## Supporting Information

# Electronic excitation spectra of cerium oxides: from ab initio dielectric response functions to Monte Carlo electron transport simulations 

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1 DOS: tests of different $U_{\text {eff }}$ values. Band structure of bulk $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$


Fig. S1 Total DOS of bulk $\mathrm{CeO}_{2}$. We show the effect of different Hubbard corrections ( $U_{\text {eff }}=0,3,5 \mathrm{eV}$ ) to deal with the localized $4 f$ states (highest peak between 2 and 4 eV ). The top of the valence band is set to 0 eV .

[^0]Figure S1 shows the total density of states (TDOS) of bulk $\mathrm{CeO}_{2}$ obtained by tuning the Hubbard correction in the commonly used range $U_{\text {eff }}=[0: 5] \mathrm{eV} .^{1]}$ Increasing $U_{\text {eff }}$ results in a blueshift of the $4 f$ states localized in the energy region 2 to 4 eV (the top of the valence band is shifted to 0 eV ), which further opens the $\mathrm{O} 2 p-\mathrm{Ce} 4 f$ gap by 0.3 eV with respect to the plain DFT-LSDA value. The broad band between -4 eV and 0 eV remains untouched by changing $U_{\text {eff }}$.


Fig. S2 Total DOS of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ bulk solid. We show the effect of different Hubbard corrections $U_{\text {eff }}=5,6 \mathrm{eV}$ to deal with the localized $4 f$ states (sharp peak near to 2 eV ).

In Figure S2 we report the TDOS of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ obtained using a Hubbard correction in the range $U_{\text {eff }}=[5: 6] \mathrm{eV}$. In Figs. S 3 and $\mathrm{S4}$ we plot the band structure of $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$, respectively, calculated using DFT $+U$.


Fig. S3 Band structure of bulk $\mathrm{CeO}_{2}$ calculated using DFT $+U$. The Fermi level (dashed red line) is set to the origin of the energy axis.


Fig. S4 Band structure of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ calculated using DFT $+U$. The Fermi level (dashed red line) is set to the origin of the energy axis.

## 2 ELF in the optical limit: tests of different $U_{\text {eff }}$ values. Momentum dispersion of the ELF



Fig. S5 ELF of bulk $\mathrm{CeO}_{2}$ in the optical limit $(q \rightarrow 0)$, calculated within the LSDA-ALDA approximation, using different values of the Hubbard correction $U_{\text {eff }}$.

In Fig. 55 we report the ELF of bulk $\mathrm{CeO}_{2}$ in the optical limit $(q \rightarrow 0)$ analysing the impact of different values of the Hubbard correction $U_{\text {eff }}$, concluding a negligible change.

The impact of different approximations on the ELF to include many-body effects using either the random phase approximation (RPA) or the ALDA kernel, with and without Local Field Effects (LFE) is shown in Fig. 4 (top panel) of the main text.

We notice that the key factor that mostly affects the accuracy is the introduction of LFE, since these effects are related to local density inhomogeneities and, thus, can be relevant in the assessment of the ELF as a function of the direction of the transferred momentum.

Furthermore, we also tested the bootstrap kernel ${ }^{[2}$, that gives results similar to RPA showing that excitonic effects are small for this system.

The ab-initio Bethe surface of bulk $\mathrm{CeO}_{2}$ up to $\sim 170 \mathrm{eV}$, including thus the 4 d transitions, is presented in Fig. S6 (top panel). It is
interesting to notice that in the $100-120 \mathrm{eV}$ range the transition peak shifts towards lower energies with increasing momentum transfer. The Bethe surface of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ is also shown in Fig. $\mathrm{S6}$ (bottom panel).


Fig. S6 ELF of bulk $\mathrm{CeO}_{2}$ along the [111] direction (top panel) and of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ along the [110] direction (bottom panel) from TDDFT calculations including LFE within ALDA.

Dependence on the direction of the momentum transfer of the ELF of bulk $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ is reported in Figs. S7 and S 8 respectively.


Fig. S7 Dependence of the ELF of bulk $\mathrm{CeO}_{2}$ on different orientations of the momentum transfer vector. Left panel: [211]. Middle panel: [110]. Right panel: [111].


Fig. S8 Dependence of the ELF of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ on different orientations of the momentum transfer vector. Left panel: [001]. Middle panel: [110]. Right panel: [100].

## 3 Refractive index and extinction coefficient

From the knowledge of the macroscopic dielectric function $\left(\bar{\varepsilon}_{1}, \bar{\varepsilon}_{2}\right)$ of a material, one can reckon also its optical properties, such as the refraction index $n$ and the extinction coefficient $\kappa$, as follows:

$$
\begin{equation*}
n=\sqrt{\frac{1}{2}\left(\sqrt{\bar{\varepsilon}_{1}^{2}+\bar{\varepsilon}_{2}^{2}}+\bar{\varepsilon}_{1}\right)}, \quad \kappa=\sqrt{\frac{1}{2}\left(\sqrt{\bar{\varepsilon}_{1}^{2}+\bar{\varepsilon}_{2}^{2}}-\bar{\varepsilon}_{1}\right)} \tag{1}
\end{equation*}
$$

where $\bar{\varepsilon}_{1,2}$ are the real and imaginary part of the macroscopic dielectric function, respectively.


Fig. S9 Left: comparison between the refractive index of bulk $\mathrm{CeO}_{2}$ calculated in this work and other experimental and computational studies ${ }^{3}$. Right: comparison between the extinction coefficient of bulk $\mathrm{CeO}_{2}$ calculated in this work and other experimental and computational studies ${ }^{3} 5$

Using Eqns. (1), and the macroscopic dielectric matrix of eqn (4) in the main text we also calculated the refractive index and the extinction coefficient of both cerium oxides. In particular, we report in Fig. S9 the refractive index (left panel) and the extinction
coefficient (right panel) of bulk $\mathrm{CeO}_{2}$, respectively, and their comparison with several other computational and experimental studies ${ }^{3 / 5}$. We find an overall good agreement between our calculations and previous experimental and computational data when including LFE, with an appreciable difference emerging in the range $10-30 \mathrm{eV}$ between the calculated and experimental refractive indexes and in the range $20-30 \mathrm{eV}$ between the calculated and experimental extinction coefficients when switching-off LFE. In fact, LFE strongly suppress the intensity of the third major peak and lead to a better agreement of our simulations with the experimental data.

Moreover, we report in Fig. $\mathrm{S10}$ the refractive index (left panel) and the extinction coefficient (right panel) of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ with and without LFE.


Fig. S10 Refractive index (left) and extinction coefficient (right) of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ with and without LFE.

## 4 Parameters for the MELF-GOS fitting of the ab initio optical ELF

Tables S1 and S2 report, respectively, the parameters used to fit the first principles optical ELF of $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$, by means of Eqns. (6)-(8) in the main text. In the column corresponding to $\Delta_{i}$, when the word "Heaviside" appears instead of a value, it means that the function $F$ is a Heaviside function for this function $i$. The criteria for the fitting, related to the fulfilment of the $f$-sum rules for individual transitions, are explained in the main text, sections 3.2 and 3.3.

Table S1 Parameters used in the MELF-GOS fit of the optical ELF of bulk $\mathrm{CeO}_{2}$ (see Eqns. (6-8) of the main text).

| Mermin | $W_{i}(\mathrm{eV})$ | $\gamma_{i}(\mathrm{eV})$ | $A_{i}\left(\mathrm{eV}^{2}\right)$ | $\Delta_{i}\left(\mathrm{eV}^{-1}\right)$ | $W_{\mathrm{th}, i}(\mathrm{eV})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1 | 6.80 | 6.80 | 5.18 | Heaviside | 2.2 |
| 2 | 15.10 | 3.54 | 44.13 | 1.873 | 5.2 |
| 3 | 25.3 | 11.43 | 107.37 | 1.873 | 14.12 |
| 4 | 33.20 | 7.62 | 125.88 | 1.873 | 14.12 |
| 5 | 33.33 | 6.26 | 214.73 | 1.873 | 16.8 |
| 6 | 42.18 | 14.97 | 259.16 | 37.80 |  |
| 7 | 77.55 | 54.42 | 185.12 | Heaviside | 2.2 |
| 8 | 113.34 | 14.97 | 303.59 | 0.184 | 100.68 |
| 9 | 122.45 | 23.13 | 148.09 | 0.184 | 100.68 |
| 10 | 326.54 | 408.17 | 108.85 | 296.18 | Heaviside |
| 11 | 122.45 |  |  |  | 206.53 |

Table S2 Parameters used in the MELF-GOS fit of the optical ELF of bulk $\mathrm{Ce}_{2} \mathrm{O}_{3}$ (see Eqns. (6-8) of the main text).

| Mermin | $W_{i}(\mathrm{eV})$ | $\gamma_{i}(\mathrm{eV})$ | $A_{i}\left(\mathrm{eV}^{2}\right)$ | $\Delta_{i}\left(\mathrm{eV}^{-1}\right)$ | $W_{\mathrm{th}, i}(\mathrm{eV})$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| 1 | 8.44 | 2.72 | 1.63 | Heaviside | 2.04 |
| 2 | 13.88 | 2.18 | 19.25 | Heaviside | 3.83 |
| 3 | 16.6 | 7.62 | 51.83 | Heaviside | 3.83 |
| 4 | 31.97 | 10.34 | 251.76 | 0.735 | 13.06 |
| 5 | 31.97 | 8.16 | 192.52 | 0.735 | 15.10 |
| 6 | 64.10 | 14.69 | 255.46 | 0.735 | 37.80 |
| 7 | 129.93 | 32.65 | 162.90 | Heaviside | 2.04 |
| 8 | 137.15 | 15.24 | 266.57 | 0.184 | 100.68 |
| 9 | 326.54 | 10.07 | 133.28 | 0.184 | 100.68 |
| 10 |  | 408.17 | 296.18 | 0.184 | 206.53 |

## Notes and references

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