

Spatial disorder dependence of the conductance of a random resistor network

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We calculate the resistance of one-, two-, and three-dimensional localized systems in the hopping regime, simulated by random-resistor networks within the r -resistor model. The analysis of the logarithm of the resistance as a function of the degree of spatial disorder reveals that in one- and two-dimensional samples the resistance increases with disorder, while in three-dimensional samples the resistance decreases. We also study the fluctuations of the conductance from sample to sample.

I. INTRODUCTION

At low temperatures, hopping is the conduction mechanism of disordered systems with localized states. In hopping transport electrons conduct by thermally effected tunneling from a filled site to a vacant site.¹ Miller and Abrahams² calculated the transition rates between localized states and showed that the problem of hopping conduction is equivalent to a random resistor network with an effective resistance between each pair of localized states.

An important feature of the effective resistances between sites, which will be given explicitly in the next section, is their exponential dependence on the distance between the sites involved and on their energies. Thus, even for a moderate range of variation of these variables the range of variation of the effective resistances is enormous. This property is used to make a substantial simplification in the complexity of the network through the use of percolation.³ Within this approximation, the values of the activation energies and the frequency and temperature dependences of the conductivity are obtained to a relatively large degree of precision.

The distribution of site energies depends on the density of states of the material considered. The two most important, and simplest, cases consist of a constant density of states and a sharp peak in the density of states removed from the Fermi level. We will concentrate on this second case, which is called the r resistor, since the spatial dependence of the resistances is the key ingredient of the problem. The energy dependence just produced an activated behavior of the conductivity with the activation energy equal to the difference between the Fermi level and the peak in the density of states.

Our main goal is to study the changes in the resistance with the degree of spatial disorder of the sites involved in the conduction process. A second aim is to analyze the magnitude of the conductance fluctuations of mesoscopic samples, also in terms of the spatial disorder. Although we work within the r -resistor model, instead of using a percolation treatment,⁴ we numerically calculate the random resistor problem by solving Kirchhoff's laws directly, which allows us to achieve a better precision in mesoscopic samples.

We assume that the only relevant modification induced by the disorder is a change in the site positions, with no variation in the wave function extension. As the wave functions are supposed to be strongly localized, the model should be specially suitable for high levels of disorder,⁵ but not for the ordered cases, which should present extended wave functions and for which hopping will not be the conduction mechanism.⁶

In the next section we detail the model used. In Sec. III, we show the results for the behavior of the logarithm of the resistance as a function of disorder, for one-, two-, and three-dimensional systems. Finally, the conclusions are discussed in Sec. IV.

II. MODEL AND NUMERICAL PROCEDURE

We assume a noninteracting and strongly localized system, each of whose states can be associated with a given site. In the Miller-Abrahams random resistor model,² the effective resistance between sites i and j is equal to¹

$$R_{ij} = R_{ij}^0 \exp \left\{ \frac{2r_{ij}}{a} \right\} \exp \left\{ \frac{\epsilon_{ij}}{kT} \right\}, \quad (1)$$

where R_{ij}^0 is a typical resistance weakly dependent on the temperature and on the other parameters of the problem, a is the localization radius, r_{ij} the distance between sites i and j , and ϵ_{ij} is given by

$$\epsilon_{ij} = \frac{1}{2} \{ |\epsilon_i - \epsilon_j| + |\epsilon_i - \mu| + |\epsilon_j - \mu| \}, \quad (2)$$

where ϵ_i and ϵ_j are the energies of states i and j , and μ is the chemical potential.

We consider a density of states with a sharp peak away from the Fermi level. In this case, the spatial and the energy contributions to the resistance of the sample decouple, and the energy problem can be solved trivially; the system presents an Arrhenius-type behavior with an activation energy equal to the difference between the Fermi energy and the energy of the peak in the density of states.³ The main problem is to find the total resistance of a sample formed by a set of resistors with resistances

depending exponentially on the distance between sites.

Let us consider a resistor network whose nodes are occupying the sites of a lattice, with unit lattice constant. The one-, two-, and three-dimensional systems studied are, respectively, a linear chain, a square lattice, and a cubic lattice, before disorder is introduced. We characterize each sample by the length L of each edge.

Disorder is introduced in the lattice by randomly displacing each site to a new coordinate according to a Gaussian distribution function, but without allowing it to enter in the Wigner cell of a neighboring site. Once we have displaced all the sites of a sample, we characterize its degree of disorder by the parameter

$$s = \frac{1}{L^{2d}} \sum_{i,j}^L \exp[i2\pi(x_i - x_j)], \quad (3)$$

where d is the dimensionality of the system, and x_i the x coordinate of the i site. The parameter s varies from $s = 1$, for the perfect lattice, to $s = 0$, for the case of complete disorder.

We choose the resistance between two nearest neighbors in the ordered case as our unit of resistance. This choice will be relevant when considering the fluctuations of the logarithm of the resistance. The natural scale for resistances is the quantum of resistance h/e^2 ,⁷ but typical values of the effective resistances in hopping conduction are not far from this quantum, which induced us into our choice of the unit of resistance.

According to the previous discussion, the effective resistance between sites i and j is given by

$$R_{ij} = \frac{\exp(2r_{ij}/a)}{\exp(2/a)}, \quad (4)$$

where the distances are measured in units of the lattice spacing. We consider resistances R_{ij} connecting only first and second neighbors i and j .

In order to obtain the conductance of the network, we first apply a voltage difference ΔV to the lattice by setting the voltage equal to zero at each site on one end surface of the lattice and assigning a constant voltage to all the sites on the opposite end. Then, for each node i (except for those for which the voltage is imposed externally) we have to obtain the voltage V_i by solving Kirchhoff's current law⁸

$$\sum_{j \neq i} \frac{V_i - V_j}{R_{ij}} = 0. \quad (5)$$

This equation is solved iteratively until a good convergence for the values of the voltages V_i at each site is finally obtained. We use periodic boundary conditions in the perpendicular directions to that of the applied voltage.

By applying Ohm's law, we obtain the current I_i through each of the resistors connected to the sites i belonging to an end face of the lattice. The total resistance R of the system is the applied voltage ΔV divided by the sum of all these currents I_i .

III. CONDUCTANCE AS A FUNCTION OF DISORDER

We divide the range of the order parameter s in 20 bins, and we average the logarithm of the resistance over those realizations whose s falls in each bin. We know that the logarithm of the resistance is a self-averaging magnitude, as we will discuss later on. In the set of figures 1, the solid line represents $\langle \log_{10} R \rangle$ and the dashed lines correspond to $\langle \log_{10} R \rangle$ plus or minus the statistical error, calculated as $\sqrt{(\langle (\log_{10} R)^2 \rangle - \langle \log_{10} R \rangle^2)/N}$, where N is the number of realizations in the bin considered.

In Fig. 1(a) we show the histogram corresponding to the logarithm of the resistance as a function of the degree of order s for a total of 2000 realizations of a one-dimensional (1D) system with $L = 300$. We can note the drastic increase of the resistance for high degrees of spatial disorder (low values of s). The resistance for the completely disordered case is about two orders of magnitude greater than the resistance of the ordered lattice. This drastic increase was to be expected since the disorder implies large gaps between sites, and so the presence of large resistances in the chain, which dominate the total resistance of the sample.

In Fig. 1(b) we present the histogram of the logarithm of the resistance as a function of the degree of order s for 600 realizations of a two-dimensional (2D) system with $L = 20$. The total resistance also increases with disorder, but in a much smoother way than in the one-dimensional system. The smaller increase is due to the fact that the large resistances can be partially avoided by an alternative route through smaller resistances.

Figure 1(c) represents the histogram of the logarithm of the resistance of 300 three-dimensional (3D) samples with $L = 6$. We can conclude from this figure that the resistance decreases with disorder. In all dimensionalities, the low density and high density regions created by the spatial disorder correspond to regions with large and with small resistances, respectively. But, in three-dimensional systems, the current can always find a path through the small resistance regions.

The results shown in Fig. 1 are very similar to those obtained for other sample sizes ($L = 100$ and 200 in 1D, for $L = 15$ in 2D, and for $L = 7$ in 3D). This indicates that we are considering samples with a size adequate for the kind of process that we are studying. The only major difference with size is for the resistance fluctuations in one-dimensional systems, as we will see later on.

In Fig. 2, we plot the average value of the resistance (dashed line), the inverse of the average value of the conductance (short dashed line), and the exponential of the average value of the logarithm of the resistance (solid line) as a function of s . In the absence of dispersion in the data, these three curves should coincide, but it is clear from the results that they do not superpose, due to the importance of the fluctuations. The resistance and the conductance are not self-averaging quantities, unlike the logarithm of the resistance.⁹ This last quantity is the one that has to be considered whenever the fluctuations are important.

Let us now consider in more detail the magnitude of

the fluctuations. First of all, we have to mention that the fluctuations are very similar for all the sizes studied and only depend on the dimensionality and on the degree of disorder. Thus, even for very large systems, the fluctuations in the conductance are comparable with

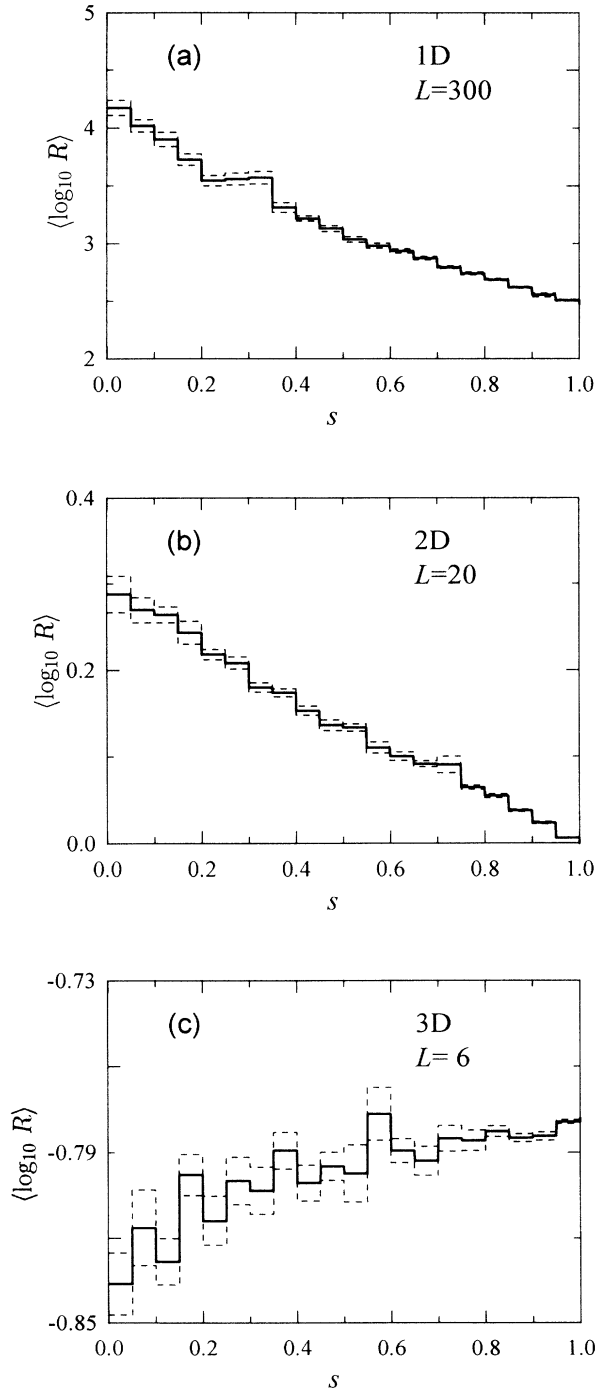


FIG. 1. Average of the logarithm of the resistance (solid line) as a function of the order parameter s ; the dashed lines show the boundaries of the statistical error, evaluated as indicated in the main text. (a) 1D sample with $L = 300$, (b) 2D sample with $L = 20$, (c) 3D sample with $L = 6$.

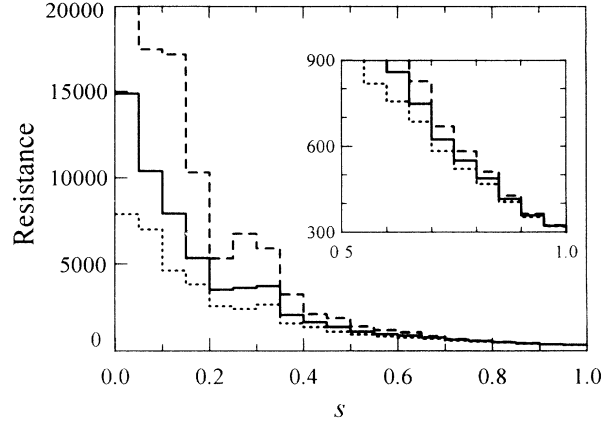


FIG. 2. Average resistance (dashed line) $\exp[\langle \log_{10} R \rangle]$ (solid line) and inverse of the average conductance (short dashed line) for a one-dimensional system with $L = 300$. A magnification of this picture for values of $s > 0.5$ is shown in the inset.

the average values of the conductance. The same is true for the resistance. The magnitude that goes to zero as the system size increases is the relative fluctuation of the logarithm of the resistance, $\delta(\log_{10} R)/\langle \log_{10} R \rangle$.

The only self-averaging magnitude is $\log_{10} R$, and is what one should study experimentally. Even for the smaller samples considered, $\delta(\log_{10} R)/\langle \log_{10} R \rangle$ is clearly smaller than $\delta R/\langle R \rangle$ and than $\delta G/\langle G \rangle$, where G stands for conductance. They would only be comparable if the individual pair resistances were sensibly smaller than the typical resistance h/e^2 . In one-dimensional systems with small disorder in the coherent regime and at zero temperature, it is known¹⁰ that $\ln\langle R \rangle = 2\langle \ln R \rangle$. Our problem corresponds to the noncoherent case, since it is thermally activated hopping, and we expect the fluctuations to be less important. We cannot compare our results with the predictions of the coherent case directly, unless we specify the activation energy (E_a) and the temperature (T), since we have to know the resistance in units of e^2/h to compare $\langle \ln R \rangle$, $\langle \ln G \rangle$, and $\ln\langle R \rangle$. We estimate that the exponent in the energy factor $e^{-E_a/kT}$ has to be approximately equal to -7 for our fluctuations to be of the same order of magnitude as in the coherent case.

IV. DISCUSSION AND CONCLUSIONS

The main conclusion of our work is that the resistances of one-dimensional and two-dimensional samples increase with the spatial disorder, while the resistance of three-dimensional samples decreases. The spatial disorder always produces regions with large resistances and regions with small resistances, and the behavior of the conductance depends on whether the current can find a conduction path through the region of small resistances. The answer is positive in three-dimensional systems, marginally negative in two-dimensional systems, and clearly nega-

tive in one-dimensional systems.

In disordered systems with a fairly constant density of states around the Fermi level, the site energy is a new random variable and the effective dimensionality of the system is four. So we expect in these cases that the conductivity drastically increases with the spatial disorder.

Our results show the importance of the reproducible, but not universal, conductance fluctuations in the localized regime. Nevertheless, it is not possible a quantitative study of these fluctuations, due to the algorithm used, primarily designed to analyze the change of the conductance with the degree of disorder. To achieve this with high enough precision, we have to consider the same number of resistances between pairs, which fixes the topology of the sample in detriment of the large scale fluctuations

in the density of sites and so in the distribution of resistances, mainly in the one-dimensional case.

Apart from its intrinsic theoretical interest, the results may be applicable to ion bombardment experiments, where important conductivity changes have been observed as a result of a possible disorder induced by the bombardment.¹¹

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¹ B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, Heidelberg, 1984).

² A. Miller and E. Abrahams, *Phys. Rev.* **120**, 745 (1960).

³ M. Pollak, in *The Metal Non-Metal Transition in Disordered Systems*, edited by L. Friedman and D. P. Tunstall (SUSSP, Edinburgh, 1978).

⁴ S. Tyč and B. I. Halperin, *Phys. Rev. B* **39**, 877 (1989).

⁵ M. Pollak and M. Ortuño, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 287.

⁶ N. F. Mott and E. A. Davis, *Electronic Properties of Non-Crystalline Materials* (Oxford University Press, Oxford,

1979).

⁷ *The Quantum Hall Effect*, 2nd ed., edited by E. Prange and M. Girvin (Springer, Heidelberg, 1990).

⁸ S. Kirkpatrick, *Rev. Mod. Phys.* **45**, 574 (1973).

⁹ B. Kramer, A. Kawabata, and M. Schreiber, *Philos. Mag. B* **65**, 595 (1992).

¹⁰ B. Kramer and A. MacKinnon, *Rep. Prog. Phys.* **56**, 1469 (1993).

¹¹ J. M. Constantini, J. L. Flament, V. Mori, L. Sinopoli, J. Trochon, J. L. Uzureau, L. Zuppiroli, L. Forro, J. Ardonceau, and D. Lesueur, *Radiat. Eff. Defects Solids* **115**, 83 (1990).