Deformations of the Almheiri – Polchinski model

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based on **arXiv:1701. 06340** [**Hideki Kyono, S.O. , Kentaroh Yoshida**]

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Motivation for the Low Dimension Model

One of the most important issues of the String theory

: to understand **the Holographic Principle**

AdS / CFT

- There is no proof of this conjecture.
- It is difficult to conform it in Quantum Level .

We need a simple toy model of Quantum Holography.

Recent Development of AdS2 / CFT1

CFT1 : **Quantum Mechanics**

Sachdev-Ye-Kitaev model [Sachdev, Ye, "93] [Kitaev, "15]

Many-Body system of Majorana Fermions

AdS2 : **2D dilaton gravity**

Almheiri - Polchinski model [Almheili, Polchinski, "14]
[Jackiw, "85] [Teitelboin, "83]

It have the Black Hole as the Vacuum Solution.

Analytical Solutions with the Matters

Motivations to study the Deformed Model

1.To obtain new insight of the AdS_2 / CFT_1.

Comparing Both Deformations
(Deformed Thermodynamics, etc…) ⟹ **New Description of the Duality**

2. To understand the Holographic Duals
of the Deformed Geometris .

by following **the Yang – Baxter Deformations**

Contents of This Talk

Action, EOM and Solutions Vacuum Case

AP model : 2D dilaton gravity model with a certain dilaton potential

$$S_\Phi = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\,\left(\Phi^2 R - (2 - 2\Phi^2)\right)$$

Conformal Gauge

$$ds^2 = -e^{2\omega}dx^+dx^- \qquad x^\pm \equiv t \pm z$$

EOM

$$\begin{cases} 4\partial_+\partial_-\omega + e^{2\omega} = 0 \\ 2\partial_+\partial_-\Phi^2 + e^{2\omega}(\Phi^2 - 1) = 0 \\ -e^{2\omega}\partial_\pm(e^{-2\omega}\partial_\pm\Phi^2) = 0 \end{cases}$$

Liouville equation is decoupled from the Dilaton

General Vacuum Solutions

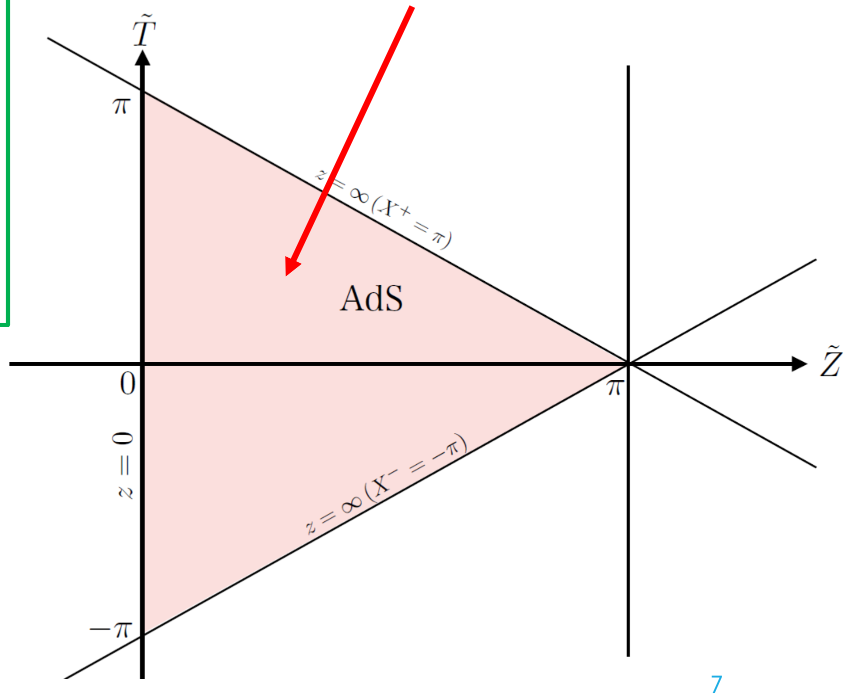
Poincare coordinates

$$\mathbf{d}s^2 = \frac{1}{z^2}(-\mathbf{d}t^2 + \mathbf{d}z^2)$$

$$\Phi^2 = 1 + \frac{a + b\,t + c\,(-t^2 + z^2)}{z}$$

a, b, c : arbitrary constants

Ricci Scalar < 0



Black Hole Solution

$$a = \frac{1}{2} \qquad b = 0 \qquad c = \frac{\mu}{2}$$

$$x^{\pm} = \frac{1}{\sqrt{\mu}} \tanh\left(\sqrt{\mu}\,(T \pm Z)\right)$$

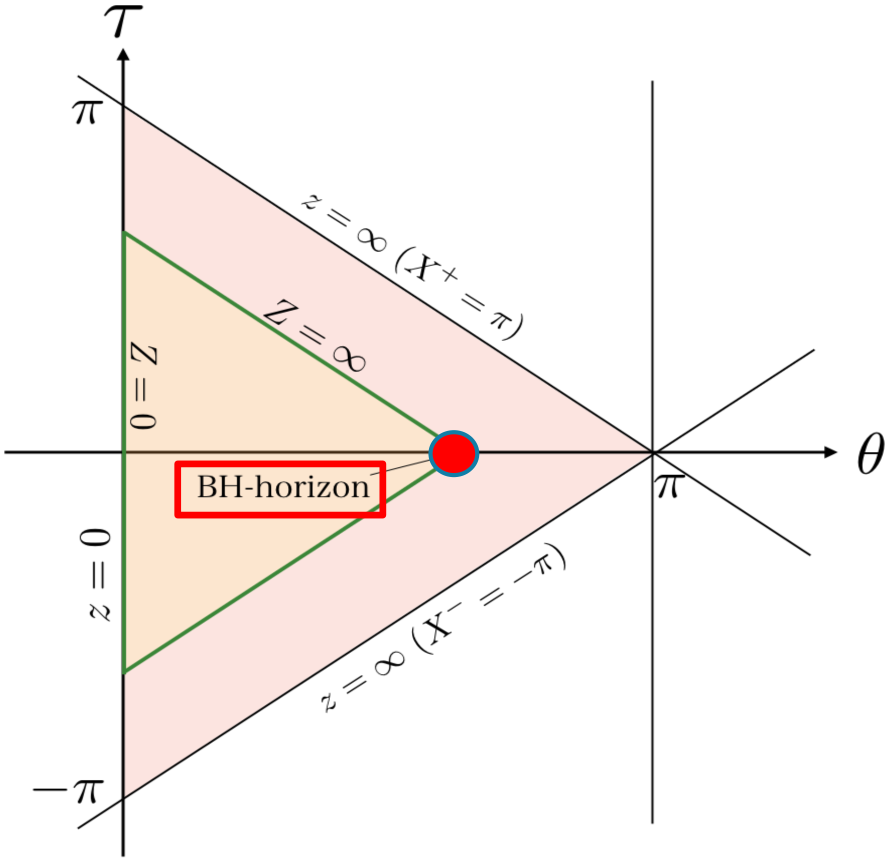
$$\mathrm{d}s^2 = \frac{4\,\mu}{\sinh(2\sqrt{\mu}\,Z)}(-\mathrm{d}T^2 + \mathrm{d}Z^2)$$

$$\Phi^2 = 1 + \sqrt{\mu}\coth(2\sqrt{\mu}\,Z)$$

$$Z = \frac{1}{2\sqrt{\mu}}\mathrm{arccoth}\left(\frac{\rho}{\sqrt{\mu}}\right)$$

$$\mathrm{d}s^2 = -4(\rho^2 - \mu)\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2 - \mu}$$

$$\Phi^2 = 1 + \rho$$



Black Hole Horizon

$$\rho = \mu \qquad (Z = \infty)$$

Bekenstein Hawking Entropy

$$ds^2 = \frac{4\mu}{\sin(2\sqrt{\mu}\,Z)}(-dT^2 + dZ^2)$$

$$= -4(\rho^2 - \mu)dt^2 + \frac{d\rho^2}{\rho^2 - \mu}$$

$$\Phi^2 = 1 + \sqrt{\mu}\coth(2\sqrt{\mu}\,Z)$$

Hawking Temperature

$$T_{\rm H} = \frac{1}{4\pi}\partial_\rho \sqrt{\frac{-g_{tt}}{g_{\rho\rho}}}\Bigg|_{\rho=\sqrt{\mu}} = \frac{\sqrt{\mu}}{\pi}$$

Bekenstein Hawking Entropy

$$S_{\rm BH} = \frac{A}{4G_{\rm eff}}$$

A : Area of the Horizon

$$\frac{1}{G_{\rm eff}} = \frac{\Phi^2}{G} \qquad A = 1$$

The Horizon is a just point.

$$S_{\rm BH} = \frac{\Phi^2}{4G}\Bigg|_{Z\to\infty} = \frac{1+\pi T_{\rm H}}{4G}$$

Entropy Evaluated on the Boundary

On – shell Action ϵ : regulator

$$S_\Phi + S_{\rm GH} = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\left[\Phi^2 R - (2-2\Phi^2)\right] + \frac{1}{8\pi G}\int dt\sqrt{-\gamma_{tt}}\,\Phi^2 K$$

$$= \int dt\left[\frac{1}{16\pi G\epsilon^2} + O(\epsilon^0)\right]$$

To cancel the divergence, **the counter term** is needed.

$$S_{\rm ct} = \frac{1}{8\pi G}\int dt\sqrt{-\gamma_{tt}}\,(1-\Phi^2)$$

The Boundary Stress Tensor

$$\langle \hat{T}_{tt}\rangle \equiv \frac{-2}{\sqrt{-\hat{\gamma}_{tt}}}\frac{\delta S_{total}}{\delta \hat{\gamma}^{tt}} = \frac{\pi T_{\rm H}^2}{8G}$$

$$dS = \frac{dE}{T_{\rm H}}$$

Entropy

$$S = \frac{\pi T_{\rm H}}{4G} + S_{T_{\rm H}=0}$$

BH entropy is reproduced.

$$S_{total} \equiv S_\Phi + S_{\rm GH} + S_{\rm ct}$$

Contents of This Talk

0. Introduction

1. Review of the AP model

2. Deformations of the AP model

   Deformations of the Model / Deformed Black Hole / Horizon Entropy / Entropy by Boundary Stress Tensor

3. Conclusion and Discussion

Deformed Metric

AdS2

$$ds^2 = \frac{1}{z^2}(-dt^2 + dz^2)$$

Yang – Baxter Deformations

Deformed Metric

$$ds^2 = \frac{-dt^2 + dz^2}{z^2 - \eta^2\left(\alpha + \beta t + \gamma(-t^2 + z^2)\right)^2}$$

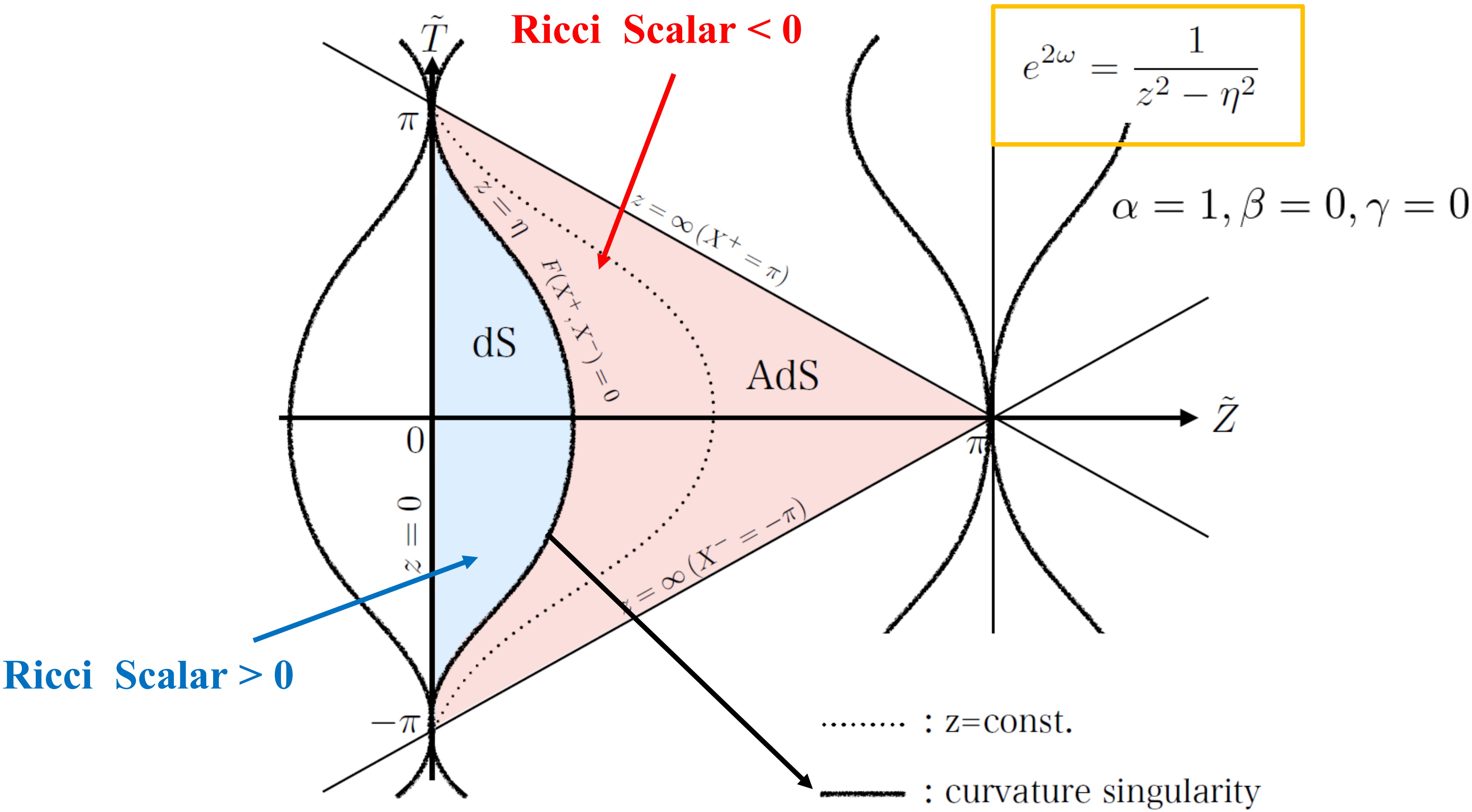
η : Parameter measuring the deformation

$\eta \to 0$ **Undeformed Limit**

α, β, γ : Deformation Parameters

We can consider the General Deformations in 2D case.

Geometry of the Deformed Background



Strategy to Obtain the Deformed Model

Assuming the General Dilaton Gravity

$$S_{\Phi} = \frac{1}{16\pi G} \int d^2x \sqrt{-g} \left(\Phi^2 R - U(\Phi) \right)$$

Deformed Metric

$$ds^2 = \frac{-dt^2 + dz^2}{z^2 - \eta^2 \left(\alpha + \beta t + \gamma(-t^2 + z^2) \right)^2}$$

Solve EOM →

- Deformed Dilaton Φ^2
- Deformed Dilaton Potential $U(\Phi)$

Deformed Dilaton Gravity Model

Deformed Dilaton Potential $\beta^2 + 4\alpha\gamma = -4c$

$$U(\Phi) = \begin{cases} -(1+4c\eta^2)\dfrac{1}{\eta}\sinh\left[2\eta(\Phi^2-1)\right] \\ +(1+4c\eta^2)\dfrac{1}{\eta}\sinh\left[2\eta(\Phi^2-1)\right] \end{cases}$$

← Ricci Scalar < 0 (first case)

← Ricci Scalar > 0 (second case)

check

$$ds^2 = \frac{-dt^2 + dz^2}{z^2 - \eta^2\left(\alpha + \beta t + \gamma(-t^2 + z^2)\right)^2}$$

$$\Phi^2 = 1 + \frac{1}{2\eta}\log\left|\frac{z + \eta\left(\alpha + \beta t + \gamma(-t^2+z^2)\right)}{z - \eta\left(\alpha + \beta t + \gamma(-t^2+z^2)\right)}\right|$$

The AP model

$$U(\Phi) = 2 - 2\Phi^2$$

$$\Phi^2 = 1 + \frac{\alpha + \beta t + \gamma(-t^2 + z^2)}{z}$$

Undeformed Limit $\eta \to 0$

Black Hole Solution

$$\alpha = \frac{1}{2} \quad \beta = 0 \quad \gamma = \frac{\mu}{2}$$

$$x^{\pm} = \frac{1}{\sqrt{\mu}} \tanh\left(\sqrt{\mu}\,(T \pm Z)\right)$$

$$\mathrm{d}s^2 = \frac{4\mu(-\mathrm{d}T^2 + \mathrm{d}Z^2)}{\sinh^2(2\sqrt{\mu}\,Z) - \eta^2\mu\cosh^2(2\sqrt{\mu}\,Z)}$$

$$\Phi^2 = 1 + \frac{1}{2\eta}\log\left|\frac{1+\eta\sqrt{\mu}\,\coth(\sqrt{\mu}\,Z)}{1-\eta\sqrt{\mu}\,\coth(\sqrt{\mu}\,Z)}\right|$$

$$r = \frac{1}{\eta}\mathrm{arctanh}\left(\eta\sqrt{\mu}\,\coth\left(2\sqrt{\mu}\,Z\right)\right)$$

$$\mathrm{d}s^2 = -4F(r)\,\mathrm{d}T^2 + \frac{\mathrm{d}r^2}{F(r)}$$

$$\Phi^2 = 1 + r$$

Undeformed Limit $\eta \to 0$

The AP model

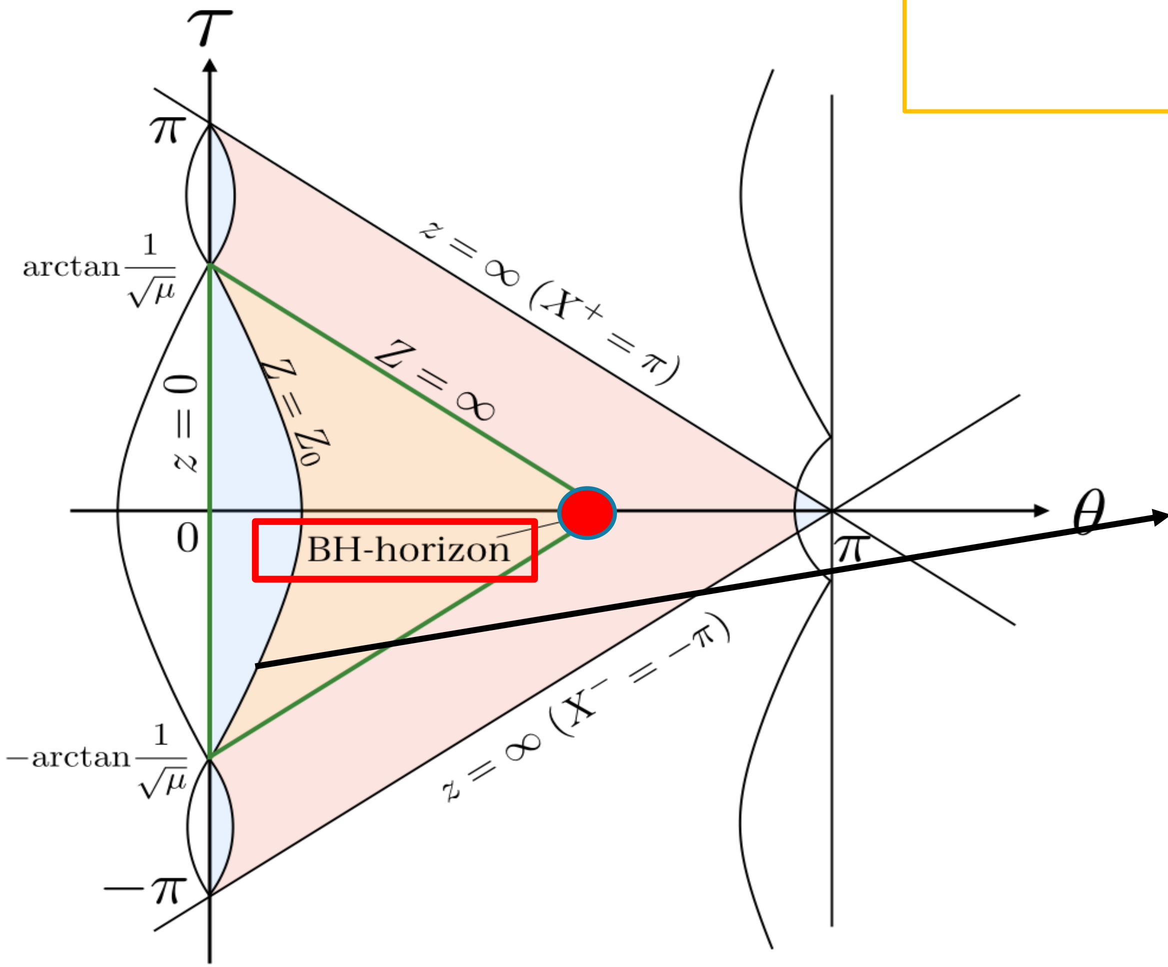
$$\mathrm{d}s^2 = \frac{4\,\mu}{\sinh(2\sqrt{\mu}\,Z)}(-\mathrm{d}T^2 + \mathrm{d}Z^2)$$

$$\Phi^2 = 1 + \sqrt{\mu}\coth(2\sqrt{\mu}\,Z)$$

$$F(r) \equiv \frac{-1 - \eta^2\mu + (1-\eta^2\mu)\cosh(2\,\eta\,r)}{2\eta^2}$$

Black Hole Solution

$$ds^2 = \frac{4\mu(-dT^2 + dZ^2)}{\sinh^2(2\sqrt{\mu}\,Z) - \eta^2\mu\cosh^2(2\sqrt{\mu}\,Z)}$$



Curvature Singularity

$$Z_0 \equiv \frac{1}{2\sqrt{\mu}}\,\text{arctanh}(\eta\sqrt{\mu})$$

Parameter Region

$$0 < \eta < \frac{1}{\sqrt{\mu}}$$

Bekenstein Hawking Entropy

Hawking Temperature

$$T_{\rm H} = \frac{1}{4\pi}\partial_r \sqrt{-\frac{g_{tt}}{g_{rr}}}\Bigg|_{r=r^*} = \frac{\sqrt{\mu}}{\pi}$$

The same as the AP model.

Bekenstein Hawking Entropy

$$S_{\rm BH} = \frac{A}{4G_{\rm eff}}$$

$$\frac{1}{G_{\rm eff}} = \frac{\Phi^2}{G} \qquad A = 1$$

$$S_{\rm BH} = \frac{\Phi^2}{4G}\Bigg|_{Z\to\infty} = \frac{\text{arctanh}(\pi T_{\rm H}\,\eta)}{4G\eta} + \frac{1}{4G}$$

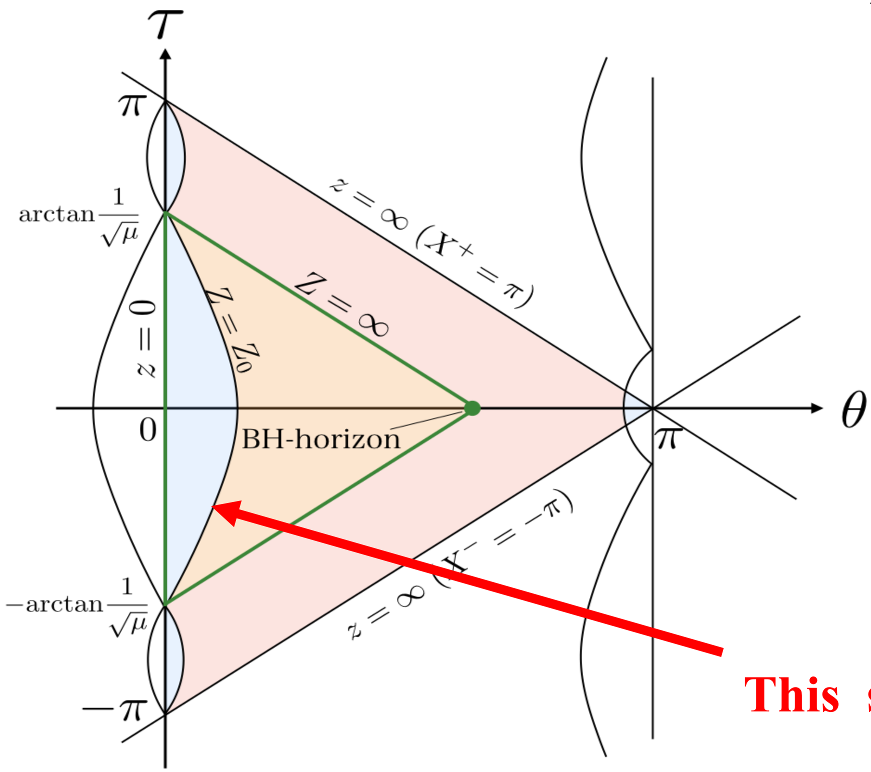
The AP model

$$S_{\rm BH} = \frac{1+\pi T_{\rm H}}{4G}$$

Undeformed Limit

$$\eta \to 0$$

Holographic Screen



Deformed Black Hole

$$ds^2 = \frac{4\mu(-dT^2 + dZ^2)}{\sinh^2(2\sqrt{\mu}\,Z) - \eta^2\mu\cosh^2(2\sqrt{\mu}\,Z)}$$

Naked Singularity are generated.

$$Z_0 \equiv \frac{1}{2\sqrt{\mu}}\,\mathrm{arctanh}(\eta\sqrt{\mu})$$

This singularity surface is Holographic screen.

cf. The deformations of $AdS_5 \times S_5$

[Kameyama , Yoshida "15]

On – shell Action on the Boundary

On – shell Action $(Z = Z_0 + \epsilon)$

$$S_\Phi + S_{\rm GH} = \frac{1}{16\pi G}\int d^2x\,\sqrt{-g}\,\left[\Phi^2 R - U(\Phi)\right] + \frac{1}{8\pi G}\int dt\,\sqrt{-\gamma}\,\Phi^2 K$$

$$= \int dt\left[\frac{1}{16\pi G\,\eta\,\epsilon} + O(\epsilon^0)\right]$$

Counter Term

$$S_{\rm ct} = -\frac{1}{8\pi G}\int dt\,\sqrt{-\gamma_{tt}}\,\sqrt{F(\Phi^2-1) - \frac{1}{\eta^2}\log(1-\eta^2\mu)}$$

$$F(\Phi^2-1) = \frac{-1-\eta^2\mu + (1-\eta^2\mu)\cosh(2\eta(\Phi^2-1))}{2\eta^2}$$

Undeformed Limit
$\eta \to 0$

$$S_{\rm ct} = \frac{1}{8\pi G}\int dt\,\sqrt{-\gamma_{tt}}\,(1-\Phi^2)$$

The AP model

Entropy Evaluated on the Boundary

The Boundary Stress Tensor

$$S_{total} \equiv S_{\Phi} + S_{\mathrm{GH}} + S_{\mathrm{ct}}$$

$$\langle \hat{T}_{tt} \rangle \equiv \frac{-2}{\sqrt{-\hat{\gamma}_{tt}}} \frac{\delta S_{total}}{\delta \hat{\gamma}^{tt}}$$

$$= -\frac{\log(1 - \pi^2 T_{\mathrm{H}}^2 \eta^2)}{8\pi G\, \eta^2}$$

$$\mathrm{d}S = \frac{\mathrm{d}E}{T_{\mathrm{H}}}$$

Entropy

$$S = \frac{\mathrm{arctanh}(\pi T_{\mathrm{H}} \eta)}{4G\eta} + S_{T_{\mathrm{H}}=0}$$

BH entropy is reproduced.

Conclusion

1. We discuss the deformations of the AP model by following the Yang–Baxter deformation.

2. We obtain the Deformed Black Hole Solution.

3. Horizon Entropy is reproduced by the boundary stress tensor with an appropriate counter term.

Discussion

Yang – Baxter deformation

: a Systematic way of Integrable Deformations

Our Deformations correspond to the q-deformations.

General Form of Dilaton Gravity and Nonlinear Gauge Theory

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ABSTRACT

We construct a gauge theory based on general nonlinear Lie algebras. The generic form of 'dilaton' gravity is derived from nonlinear Poincaré algebra, which

nce we consider nonlinear extension* of th
orentz structure of the genuine Poincaré a

$$[J, J] = 0, \quad [J, P_a] = \epsilon_{ab}\eta^{bc}P_c,$$
$$[P_a, P_b] = -\epsilon_{ab}\mathcal{W}(J),$$

mensional Minkowski metric. Note that th
ginal Poincaré algebra.

Dilaton Potential

$$[P_a, P_b] = \epsilon_{ab} \sinh(J)$$

q-deformed sl(2) Algebra