_3(3) NS5(3) 5^2_2(3) KK5(6)
⋮	⋮	⋮	⋮	⋮

All branes are defect branes of codim-2.

7-branes in F-theory

Greene, Shapere, Vafa, Yau: NPB337 (1990) 1
Vafa: hep-th/9602022

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	$\times SL(2,\mathbb{Z})$	$2 \subset (1,3)$	NS5(1) 5^2_2(1)	KK5(2)
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⋮	⋮	⋮	⋮	⋮

All branes are defect branes of codim-2.

3D T_3 theory and its mirror dual

Benini, Tachikawa, Xie: arXiv:1007.0992

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10B	$SL(2,\mathbb{Z})$	$2 \subset 3$	–	$D7_{(1)}$ $7_{3(1)}$
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8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	$6 \subset (8,1)$ $2 \subset (1,3)$	$D6_{(2)}$ $6^1_{3(2)}$ $KK5_{(2)}$ $NS5_{(1)}$ $5^2_{2(1)}$	$D7_{(1)}$ $7_{3(1)}$ $D5_{(1)}$ $5^2_{3(1)}$ $NS5_{(1)}$ $5^2_{2(1)}$ $KK5_{(2)}$
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⋮	⋮	⋮	⋮	⋮

All branes are defect branes of codim-2.

$SL(2,\mathbb{Z})$ doublet

Benini, Benvenuti, Tachikawa: arXiv:0906.0359
TK: arXiv:1410.8403
Sasaki, Yata, TK: arXiv:1411.3457

Towards SCFT in 4D

5-brane web with N_f D7-branes gives rise to 5D SCFT with E_{N_f+1} symmetry.

In case of $N_f = 5, 6, 7$,

3D SCFT with E_{N_f+1} is obtained from brane config with exotic 5-branes.

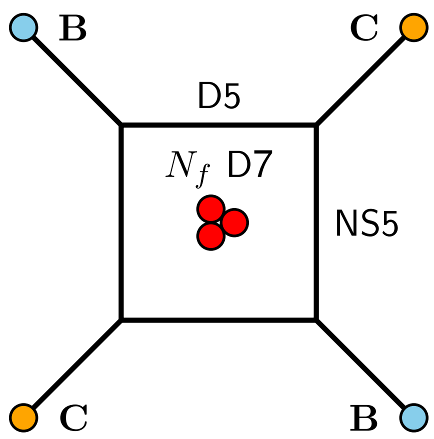
Is it possible to find 4D SCFT from brane config with exotic 6-branes?

cf.) 4D theory is given by D4-NS5-D6 system in IIA and its M-theory lift.

Witten: hep-th/9703166

Gaiotto: arXiv:0904.2715

Begin with brane configurations with 7-branes :



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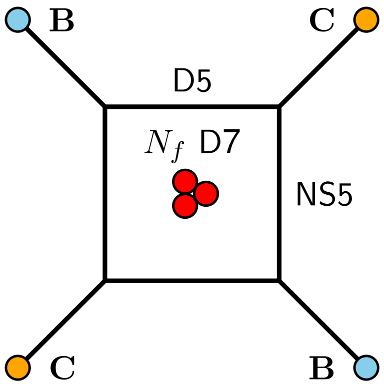
Brane-web with exotic 6-branes from F-theory :



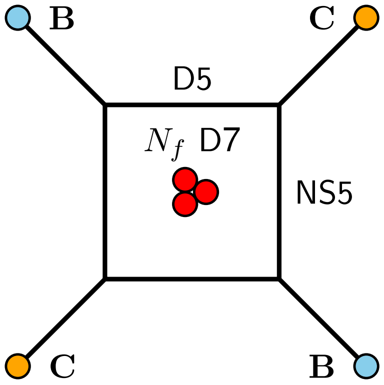
IIB	0	1	2	3	4	5	6	7	8	9
N_f D7	–	–	–	–	–	–	–	–		
D5	–	–	–	–	–				–	
NS5	–	–	–	–	–					–
$(1,1)_5$	–	–	–	–	–				angle	

—— 5D theory ——

[D7, 7_3] has $SL(2,\mathbb{Z})$ monodromy in 10D

↓ T-dualize and smear along x^4

Brane-web with exotic 6-branes from F-theory :

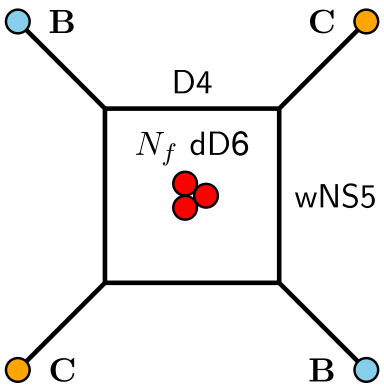


IIB	0	1	2	3	4	5	6	7	8	9
N_f D7	—	—	—	—	—	—	—	—		
D5	—	—	—	—	—				—	
NS5	—	—	—	—	—					—
$(1,1)_5$	—	—	—	—	—				angle	

— 5D theory —

[D7, 7_3] has $SL(2,\mathbb{Z})$ monodromy in 10D

↓ T-dualize and smear along x^4

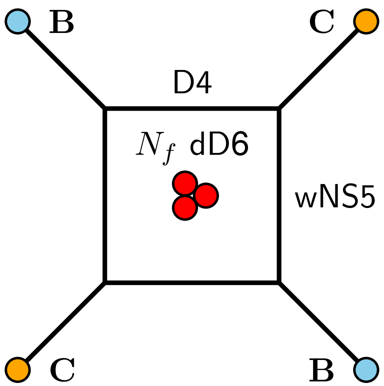


IIA	0	1	2	3	④	5	6	7	8	9
N_f dD6	—	—	—	—		—	—	—		
D4	—	—	—	—					—	
wNS5	—	—	—	—	—					—
$(1,1)_4$	—	—	—	—	—				angle	

— 4D theory — (smeared)

[dD6, 6_3^1] has $SL(2,\mathbb{Z})$ monodromy in 9D

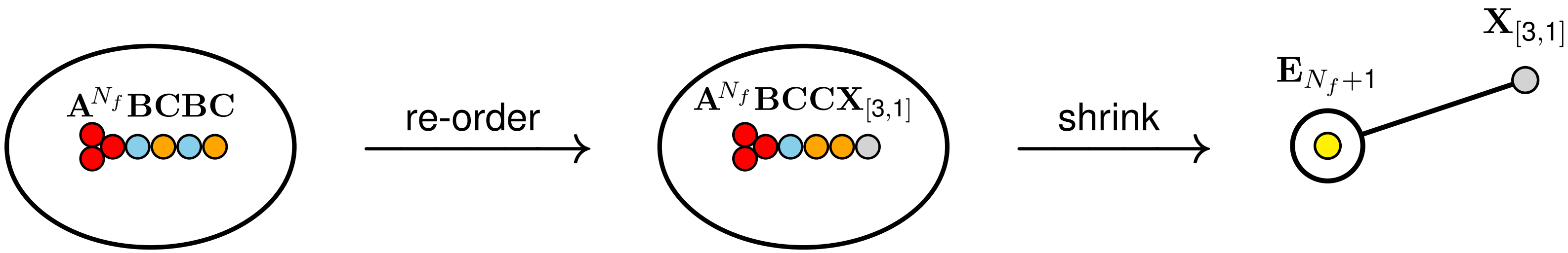
$N_f = 5, 6, 7$ case :



IIA	0	1	2	3	④	5	6	7	8	9
N_f dD6	–	–	–	–		–	–	–		
D4	–	–	–	–					–	
wNS5	–	–	–	–	–					–
$(1,1)_4$	–	–	–	–	–				angle	
	— 4D theory —				(smeared)					

[dD6, 6^1_3] has $SL(2,\mathbb{Z})$ monodromy in 9D

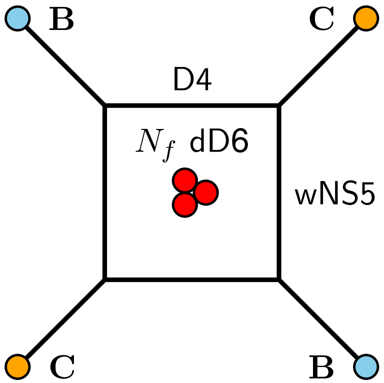
↓ Hanany-Witten transition



Brane realization of Minahan-Nemeschansky theory

Benini, Benvenuti, Tachikawa: arXiv:0906.0359
TK: arXiv:1602.08606

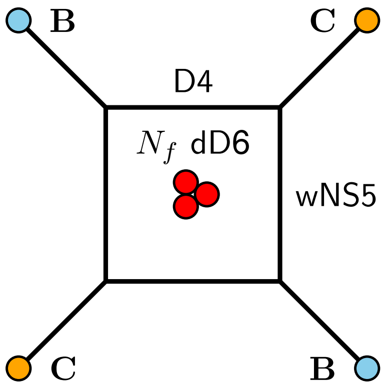
$N_f \leq 4$ in two configurations :



IIA	0	1	2	3	④	5	6	7	8	9
N_f dD6	—	—	—	—		—	—	—		
D4	—	—	—	—					—	
wNS5	—	—	—	—	—					—
$(1,1)_4$	—	—	—	—	—				angle	
	— 4D theory —				(smeared)					

$SL(2, \mathbb{Z})$ monodromy
(F-theoretical)

$N_f \leq 4$ in two configurations :

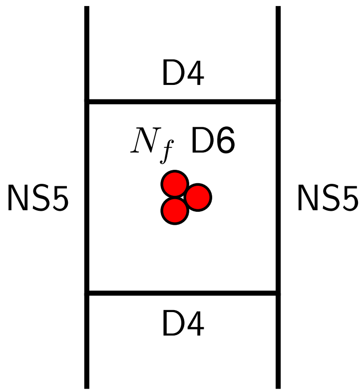


IIA	0	1	2	3	④	5	6	7	8	9
N_f dD6	–	–	–	–		–	–	–		
D4	–	–	–	–					–	
wNS5	–	–	–	–	–					–
$(1,1)_4$	–	–	–	–	–				angle	

— 4D theory — (smeared)

$SL(2,\mathbb{Z})$ monodromy
(F-theoretical)

vs



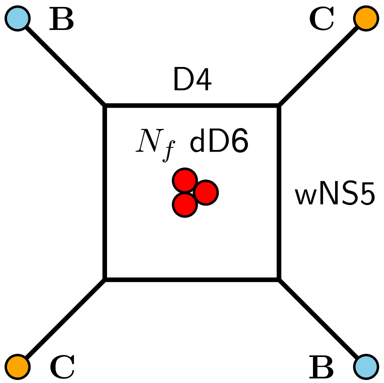
IIA	0	1	2	3	4	5	6	7	8	9
N_f D6	–	–	–	–		–	–	–		
D4	–	–	–	–					–	
NS5	–	–	–	–	–					–
$(1,1)_4$	–	–	–	–	–				angle	

— 4D theory —

conventional config
(M-theoretical)
Witten: hep-th/9703166

difference : x^4-dependence

$N_f \leq 4$ in two configurations :

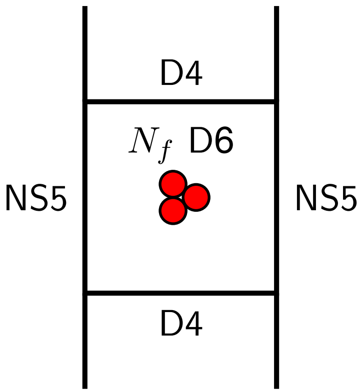


isolated SCFT symmetry as reduction from 5D (?)

N_f	0	1	2	3	4	$\cdots$
symmetry	A_1	$A_1 \times U(1)$	$A_2 \times A_1$	A_4	D_5	$\cdots$

E_{N_f+1}-type symmetry

vs

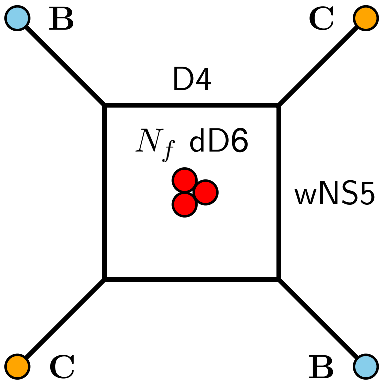


established SCFT symmetry

N_f	0	1	2	3	4	$\cdots$
symmetry	—	D_1	D_2	D_3	D_4	—

difference : x^4-dependence

$N_f \leq 4$ in two configurations :

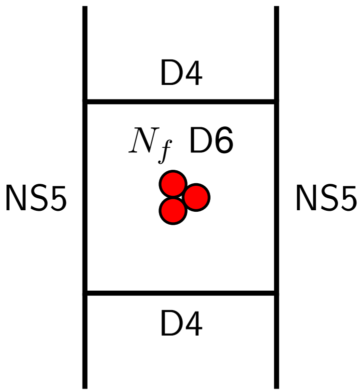


isolated SCFT symmetry as reduction from 5D (!?)

N_f	0	1	2	3	4	$\cdots$
symmetry	A_1	$A_1 \times U(1)$	$A_2 \times A_1$	A_4	D_5	$\cdots$

E_{N_f+1}-type symmetry

$\uparrow$ "Integrate out" x^4-dependence

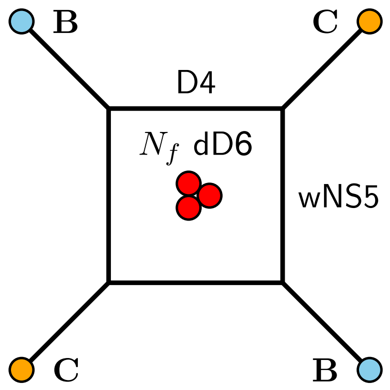


established SCFT symmetry

N_f	0	1	2	3	4	$\cdots$
symmetry	—	D_1	D_2	D_3	D_4	—

difference : x^4-dependence

$N_f \leq 4$ in two configurations :



isolated SCFT symmetry as reduction from 5D (!?)

N_f	0	1	2	3	4	$\cdots$
symmetry	A_1	$A_1 \times U(1)$	$A_2 \times A_1$	A_4	D_5	$\cdots$

E_{N_f+1}-type symmetry

elliptic curve (geometrical) $\leftrightarrow$ Seiberg-Witten curve (field theoretical) in 4D

$$D_5 : \quad y^2 = x^3 + ux^2 + \Lambda^{-2} u^4$$

Field theory from 4 dD6-branes on D_5 singularity flows to IR free.

Argyres, Lotito, Lü, Martone: arXiv:1505.04814

Summary

- Exotic branes are string theory origins of charged particles in 3D.
  They are codim-2 objects in D-dim.

- There exist $SL(2,\mathbb{Z})$ doublets in $D \leq 10$.

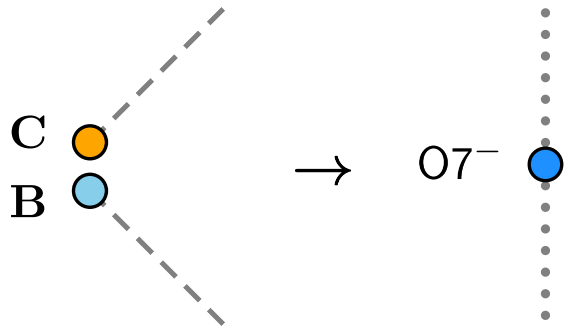
- Exotic 5-branes contribute to 3D T_3-theory and its mirror dual.

- Isolated SCFT in 4D could be (?) obtained by virtue of exotic 6-branes.
  (E_{N_f+1}-type symmetry except for $E_5 = D_5$)

- SCFT in 3D, 2D ...

Orientifold $O7^{-}$-plane originates from $\mathbf{C}$- and $\mathbf{B}$-branes in F-theory :

$$\mathbf{C} = [\mathrm{D7}, +7_3]$$
$$\mathbf{B} = [\mathrm{D7}, -7_3]$$



Sen: hep-th/9605150

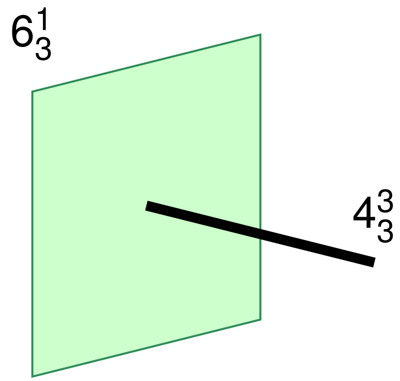
Is it possible to construct orientifold planes by exotic $[a,b]$-branes?

$$[\mathrm{dD6}, -6^1_3] \quad + \quad [\mathrm{dD6}, +6^1_3] \quad \stackrel{?}{\longrightarrow} \quad \mathrm{O6}^{-}\text{-plane}$$
$$\mathbf{T} \downarrow \qquad\qquad\qquad\qquad \downarrow \mathbf{T}$$
$$[\mathrm{dD5}, -5^2_3] \quad + \quad [\mathrm{dD5}, +5^2_3] \quad \stackrel{?}{\longrightarrow} \quad \mathrm{O5}^{-}\text{-plane}$$
$$\mathbf{S} \downarrow \qquad\qquad\qquad\qquad \downarrow \mathbf{S}$$
$$[\mathrm{dNS5}, -5^2_2] \quad + \quad [\mathrm{dNS5}, +5^2_2] \quad \stackrel{?}{\longrightarrow} \quad \mathrm{ON5}^{-}_{\mathrm{B}}\text{-plane}$$

Hanany, Kol: hep-th/0003025

The configurations discussed here are deduced from F-theory.

It is interesting to consider "exotic branes intersecting exotic branes" such as



TK: arXiv:1601.02175

This does not originate from conventional brane configurations.

Exceptional Field Theory associated with U-duality

Thanks

Appendix

Consider charged particles in $D \geq 4$ maximal supergravity :

# of charged particles $=$ # of $U(1)$ gauge one-form fields

They can be regarded as standard branes (Dp, NS5, KK5, F1, P)
(un)wrapped on torus T^{10-D} in type II string theory

D	#	IIA						IIB					
10A	1	D0(1)						–					
10B	0	–						–					
9	3	F1(1)	D0(1)	P(1)				D1(1)	F1(1)	P(1)			
8	3×2	D2(1)	F1(2)	D0(1)	P(2)			D1(2)	F1(2)	P(2)			
7	10	D2(3)	F1(3)	D0(1)	P(3)			D3(1)	D1(3)	F1(3)	P(3)		
6	16	D4(1)	D2(6)	F1(4)	D0(1)	P(4)		D3(4)	D1(4)	F1(4)	P(4)		
5	27	NS5(1)	D4(5)	D2(10)	F1(5)	D0(1)	P(5)	D5(1)	NS5(1)	D3(10)	D1(5)	F1(5)	P(5)
4	28×2	D6(1)	NS5(6)	KK5(6)	D4(15)	D2(15)	F1(6)	D5(6)	NS5(6)	KK5(6)	D3(20)	D1(6)	F1(6)
		D0(1)	P(6)					P(6)					

Transformations via string duality :

$$\mathbf{T}_y : \quad R_y \to \frac{\ell_s^2}{R_y}, \quad g_s \to \frac{\ell_s}{R_y} g_s$$

$$\mathbf{S} : \quad g_s \to \frac{1}{g_s}, \quad \ell_s^2 \to g_s \, \ell_s^2$$

R_y : compact radius of y-direction

g_s : string coupling constant

ℓ_s : string length

$$\text{tension of } b_n^c \quad = \quad \frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2}{g_s^n \, \ell_s^{b+2c+1}}$$

$$\text{tension of } b_n^{(d,c)} \quad = \quad \frac{R_1 R_2 \cdots R_b \, (R_{b+1} \cdots R_{b+c})^2 \, (R_{b+c+1} \cdots R_{b+c+d})^3}{g_s^n \, \ell_s^{b+2c+3d+1}}$$

D	U-duality	#	IIA	IIB
10A	1	–	–	–
10B	$SL(2,\mathbb{Z})$	$2 \subset 3$	–	D7(1) 7_3(1)
9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$	$2 \subset 3$	D6(1) 6^1_3(1)	D7(1) 7_3(1)
8	$SL(3,\mathbb{Z})$ $\times SL(2,\mathbb{Z})$	$6 \subset (8,1)$ $2 \subset (1,3)$	D6(2) 6^1_3(2) KK5(2) NS5(1) 5^2_2(1)	D7(1) 7_3(1) D5(1) 5^2_3(1) NS5(1) 5^2_2(1) KK5(2)
7	$SL(5,\mathbb{Z})$	$20 \subset 24$	D6(3) 6^1_3(3) NS5(3) 5^2_2(3) KK5(6) D4(1) 4^3_3(1)	D7(1) 7_3(1) D5(3) 5^2_3(3) NS5(3) 5^2_2(3) KK5(6)
6	$SO(5,5;\mathbb{Z})$	$40 \subset 45$	D6(4) 6^1_3(4) NS5(6) 5^2_2(6) KK5(12) D4(4) 4^3_3(4)	D7(1) 7_3(1) D5(6) 5^2_3(6) NS5(6) 5^2_2(6) KK5(12) D3(1) 3^4_3(1)
5	$E_{6(6)}(\mathbb{Z})$	$72 \subset 78$	D6(5) 6^1_3(5) NS5(10) 5^2_2(10) KK5(20) D4(10) 4^3_3(10) D2(1) 2^5_3(1)	D7(1) 7_3(1) D5(10) 5^2_3(10) NS5(10) 5^2_2(10) KK5(20) D3(5) 3^4_3(5)
4	$E_{7(7)}(\mathbb{Z})$	$126 \subset 133$	D6(6) 6^1_3(6) NS5(15) 5^2_2(15) KK5(30) D4(20) 4^3_3(20) D2(6) 2^5_3(6) F1(1) 1^6_4(1)	D7(1) 7_3(1) D5(15) 5^2_3(15) NS5(15) 5^2_2(15) KK5(30) D3(15) 3^4_3(15) D1(1) 1^6_3(1) F1(1) 1^6_4(1)
3	$E_{8(8)}(\mathbb{Z})$	$240 \subset 248$	D6(7) 6^1_3(7) NS5(21) 5^2_2(21) KK5(42) D4(35) 4^3_3(35) D2(21) 2^5_3(21) F1(7) 1^6_4(7) D0(1) 0^7_3(1) P(7) $0^{(1,6)}_4$(7)	D7(1) 7_3(1) D5(21) 5^2_3(21) NS5(21) 5^2_2(21) KK5(42) D3(35) 3^4_3(35) D1(7) 1^6_3(7) F1(7) 1^6_4(7) P(7) $0^{(1,6)}_4$(7)

"Dp", "NS5" etc. are defect branes such as dDp, dNS5, respectively.

# of defect branes of codim-2 in D-dim $\quad (\equiv \ n_{\mathrm{D}})$

$= \quad$ # of supersymmetric Wess-Zumino couplings

$= \quad \dim G - (\mathrm{rank}\, T + 1)$

$= \quad \dim G - \mathrm{rank}\, G$

$= \quad 2 \times \underbrace{\text{\# of } (D-2)\text{-form central charges in maximal SUSY algebra}}_{\dim H}$

D	n_{D}	G	H	T
10A	0	$\mathbb{R}^+$	1	1
10B	2	$SL(2,\mathbb{R})$	$SO(2)$	1
9	2	$SL(2,\mathbb{R}) \times \mathbb{R}^+$	$SO(2) \times \mathbb{R}^+$	$SO(1,1)$
8	$6+2$	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	$SO(2,2)$
7	20	$SL(5,\mathbb{R})$	$SO(5)$	$SO(3,3)$
6	40	$SO(5,5)$	$SO(5) \times SO(5)$	$SO(4,4)$
5	72	$E_{6(6)}$	$USp(8)$	$SO(5,5)$
4	126	$E_{7(7)}$	$SU(8)$	$SO(6,6)$
3	240	$E_{8(8)}$	$SO(16)$	$SO(7,7)$

Bergshoeff, Ortín, Riccioni: arXiv:1109.4484

IIB action in Einstein frame :

$$S_{\rm IIB} = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G_{\rm E}}\left\{R_{\rm E} - \frac{\partial_M\bar{\rho}\,\partial^M\rho}{2(\rho_2)^2} - \frac{1}{2}F^i_{(3)}\cdot\mathcal{M}_{ij}\,F^j_{(3)} - \frac{1}{4}|\widetilde{F}_{(5)}|^2\right\}$$
$$-\frac{\epsilon_{ij}}{8\kappa^2}\int C_{(4)}\wedge F^i_{(3)}\wedge F^j_{(3)}$$

$$\rho \equiv C + {\rm i}e^{-\phi} \equiv \rho_1 + {\rm i}\rho_2\,, \qquad \mathcal{M}_{ij} \equiv \frac{1}{\rho_2}\begin{pmatrix} 1 & -\rho_1 \\ -\rho_1 & |\rho|^2 \end{pmatrix}$$

$$F^i_{(3)} \equiv \begin{pmatrix} dC_{(2)} \\ dB_{(2)} \end{pmatrix}, \qquad \widetilde{F}_{(5)} \equiv dC_{(4)} - \frac{1}{2}C_{(2)}\wedge H_{(3)} + \frac{1}{2}B_{(2)}\wedge F_{(3)}$$

$SL(2,\mathbb{Z})$ S-duality

$$\rho \to \frac{a\rho + b}{c\rho + d}, \qquad \Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$F^i_{(3)} \to \Lambda^i{}_j\,F^j_{(3)}\,, \qquad \widetilde{F}_{(5)} \to \widetilde{F}_{(5)}\,, \qquad G^{\rm E}_{MN} \to G^{\rm E}_{MN}$$

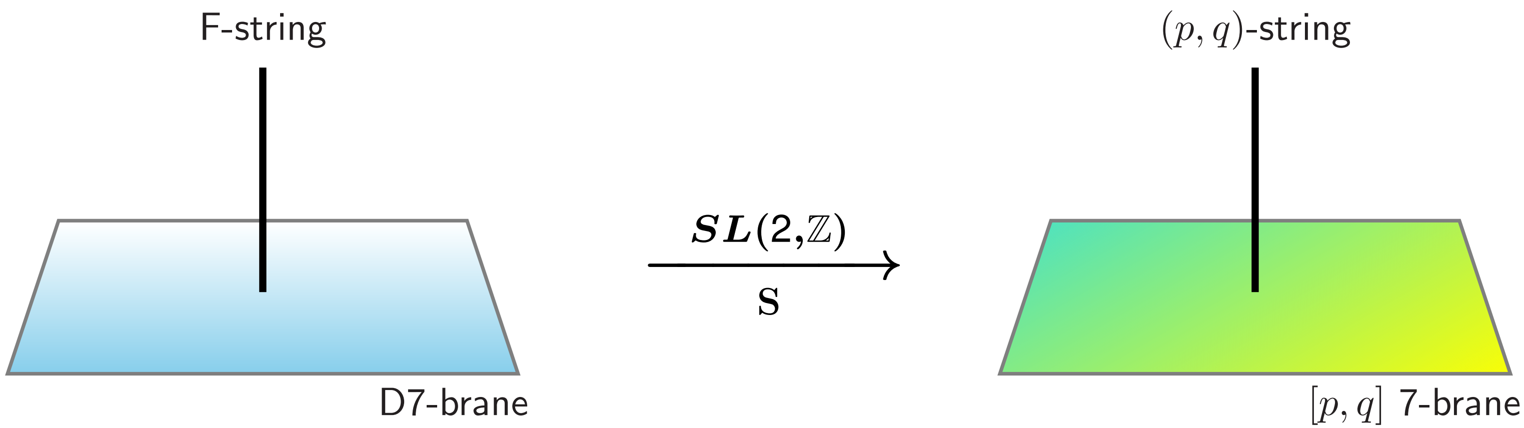
$$\mathcal{M} \to \Lambda^{-{\rm T}}\mathcal{M}\Lambda^{-1}$$

F-string : couple to $B_{(2)}$

D-string : couple to $C_{(2)}$

$$\rho(z) = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{r}$$

D7(1234567) : couple to $\rho(z) = C + \mathrm{i}e^{-\phi}$ $\quad (z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta})$



$(1,0)$-string $=$ F1 $\qquad [1,0]$ 7-brane $=$ D7(1234567)

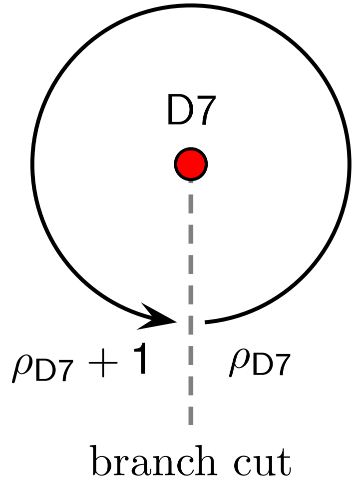
$(0,1)$-string $=$ D1 $\qquad [0,1]$ 7-brane $=$ 7_3(1234567)

Open D-string is ending on 7_3(1234567).

This is a setup in F-theory. We perform $\mathbf{T}_{67}$- and $\mathbf{S}$-duality and reduce 67-directions.

When ρ moves around D7-brane counterclockwise,

it receives a magnetic "charge" of D7-brane (monodromy) $\rightarrow$ $\rho + 1$



$$\rho \rightarrow \rho + 1 = \frac{a\rho + b}{c\rho + d} \equiv M_{[1,0]} \cdot \rho \,, \quad M_{[1,0]} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2, \mathbb{Z})$$

or equivalently

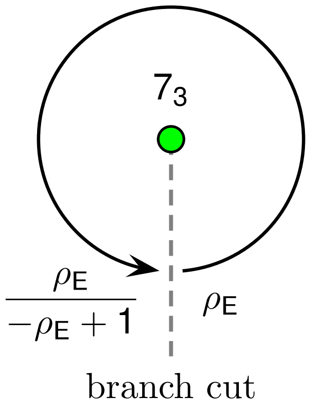
$$K_{[1,0]} \cdot (\rho + 1) = \rho \,, \quad K_{[1,0]} = (M_{[1,0]})^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Monodromy matrix $K_{[1,0]}$ is $SL(2,\mathbb{Z})$ transformed to $K_{[p,q]}$ for $[p,q]$ 7-brane :

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to g \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}, \quad g = \begin{pmatrix} p & r \\ q & s \end{pmatrix} \in SL(2,\mathbb{Z})$$

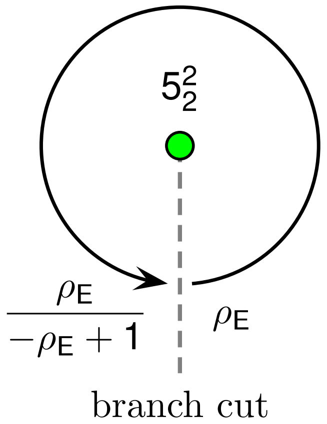
$$K_{[p,q]} = g\, K_{[1,0]}\, g^{-1} = \begin{pmatrix} 1+pq & -p^2 \\ q^2 & 1-pq \end{pmatrix}$$

ex) monodromy $K_{[0,1]}$ for 7_3-brane : $\quad K_{[0,1]} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$



$$\rho_E = -\frac{1}{\rho_{D7}}$$

exotic 5^2_2-brane : (T_{ab}-dualized from defect NS5-brane)



$$SL(2,\mathbb{Z}) : \quad -\frac{1}{\rho_E} \to -\frac{1}{\rho_E} + 1 \qquad \text{where} \quad -\frac{1}{\rho_E} = \rho_{NS5}$$

We cannot remove this shift

by
- B-field gauge transformation
- coordinate transformations

The property of 5^2_2-brane is from that of defect NS5-brane

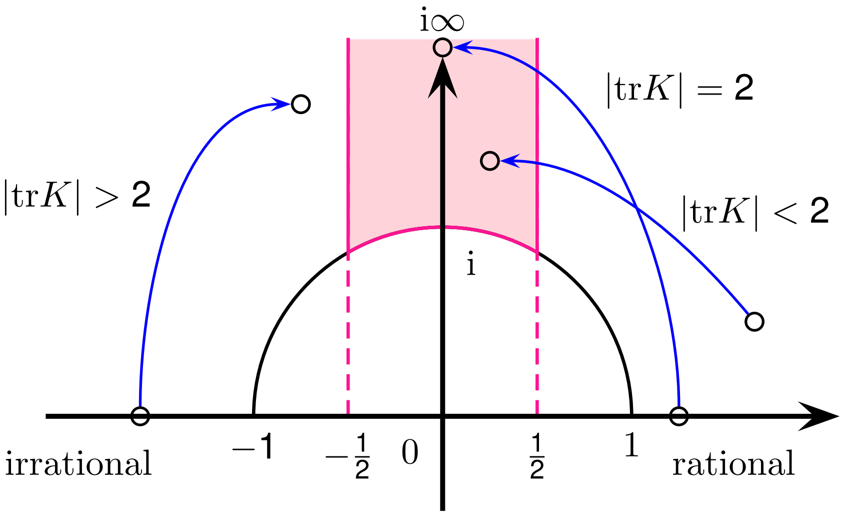
via $SL(2,\mathbb{Z}) \subset SO(2,2;\mathbb{Z})$

T_{ab}-duality

$|\mathrm{tr}\boldsymbol{K}|$ is a good character to classify 7-branes :

$$\boldsymbol{K} \cdot \boldsymbol{\rho}_* = \frac{a\rho_* + b}{c\rho_* + d} = \rho_* \,, \qquad \boldsymbol{K} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$$\therefore \quad \rho_* = \frac{1}{2c}\Big\{(a-d) \pm \sqrt{(\mathrm{tr}\boldsymbol{K})^2 - 4}\Big\}$$



$|\mathrm{tr}\boldsymbol{K}| = 2$: parabolic (collapsible)

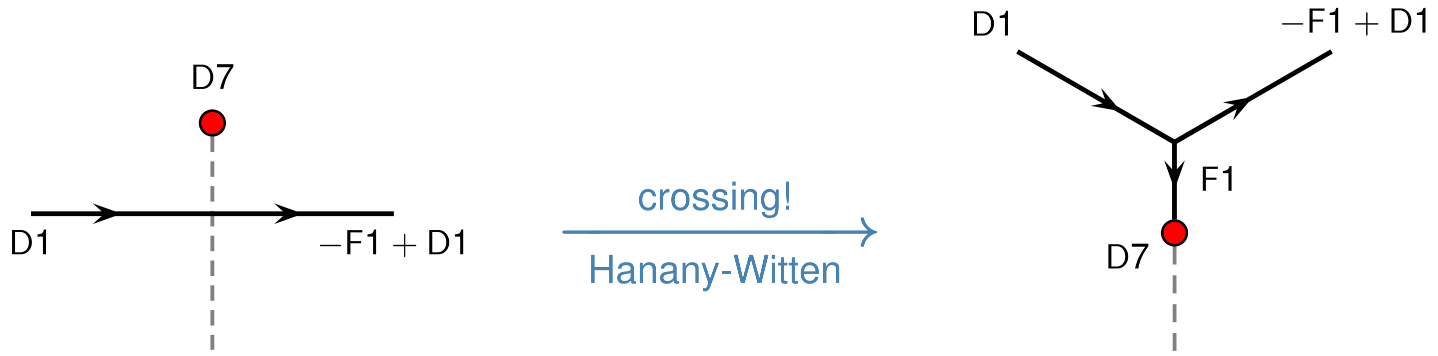
$|\mathrm{tr}\boldsymbol{K}| < 2$: elliptic (collapsible)

$|\mathrm{tr}\boldsymbol{K}| > 2$: hyperbolic (non-collapsible)

tr$K$	monodromy	branes	collapsible?	symmetry	type
+2	$T^{-n} = K_A^n$	A^n	yes	A_{n-1} $(n \geq 1)$	I_n
	$\mathbb{1} = T^0 = K_C K_B K_C K_B K_A^8$	$\widehat{\mathrm{E}}_9 \equiv \mathrm{A}^8\mathrm{BCBC}$	yes	$\widehat{E}_9$	I_0
	$T^{\lvert n\rvert} = K_C K_B K_C K_B K_A^{8-\lvert n\rvert}$	$\mathrm{A}^{8-\lvert n\rvert}\mathrm{BCBC}$	no	$\widehat{E}_{9-\lvert n\rvert}$ $(n \leq -1)$	
+1	$ST \sim K_C K_A$	$\mathrm{H}_0 \equiv \mathrm{AC}$	yes	H_0	II
	$(ST)^{-1} \sim K_C^2 K_B K_A^7$	$\mathrm{E}_8 \equiv \mathrm{A}^7\mathrm{BC}^2$	yes	E_8	II^*
0	$S \sim K_C K_A^2$	$\mathrm{H}_1 \equiv \mathrm{A}^2\mathrm{C}$	yes	H_1	III
	$-S \sim K_C^2 K_B K_A^6$	$\mathrm{E}_7 \equiv \mathrm{A}^6\mathrm{BC}^2$	yes	E_7	III^*
−1	$-(ST)^{-1} \sim K_C K_A^3$	$\mathrm{H}_2 \equiv \mathrm{A}^3\mathrm{C}$	yes	H_2	IV
	$-ST \sim K_C^2 K_B K_A^5$	$\mathrm{E}_6 \equiv \mathrm{A}^5\mathrm{BC}^2$	yes	E_6	IV^*
−2	$-T^{-n} = K_C K_B K_A^{n+4}$	$\mathrm{D}_{n+4} \equiv \mathrm{A}^{n+4}\mathrm{BC}$	yes	D_{n+4} $(n \geq 1)$	I_n^*
	$-\mathbb{1} = -T^0 = K_C K_B K_A^4$	$\mathrm{D}_4 \equiv \mathrm{A}^4\mathrm{BC}$	yes	D_4	I_0^*
	$-T = K_C K_B K_A^3$	$\mathrm{A}^3\mathrm{BC}$	no	D_3	
	$-T^2 = K_C K_B K_A^2$	$\mathrm{A}^2\mathrm{BC}$	no	D_2	
	$-T^3 = K_C K_B K_A$	ABC	no	D_1	
	$-T^4 = K_C K_B$	BC	no	–	

Consider a D-string crossing the branch cut of D7-brane from the left.

A new string and a junction appear by Hanany-Witten transition.



Note: D7-brane is stretched in 1234567-directions.

This is a string junction in F-theory.

Gaberdiel and Zwiebach: hep-th/9709013
DeWolfe and Zwiebach: hep-th/9804210
etc..

Consider a D3-brane wrapped on two-torus (wD3) and defect NS5-brane (dNS5).

If dNS5 goes across wD3,

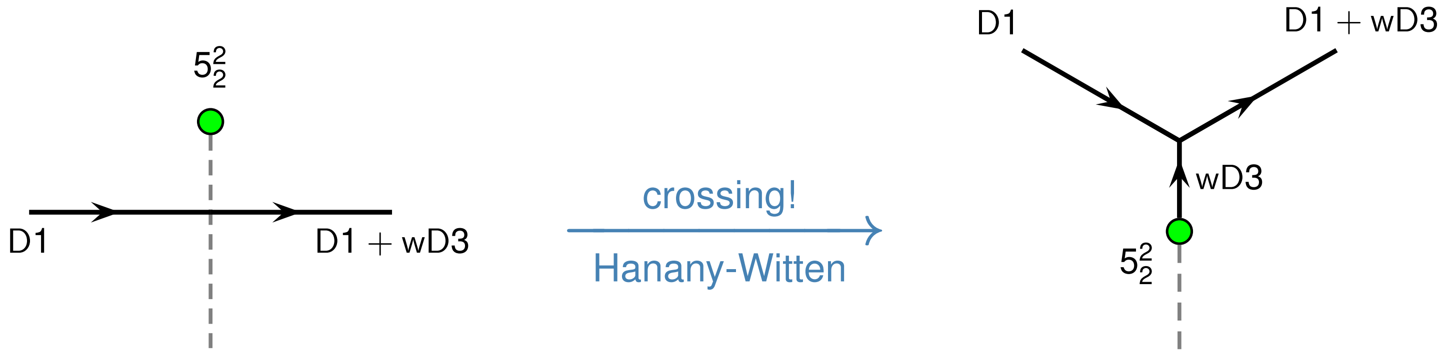
a new D-string and a junction appear by Hanany-Witten transition.

Consider a D-string and 5^2_2-brane.

D-string charge is jumped by monodromy.

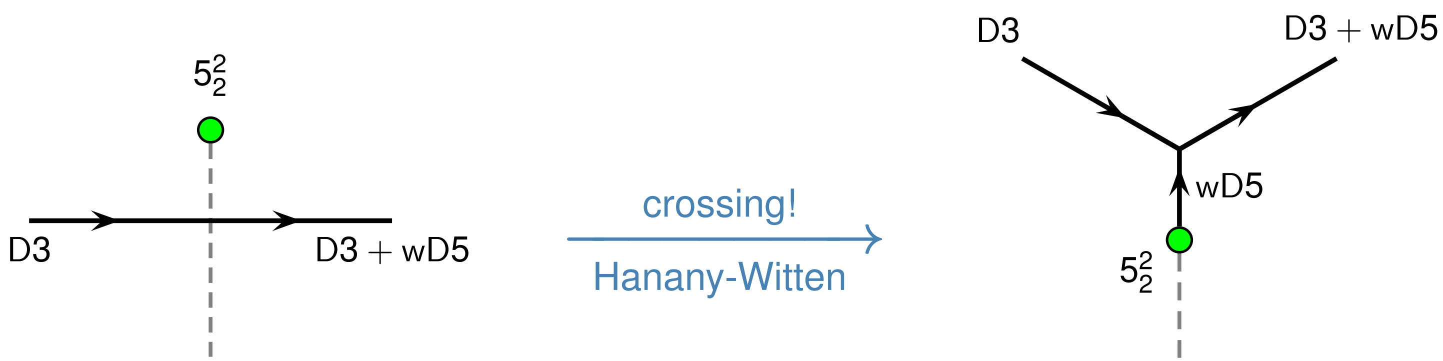
If 5^2_2-brane goes across D-string,

a new wD3 and a junction appear (Hanany-Witten transition).



- defect NS5-brane and D5-brane wrapped on two-torus :

- 5^2_2-brane and D3-brane :



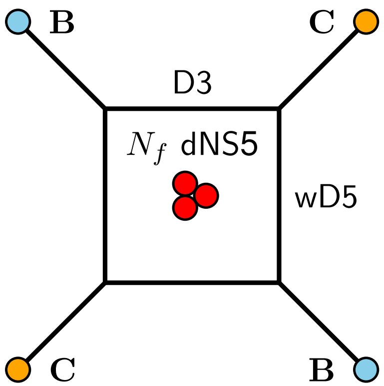
N_f	0	1	2	3	4	5	6	7
symmetry	$E_1 = A_1$	$E_2 = A_1 \times U(1)$	$E_3 = A_2 \times A_1$	$E_4 = A_4$	$E_5 = D_5$	E_6	E_7	E_8

type	symmetry	regular SW curve in 4D	fixed point τ_0
II^*	E_8	$y^2 = x^3 + u^5$	$e^{i\pi/3}$
III^*	E_7	$y^2 = x^3 + u^3 x$	i
IV^*	E_6	$y^2 = x^3 + u^4$	$e^{i\pi/3}$
I_0^*	D_4	$y^2 = \prod_{i=1}^{3}(x - e_i(\tau)u)$	τ
IV	H_2	$y^2 = x^3 + u^2$	$e^{i\pi/3}$
III	H_1	$y^2 = x^3 + ux$	i
II	H_0	$y^2 = x^3 + u$	$e^{i\pi/3}$
I_n^* $(n \geq 1)$	D_{n+4}	$y^2 = x^3 + ux^2 + \Lambda^{-2n} u^{n+3}$	$i\infty$
I_n $(n \geq 1)$	A_{n-1}	$y^2 = (x-1)(x^2 + \Lambda^{-n} u^n)$	$i\infty$

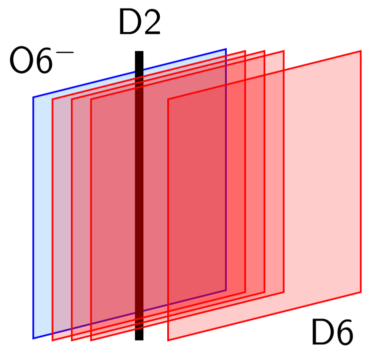
Argyres, Lotito, Lü, Martone: arXiv:1505.04814

3D $\mathcal{N} = 4$ SCFT from configs with N_f branes :

[dNS5, 5^2_2] is $SL(2,\mathbb{Z})$ doublet



vs



Seiberg: hep-th/9606017

dNS5 config :

N_f	0	1	2	3	4	5	6	7
symmetry	A_1	$A_1 \times U(1)$	$A_2 \times A_1$	A_4	D_5	E_6	E_7	E_8

vs

established :

N_f	0	1	2	≥ 3
symmetry	–	–	A_1	$D_{N_f-2} \times A_1$

MN-type		
E_6	E_7	E_8

for $N_f = 5, 6, 7$: Benini, Tachikawa, Xie: arXiv:1007.0992

- D7(1234567) : $z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta}$

$$\begin{aligned} ds^2 &= \frac{1}{(\rho_2)^{1/2}} dx^2_{01234567} + (\rho_2)^{1/2} |f|^2\, dz d\bar{z} \\ e^{2\phi} &= (\rho_2)^{-2} \\ C_{(0)} &= \rho_1 \\ C_{(8)} &= -\frac{1}{\rho_2} dx^0 \wedge dx^1 \wedge \cdots \wedge dx^7 \\ \rho(z) &= \rho_1 + \mathrm{i}\rho_2 \;=\; C_{(0)} + \mathrm{i}e^{-\phi} \;=\; \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{r} \end{aligned}$$

Sakatani: arXiv:1412.8769

- $7_3(1234567)$:

$$
\begin{aligned}
ds'^2 &= \frac{1}{(\rho'_2)^{1/2}}\, dx^2_{01234567} + (\rho'_2)^{1/2} |f'|^2\, dz d\bar{z} \\
e^{2\phi'} &= (\rho'_2)^{-2} \\
C'_{(0)} &= \rho'_1\,, \qquad C'_{(8)} \;=\; -\frac{1}{\rho'_2}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^7 \\
\rho'(z) &= \rho'_1 + i\rho'_2 \;=\; C'_{(0)} + i e^{-\phi'} \;=\; -\frac{1}{\rho_{D7}} \\
\rho'_1 &= -\frac{\rho_1}{|\rho|^2}\,, \qquad \rho'_2 \;=\; \frac{\rho_2}{|\rho|^2} \\
\rho'_2 |f'|^2 &= \rho_2 |f|^2\,, \qquad |f'| \;=\; |\rho||f|
\end{aligned}
$$

- defect NS5(12345) smeared along 67-directions :

$$z = x^8 + \mathrm{i} x^9 = r\, \mathrm{e}^{\mathrm{i}\theta}$$

$$\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}x_{012345}^2 + \rho_2\, \mathrm{d}x_{67}^2 + \rho_2 |f|^2\, \mathrm{d}z \mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \rho_2 \\
B_{(2)} &= \rho_1\, \mathrm{d}x^6 \wedge \mathrm{d}x^7 \,, \qquad B_{(6)} = \frac{1}{\rho_2} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^5 \\
\rho(z) &= \rho_1 + \mathrm{i}\rho_2 = B_{67}^{(2)} + \mathrm{i}\mathrm{e}^{2\phi} = B_{67}^{(2)} + \mathrm{i}\sqrt{\det G_{mn}} = \frac{\theta}{2\pi} + \frac{\mathrm{i}}{2\pi} \log \frac{\Lambda}{r} \\
\tau &= (\text{complex structure of } T_{67}^2) = \mathrm{i} \\
m, n &= 6, 7
\end{aligned}$$

- 5^2_2(12345,67) :

$$
\begin{aligned}
ds'^2 &= dx^2_{012345} + \rho'_2\, dx^2_{67} + \rho'_2 |f'|^2\, dz d\bar{z} \\
e^{2\phi'} &= \rho'_2 \\
B'_{(2)} &= \rho'_1\, dx^6 \wedge dx^7 \,, \qquad B'_{(6)} \;=\; \frac{1}{\rho'_2}\, dx^0 \wedge dx^1 \wedge \cdots \wedge dx^5 \\
\rho'(z) &= \rho'_1 + i\rho'_2 \;=\; B'^{(2)}_{67} + i e^{2\phi'} \;=\; B'^{(2)}_{67} + i\sqrt{\det G'_{mn}} \;=\; -\frac{1}{\rho_{\text{NS5}}} \\
\rho'_1 &= -\frac{\rho_1}{|\rho|^2} \,, \qquad \rho'_2 \;=\; \frac{\rho_2}{|\rho|^2} \\
\tau' &= (\text{complex structure of } \widetilde{T}^2_{67}) \;=\; i \;=\; -\frac{1}{\tau_{\text{NS5}}} \\
\rho'_2 |f'|^2 &= \rho_2 |f|^2 \,, \qquad m, n = 6, 7
\end{aligned}
$$

- defect KK5(12345,7) smeared along 67-directions :

$$\begin{aligned}
ds^2 &= dx^2_{012345} + \tau_2\, dx^2_6 + \frac{1}{\tau_2}\big(dx^7 - \tau_1\, dx^6\big)^2 + \tau_2 |f|^2\, dz d\bar{z} \\
e^{2\phi} &= 1\,, \qquad B_{(2)} \;=\; 0\,, \qquad B_{(6)} \;=\; 0 \\
\rho &= \rho_1 + i\rho_2 \;=\; B^{(2)}_{67} + i\sqrt{\det G_{mn}} \;=\; i \\
\tau(z) &= (\text{complex structure of } T^2_{67}) \;=\; \tau_1 + i\tau_2 \;=\; \frac{\theta}{2\pi} + \frac{i}{2\pi}\log\frac{\Lambda}{r} \\
m, n &= 6, 7
\end{aligned}$$

- defect D$p(12\cdots p)$ smeared along $a_1 \cdots a_{7-p}$-directions : $z = x^8 + \mathrm{i}x^9 = r\,\mathrm{e}^{\mathrm{i}\theta}$

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{(\rho_2)^{1/2}}\,\mathrm{d}x^2_{012\cdots p} + (\rho_2)^{1/2}\,\mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\rho_2)^{1/2}|f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= (\rho_2)^{\frac{3-p}{2}} \\
C_{(7-p)} &= \rho_1\,\mathrm{d}x^{a_1}\wedge\cdots\wedge\mathrm{d}x^{a_{7-p}} \\
C_{(p+1)} &= -\frac{1}{\rho_2}\,\mathrm{d}x^0\wedge\mathrm{d}x^1\wedge\cdots\wedge\mathrm{d}x^p \\
\rho(z) &= \rho_1 + \mathrm{i}\rho_2 \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi} \;=\; C^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{-\phi}(\det G_{mn})^{1/2} \\
m, n &= a_1, \ldots, a_{7-p}
\end{aligned}
$$

- $p_3^{7-p}(12\cdots p, a_1 \cdots a_{7-p})$:

$$
\begin{aligned}
\mathrm{d}s'^2 &= \frac{1}{(\rho_2')^{1/2}}\, \mathrm{d}x^2_{012\cdots p} + (\rho_2')^{1/2}\, \mathrm{d}x^2_{a_1\cdots a_{7-p}} + (\rho_2')^{1/2} |f'|^2\, \mathrm{d}z \mathrm{d}\overline{z} \\
\mathrm{e}^{2\phi'} &= (\rho_2')^{\frac{3-p}{2}} \\
C'_{(7-p)} &= \rho_1'\, \mathrm{d}x^{a_1} \wedge \cdots \wedge \mathrm{d}x^{a_{7-p}}\,, \qquad C'_{(p+1)} \;=\; -\frac{1}{\rho_2'}\, \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \cdots \wedge \mathrm{d}x^p \\
\rho'(z) &= \rho_1' + \mathrm{i}\rho_2' \;=\; C'^{(7-p)}_{a_1\cdots a_{7-p}} + \mathrm{i}\mathrm{e}^{\frac{4}{3-p}\phi'} \;=\; -\frac{1}{\rho_{\mathrm{D}p}} \\
\rho_1' &= -\frac{\rho_1}{|\rho|^2}\,, \qquad \rho_2' \;=\; \frac{\rho_2}{|\rho|^2} \\
\rho_2' |f'|^2 &= \rho_2 |f|^2\,, \qquad |f'| \;=\; |\rho||f|\,, \qquad m, n = a_1, \ldots, a_{7-p}
\end{aligned}
$$

- defect F1(1) smeared along 234567-directions :

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{1}{\rho_2}\,\mathrm{d}x_{01}^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\
\mathrm{e}^{2\phi} &= \frac{1}{\rho_2} \\
B_{(6)} &= \rho_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B_{(2)} \;=\; -\frac{1}{\rho_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1 \\
\rho(z) &= \rho_1 + \mathrm{i}\rho_2 \;=\; B^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi}
\end{aligned}
$$

- 1^6_4(1,234567) :

$$\mathrm{d}s'^2 = \frac{1}{\rho'_2}\,\mathrm{d}x^2_{01} + \mathrm{d}x^2_{234567} + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z}$$

$$\mathrm{e}^{2\phi'} = \frac{1}{\rho'_2}$$

$$B'_{(6)} = \rho'_1\,\mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \cdots \wedge \mathrm{d}x^7\,, \qquad B'_{(2)} = -\frac{1}{\rho'_2}\,\mathrm{d}x^0 \wedge \mathrm{d}x^1$$

$$\rho'(z) = \rho'_1 + \mathrm{i}\rho'_2 = B'^{(6)}_{234567} + \mathrm{i}\mathrm{e}^{-2\phi'} = -\frac{1}{\rho_{\mathrm{F1}}}$$

$$\rho'_2|f'|^2 = \rho_2|f|^2\,, \qquad |f'| = |\rho||f|$$

- defect P smeared along 1234567-directions :

$$\begin{aligned} \mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho_2\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\ \mathrm{e}^{2\phi} &= \frac{1}{\rho_2}\,, \quad B_{(2)} = 0\,, \quad B_{(6)} = 0 \\ \rho(z) &= \rho_1 + \mathrm{i}\rho_2 = \mathrm{i}\mathrm{e}^{-2\phi} \end{aligned}$$

- $0_4^{(1,6)}$(,234567,1) :

$$\begin{aligned} \mathrm{d}s^2 &= -2\mathrm{d}x^0\mathrm{d}x^1 + \rho'_2\,\mathrm{d}x_1^2 + \mathrm{d}x_{234567}^2 + |f'|^2\,\mathrm{d}z\mathrm{d}\bar{z} \\ \mathrm{e}^{2\phi'} &= \frac{1}{\rho'_2} = \frac{|\rho|^2}{\rho_2}\,, \quad B'_{(2)} = 0\,, \quad B'_{(6)} = 0 \\ \rho'(z) &= \rho'_1 + \mathrm{i}\rho'_2 = \mathrm{i}\mathrm{e}^{-2\phi'} = -\frac{1}{\rho_{\mathrm{P}}}\,, \quad \rho'_2|f'|^2 = \rho_2|f|^2\,, \quad |f'| = |\rho||f| \end{aligned}$$