



### Funciones reales de variable real

78. Encuentre los conjuntos de números  $x \in \mathbb{R}$  que verifican las siguientes desigualdades:

- (a)  $2 + 3x < 5$ ; (b)  $3x - 2 \leq 1 + 6x$ ; (c)  $\frac{1}{2}(2x + 3) < 6$ ;  
 (d)  $x^2 - 1 < 0$ ; (e)  $x^2 + 9x + 20 < 0$ ; (f)  $x(2x - 1)(3x - 5) \leq 0$ ;  
 (g)  $x^3 - 2x^2 + x \geq 0$ ; (h)  $1 - 3x^2 < \frac{1}{2}(2 - x^2)$ ; (i)  $6x^2 + 4x + 4 \leq 4(x - 1)^2$ ;  
 (j)  $\frac{x}{x-5} \geq 0$ ; (k)  $\frac{x}{x-5} > \frac{1}{4}$ ; (l)  $\frac{3x^2-1}{1+x^2} \geq 0$ ;  
 (m)  $|x| < 2$ ; (n)  $|x| \geq 1$ ; (o)  $|x - 3| < \frac{1}{4}$ ;  
 (p)  $|5x - 1| > 9$ ; (q)  $|x^2 - 1| < 1$ ; (r)  $0 < |x - \frac{1}{2}| < 2$ ;  
 (s)  $|x^2 - 4| > 3$ ; (t)  $|x^2 - 4x + 1| > 1$ .

79. Calcule las derivadas de las siguientes funciones

- (1)  $y = \frac{x-1}{x+1}$  (2)  $y = \frac{1-x^3}{1+x^3}$  (3)  $y = \frac{2x^2-x+1}{x^2+x-1}$   
 (4)  $y = x - 2\frac{1}{x} + x^{-2} + 5\frac{1}{x^3}$  (5)  $y = (3 - x^4) \left(1 + \frac{3}{x^4}\right)$  (6)  $y = \frac{x+\sqrt{x}}{x+\sqrt[3]{x}}$   
 (7)  $y = \ln(x^2 - 2x)$  (8)  $y = \log_2(2x^3 + 3x^2)$  (9)  $y = \sqrt{\ln x}$   
 (10)  $y = x^n \ln x$  (11)  $y = \frac{\ln x}{x^n}$  (12)  $y = \frac{x^n}{\ln x}$   
 (13)  $y = \left(\ln \frac{1+x}{1-x}\right)^2$  (14)  $y = \frac{e^x}{x+1}$  (15)  $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$   
 (16)  $y = 3^{\frac{x}{\ln x}}$  (17)  $y = \frac{\sin x}{1 + \cos x}$  (18)  $y = \ln(\tan x)$   
 (19)  $y = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}$  (20)  $y = \sqrt{1 + 2 \tan x}$  (21)  $y = \frac{x - \cos x}{\sin x} e^x$   
 (22)  $y = \frac{\arccos x}{\sqrt{1-x^2}}$  (23)  $y = \arctan x - \frac{x}{1+x^2}$  (24)  $y = \sqrt{x} \arctan x$

80. Estudie y represente gráficamente las siguientes funciones:

- (1)  $f(x) = \frac{|x|-x}{2}$  (2)  $f(x) = \frac{x^2+|x|}{2}$  (3)  $f(x) = \frac{x^2-1}{x-2}$   
 (4)  $f(x) = \frac{x^2-x}{x-1}$  (5)  $f(x) = x + \sqrt{x^2 - 1}$  (6)  $f(x) = xe^{\frac{1}{x}}$   
 (7)  $f(x) = \ln(x^2 - 4)$  (8)  $f(x) = \frac{(x-1)^2}{(x+1)^2}$  (9)  $f(x) = |x^2 - 4x + 3|$   
 (10)  $f(x) = \frac{e^x}{x^2}$  (11)  $f(x) = \frac{x^2-3x+2}{x^2+3x+2}$  (12)  $f(x) = \frac{(x+1)(x+2)x}{(x-1)(x+3)}$   
 (13)  $f(x) = \ln(|x+1|)$  (14)  $f(x) = \sin^2 x$  (15)  $f(x) = \cos x - \frac{1}{2} \cos 2x$

81. Estudie y represente gráficamente las funciones:

$$a) f(x) = \begin{cases} e^x & \text{si } x \leq 0 \\ 1 - x^2 & \text{si } 0 < x \leq 1 \\ x & \text{si } x > 1 \end{cases}$$

$$b) f(x) = \begin{cases} \ln(-x) & \text{si } x < -2 \\ \text{sen } \pi x & \text{si } -2 \leq x \leq 2 \\ 0 & \text{si } 2 < x < 4 \\ x^2 - 12 & \text{si } x \geq 4 \end{cases}$$

82. Estudie según los valores de los parámetros, la continuidad de las funciones siguientes:

$$a) f(x) = \begin{cases} \text{sen } 3x & \text{si } x \leq \frac{\pi}{2} \\ 2k + \cos 2x & \text{si } x > \frac{\pi}{2} \end{cases}$$

$$b) f(x) = \begin{cases} \cos x & \text{si } -\pi \leq x \leq 0 \\ a + x^2 & \text{si } 0 < x < 1 \\ \frac{b}{x} & \text{si } 1 \leq x \leq \pi \end{cases}$$

### Soluciones del ejercicio 79

$$(1) y' = \frac{2}{(x+1)^2}$$

$$(2) y' = -\frac{6x^2}{(x^3+1)^2}$$

$$(3) y' = \frac{3x(x-2)}{(x^2+x-1)^2}$$

$$(4) y' = \frac{2}{x^2} - \frac{2}{x^3} - \frac{15}{x^4} + 1$$

$$(5) y' = -\frac{4(x^8+9)}{x^5}$$

$$(6) y' = -\frac{3\sqrt[3]{x^2} - 4\sqrt{x} - 1}{6\sqrt[5]{x^6}(\sqrt[3]{x^2}+1)^2}$$

$$(7) y' = \frac{2(x-1)}{x(x-2)}$$

$$(8) y' = \frac{6(x+1)}{x(2x+3)\ln 2}$$

$$(9) y' = \frac{1}{2x\sqrt{\ln x}}$$

$$(10) y' = x^{n-1}(1+n\ln x)$$

$$(11) y' = \frac{1-n\ln x}{x^{n+1}}$$

$$(12) y' = x^{n-1} \left( \frac{n}{\ln x} - \frac{1}{(\ln x)^2} \right)$$

$$(13) y' = \frac{4\ln\left(\frac{x+1}{1-x}\right)}{1-x^2}$$

$$(14) y' = \frac{xe^x}{(x+1)^2}$$

$$(15) y' = \frac{4}{(e^x + e^{-x})^2}$$

$$(16) y' = 3^{\frac{x}{\ln x}} \ln 3 \frac{-1 + \ln x}{(\ln x)^2}$$

$$(17) y' = \frac{1}{1 + \cos x}$$

$$(18) y' = \frac{1}{\text{sen } x \cos x}$$

$$(19) y' = \frac{2 \text{sen } x}{\cos^3 x} - \frac{2 \cos x}{\text{sen}^3 x}$$

$$(20) y' = \frac{1 + \tan^2 x}{\sqrt{1 + 2 \tan x}}$$

$$(21) y' = \left( \frac{\text{sen } x - x \cos x + 1}{\text{sen}^2 x} + \frac{x - \cos x}{\text{sen } x} \right) e^x$$

$$(22) y' = \frac{-\sqrt{1-x^2} - x \arccos x}{\sqrt{(1-x^2)^3}}$$

$$(23) y' = \frac{2x^2}{(1+x^2)^2}$$

$$(24) y' = \frac{\arctan x}{2\sqrt{x}} + \frac{\sqrt{x}}{1+x^2}$$